Shear wave velocity dependence on fluid saturation

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ABSTRACT
Sets of aligned open fractures in a background porous rock may result in an effective medium that is anisotropic with respect to wave propagation. Using Brown and Korringa's fluid substitution theory we show that the velocities of vertically propagating shear waves in a fractured, anisotropic rock may be dependent on fluid saturation. The shear-wave sensitivity to fluids originates from the effective symmetry of the fractured rock but it is influenced by the “isotropic” porosity, the mineralogy of the rock and the bulk modulus of the saturating fluid. Transversely isotropic (TI) and orthorhombic symmetry rocks will only show changes with saturation of vertical shear waves when the symmetry planes of the rock are tilted with respect to the vertical direction. However, monoclinic symmetry rocks in general will present fluid sensitive shear waves.

We concentrate on monoclinic symmetry rocks and quantify the changes in the rock compliances with saturation for the cases of: (i) one set of vertical, micro-corrugated fractures in an isotropic background (ii) one set of dipping, rotationally invariant fractures in a transversely isotropic background with a vertical symmetry axis, and (iii) two vertical sets of non-orthogonal penny-shaped cracks in an isotropic background. The results for the cases of micro-corrugated fractures and dipping, rotationally invariant fractures indicate that the shear-wave splitting parameter increases with the compressibility of the saturating fluid, which is consistent with recent observations that have been reported from multicomponent seismic experiments.

In the case of two vertical sets of non-orthogonal, penny-shaped cracks, the shear-wave splitting parameter is not sensitive to fluid saturation. However, the azimuthal variations of the normal-moveout (NMO) velocities of the $P_\perp$, $S_1$- and $S_2$-wave modes are sensitive to fluid saturation with the $S_1$ mode being the most sensitive.

Key words: shear waves, anisotropy, rock physics, monitoring, fractures

Introduction
Characterization of fractured reservoirs is an important area of research due to its economical impact. A large portion of the oil and natural gas in the world is trapped in tight reservoirs that require large permeabilities provided by natural fractures to produce reservoir fluids economically (Nelson, 1985). If the fractures have preferential orientations, the reservoir becomes anisotropic with respect to wave propagation, and by characterizing this anisotropy we can obtain information about the properties of the fracture network.

The migration of fluids throughout the fractured reservoir is an information that can be obtained from time-lapse seismic measurements. Given the range of frequencies common in surface seismic data we are specially interested in theories that can describe the influence of the fluid compressibility on the compliance of the rock when subject to varying stresses in the 10-80Hz range.

Gasman's (1951) pioneering work allows us to obtain the bulk and shear moduli of the fluid saturated rock from the (assumed known) dry rock compliance, rock porosity, mineral and fluid compressibility. This can be done under the assumptions that the rock is (1) isotropic, (2) monomineralic, and (3) that the pressure is equilibrated throughout the pore space. This theory predicts that the rock's bulk modulus ($K$) changes with sat-
uation but the shear modulus (µ) remains fixed. Hence, arises the commonly accepted notion that P-waves are sensitive to pore fluids while, except for density effects, S-waves are not.

However, interpretation of recent multicomponent data acquired over fractured reservoirs at Natih field, Oman, and Vacuum field, United States, have suggested that the slow shear-velocity decreases in the presence of high compressibility fluids (Guest et al., 1998), (Duranti et al., 1999). The apparent contradiction of these results with the generally accepted view that shear waves are insensitive to fluids relies in that Gassmann’s isotropic rock assumption is violated when aligned sets of fractures make the rock anisotropic.

Through a heuristic explanation, Bakulin et al. (2000) suggested that changes with saturation in the shear modulus of vertically propagating shear waves are possible in a rock with monoclinic symmetry, generated by one set of micro-corrugated fractures in an isotropic background. This model has a vertical symmetry plane perpendicular to the fractures. In such fracture system, the normal displacements, which are sensitive to the fluid content, are coupled to the the tangential displacements along the fracture faces which determine shear-wave splitting.

Using Brown and Kinning’s (1975) anisotropic generalization of Gassman’s equations we show that under conditions of equilibrated pore pressures, fractured rocks with symmetry lower or equal to monoclinic have “shear moduli” that are sensitive to pore fluids. Therefore, the model of micro-corrugated fractures proposed by Bakulin et al. (2000) belongs to a larger group of rocks for which Brown and Kinning’s theory predicts changes of shear-wave velocities with saturation.

The analysis is done for another monoclinic rock with a vertical symmetry plane proposed by Grechka and Tsvankin (2000), in which one set of dipping rotationally invariant fractures is embedded in a transversely isotropic (VTI) background. In this case, we find similar results to those for the micro-corrugated fractures where the shear-wave splitting parameter increases with increasing compressibility of the saturating fluid.

Finally, we study the case of a rock generated by two sets of vertical, non-orthogonal, penny-shaped cracks in an isotropic background (Bakulin et al., 2000). In contrast to the previous models this modeled rock is monoclinic with a horizontal symmetry plane, in which the velocities of vertically propagating shear waves do not change with saturation. However, the azimuthal variation of the normal-moveout (NMO) velocities of the P, S1 and S2 modes is sensitive to pore fluids with the S1-wave being the most sensitive.

Fluid Substitution Theory

Brown and Kinning (B&K) generalized Gassmann’s work by relaxing the conditions of isotropy and monomineralic rock. In this way, Gassmann’s scalar equations that relate the bulk and shear moduli of the dry and saturated cases, are replaced by a tensor equation that relates the compliances of the dry and saturated rock. Since the complications related to multimineralic rocks are beyond the scope of this work, we will only deal with the B&K formulation for the monomineralic case.

Brown and Kinning’s equation can be written as:

\[ S_{ijkl}^{dry} - S_{ijkl}^{sat} = \frac{(S_{ijaa}^{dry} - S_{ijaa}^{sat})(S_{klaa}^{dry} - S_{klaa}^{sat})}{(S_{aαββ}^{dry} - S_{aαββ}^{sat}) + (C_f - S_{aαββ}^{sat})}\phi, \]

where \( S_{ijkl}^{dry} \) and \( S_{ijkl}^{sat} \) are the compliances of the dry and saturated rock, \( S_{ijkl}^{sat} \) is the compliance of the mineral material, \( C_f \) is the compressibility of the fluid, and \( \phi \) is the total connected porosity. The quantities \( S_{ijaa}^{dry} \) and \( S_{aαββ}^{sat} \) in the denominator are the compliances of the dry rock and mineral material, respectively. Due to the double index contractions both quantities are scalars and they can be defined as \( S_{ijaa}^{dry} \equiv C_{dry} \) and \( S_{aαββ}^{sat} \equiv C_0 \).

Equation (1) assumes the rock is subjected to slowly varying stresses such that the pressure is equilibrated throughout the pore space. This condition can be satisfied when the fluid substitution occurs in a well connected pore space. However, the effect of fluids in disconnected cavities could be included in equation (1) as part of the “solid” through effective mineral (\( C_0 \)) and dry (\( C_{dry} \)) compressibilities.

We believe that greater intuition into the meaning of Brown and Kinning’s equation can be achieved by rewriting equation (1) in the conventional (2-subscript) 6x6 matrix notation. By doing this, it is clear that the terms with contracted indices \( (S_{ijaa}) \) in the numerator of equation (1) represent the sums of elements of the first three rows of the 6x6 compliance matrix. Therefore, by defining \( V_f \) as the sum of the first three elements of the \( K-th \) column of the compliance matrix, as shown in Figure 1a, B&K’s equation can be rewritten as:

\[ S_{ijkl}^{dry} - S_{ijkl}^{sat} = \frac{(V_f^{dry} - V_f^{sat})(V_f^{dry} - V_f^{sat})}{(C_f - C_0) + (C_f - C_0)\phi}. \]

Although equations (1) and (2) are equivalent, now it is easier to see when changes in vertical shear-wave velocities may be expected with saturation based on the symmetry of the rock. Figure 1a shows a compliance matrix of general symmetry subdivided into four blocks, two diagonal and two off-diagonal. For symmetries in which the off-diagonal blocks of the compliance matrix are zero, vertical shear-wave velocities will depend only on the \( S_{44} \).
and $S_{55}$ compliances. Since in these case the sums $V_4$ and $V_5$ are zero (see Figure 1b), from equation (2) we see that $S_{kk}^{yy} = S_{kk}^{xx}$ for $k = 4, 5$. This means that for these symmetries the vertically propagating shear waves do not depend on fluid saturation.

For isotropic rocks, the off-diagonal blocks of the compliance matrix are zero and $1/\mu = S_{44} = S_{55}$, where $\mu$ is the shear modulus. Therefore, from the above explanation, Gasman's result that states that the shear modulus does not depend on fluid saturation is obtained immediately. In the case of transverse isotropy and orthorhombic symmetry rocks whose symmetry planes are not tilted with respect to the vertical axis, the off-diagonal compliance blocks are also zero and the vertical shear waves are not sensitive to saturation (see Figure 1b). This result has been described in a less explicit form by Brown and Karrina (1975).

If an anisotropic rock has a symmetry lower than orthorhombic (e.g. monoclinic) in which the sums $V_4$ and/or $V_5$ are non-zero, the vertically propagating shear waves may be sensitive to the pore fluids. This will occur because either $S_{44}$ or $S_{55}$ will be fluid sensitive and also because the S-wave velocities will depend on other compliances besides $S_{44}$ and $S_{55}$.

The arguments above have been stated under the assumption that the symmetry planes of the anisotropic rock are not tilted with respect to the vertical propagation axis. If we allow tilt, even a transversely isotropic rock may present fluid sensitive, vertical shear waves. This can be understood considering the compliance matrix of a horizontal transversely isotropic rock (HTI), with symmetry axis in the $X_1$ direction, that has been rotated by an angle $\theta$ around the $X_2$ axis according to the so called Bond transformation. The rotated compliance is calculated as $S_{rot} = N(\theta)S^N N^T(\theta)$, where $S^N$ is the HTI compliance and $N(\theta)$ is the rotation matrix ($N(\theta)$ can be found explicitly in Mayko et al. (1999)).

After rotation, the tilted HTI compliance has the following non-zero terms,

$$ S_{rot} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & 0 & S_{15} & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & S_{35} & 0 \\ 0 & 0 & 0 & S_{44} & S_{46} & S_{56} \\ S_{15} & 0 & S_{35} & 0 & S_{55} & 0 \\ 0 & 0 & 0 & S_{46} & S_{66} & \end{pmatrix} \quad (3) $$

By the arguments stated previously it is clear that the $S_{55}$ component and one of the vertically propagating shear-wave velocities will change with saturation. This observation indicates that vertical shear waves that are sensitive to pore fluids may be a common phenomenon. The fact that it has seldom been reported from surface seismic experiments is probably due to the small number of multicomponent, time-lapse experiments that have been acquired.

In the next sections we will concentrate on fractured rocks with monoclinic symmetry which will show changes in shear-wave velocities with saturation even when the symmetry planes are not tilted with respect to the vertical propagation direction.

### Methodology

The results presented in the following sections compare the rock compliances and vertical shear-wave velocities of the dry and saturated rock. First we model the compliance and compressibility of the mineral material and the dry rock. Then we use equation (2) to cal-

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**Figure 1.** Illustration of how the symmetry of the rock determines changes in the shear compliances with saturation. (a) Compliance matrix of general symmetry indicating elements that contribute to the sums $V_k$ in equation (2). (b) The change with saturation of the components $S_{44}$, $S_{55}$ and $S_{66}$ depends on the sum of the elements in columns $V_4$, $V_5$ and $V_6$, which vanish for isotropic, VTI, HTI and non-tilted orthorhombic media. In monoclinic rocks the sums $V_4$, $V_5$ and $V_6$ may be non-zero resulting in fluid dependent shear compliances.
culate the compliance of the 100% brine saturated rock and compare the results for both saturations. It is assumed that for all the models the compliance matrix of the dry, fractured rock can be calculated using Schoenberg’s linear slip theory (Schoenberg and Muir, 1989), (Schoenberg and Sayers, 1995), as the sum of a background compliance and an excess fracture system compliance matrix:

\[ S^{dry} = S^{dry}_{back} + S^{dry}_{frac}. \]  

(4)

If the background is isotropic, \( S^{dry}_{back} \) is calculated from empirical relations of \( V^p_{dry}(\phi_p) \), \( V^S_{dry}(\phi_p) \) and density \( \rho^{dry}(\phi_p) \), all functions of the “isotropic” porosity \( \phi_p \). The total porosity of the rock is \( \phi_t = \phi_p + \phi_c \), where \( \phi_c \) is the crack (fracture) porosity that only influences the fracture system compliance \( S^{dry}_{frac} \).

In all the models the mineral material of the background rock is isotropic. The calculations are done for quartz, calcite, and dolomite. The values of the bulk (\( K \)) and shear moduli (\( \mu \)) are taken from Mavko et al. (1999) (see table 1).

<table>
<thead>
<tr>
<th>Mineral material</th>
<th>( K ) (GPa)</th>
<th>( \mu ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>36.5</td>
<td>45.6</td>
</tr>
<tr>
<td>Calcite</td>
<td>70.2</td>
<td>29.0</td>
</tr>
<tr>
<td>Dolomite</td>
<td>94.9</td>
<td>45.0</td>
</tr>
</tbody>
</table>

The calculations have been done for a fluid substitution that occurs throughout the entire pore space including all the “isotropic” and fracture porosity. Since the permeability of the isotropic background is typically smaller than the permeability of the fractures, it is assumed that sufficient time is allowed to have a complete fluid substitution in the background matrix.

One Set of Micro-corrugated Fractures in an Isotropic Background

Using Schoenberg’s linear slip theory, Bakulin et al. (2000) have studied the anisotropy produced by one system of micro-corrugated fractures embedded in an isotropic background, with the normals in the \( X_1 \) direction. By micro-corrugated it is meant that the fracture surfaces are irregular at a scale much smaller than a seismic wavelength. The irregularity can be idealized as sawtooth profiles that are offset from one fracture surface to the other as shown in Figure 2. In this idealization the “roughness” of the fracture surface in the \( X_3 \) direction causes a coupling between normal and tangential displacements when the fracture is stressed.

The symmetry of this effective medium is monoclinic with a vertical symmetry plane, and its compliance matrix can be written as the sum of the isotropic compliance of the background plus the fracture-system compliance,

\[ S_{rock} = S_{iso} + \begin{pmatrix} Z_N & 0 & 0 & Z_{NV} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ Z_{NV} & 0 & 0 & Z_V & 0 \\ 0 & 0 & 0 & 0 & Z_H \end{pmatrix}. \]  

(5)

Here \( Z_N \) is the normal compliance of the fractures that relates the normal displacements to the normal stresses applied to the fracture in the \( X_1 \) direction. The \( Z_V \) and \( Z_H \) elements are the tangential compliances that relate tangential displacements and stresses in the \( X_2 \) and \( X_3 \) directions, respectively. The \( Z_{NV} \) compliance is responsible for the coupling of normal displacements to tangential applied stresses, or conversely, the coupling of tangential displacements to applied normal stresses (see Figure 2).

From equation (2) and (5) it is clear that the \( S_{44} \) and \( S_{66} \) shear compliances of the fractured rock will not change with saturation because the sums \( V_4 \) and \( V_6 \) of \( S_{rock} \) in equation (2) are zero. However, \( S_{55} \) will change with saturation because \( V_5 = Z_{NV} \) and the right side of equation (2) will be non-zero. From this analysis we see that, under the assumption that the background rock is isotropic, Brown and Koning’s equation requires that the term \( Z_{NV} \) of the dry rock be non-zero to have changes in the \( S_{55} \) shear compliance. This result stands in contrast with the heuristic proposal by Bakulin et al.
(2000) in which they suggest that the reason why shear-wave velocities change with saturation is that the term $Z_{NV}$ is zero for dry rocks and non-zero for saturated rocks.

To obtain more information on how the compliance terms $Z_{NV}$ and $Z_V$ change with saturation, I have explicitly derived relations between the compliances of the dry and saturated rocks using equations (2) and (5) (See Appendix A). The difference between the dry and saturated values of the compliances is a function of the mineral and fluid compressibility, the total rock porosity and the values of the dry-rock normal and coupling compliances ($Z_{NV}^{dry}$ and $Z_{NV}^{sat}$).

Even though there is no micro-structural theory from which the values of the fracture compliances can be estimated for this model, bounds can be obtained from the constraints imposed by the fracture system compliance. Since the stability condition requires that the fracture system compliance matrix be positive definite, the compliances must satisfy $Z_N Z_V \geq Z_{NV}^2$. Furthermore, numerical calculations done by Nakagawa et al. (1999) on a single micro-corrugated crack suggest that $Z_{NV}^{dry} \approx Z_{NV}^{sat}$ and in most cases $Z_{NV}^{dry} \geq Z_{NV}^{sat}$.

The changes in the compliances with fluid saturation show that for this fracture model there is a weak dependence on the compressibility of the mineral material. This can be understood in light of equations (A6) and (A7) where we see that if the compressibility of the fluid is much larger than that of the mineral the contribution of the mineral compressibility is small.

Figure 3 shows the difference between the dry and saturated values of $Z_{NV}$ and $Z_V$ for a porous, fractured limestone (calcite matrix) that is 100% saturated with brine ($C_f = 0.44 \ GPa^{-1}$). These curves are calculated using the equations derived in Appendix A assuming that the background is an isotropic, porous limestone matrix that can be modeled as dilute collection of spherical pores with a porosity $\phi_p \leq 0.1$ (Mavko et al., 1999).

From Figure 3 it is clear that the change in the compliances is strongly dependent on the value of $Z_{NV}^{dry}$. As mentioned before, if $Z_{NV}^{dry} = 0$ the symmetry of the rock is no longer monoclinic and the compliances are not sensitive to the change of saturation.

An interesting observation is that if the coupling term is independent of the “isotropic” porosity ($\phi_p$), larger changes should be expected with saturation for smaller values of $\phi_p$. However, from Hudson’s theory of penny-shaped cracks we know that in transversely isotropic media the normal and tangential compliances of the fracture system decrease as the background rock becomes stiffer or its porosity becomes smaller (Schoenberg and Douma, 1988). If we assume that this type of dependence on porosity applies to the coupling term $Z_{NV}$ for the monoclinic rock, we cannot conclude that the changes with saturation will be as large as shown for small values of $\phi_p$. Since there is no inclusion theory for micro-corrugated cracks (the equivalent of Hudson’s theory for penny-shaped cracks), it is unclear whether changes with saturation should be larger for smaller background porosities in these type of fractures.

Bakulin et al. (2000) have calculated the shear-wave splitting parameter for this monoclinic rock in terms of the vertical velocities $V_{S1}$ and $V_{S2}$ of split shear waves as:

![Figure 3. Difference between the dry and saturated values of (a) the fracture coupling compliance, $Z_{NV}$, and (b) the fracture tangential compliance, $Z_V$, as a function of $Z_{NV}^{dry}$. The curves are calculated using the equations derived in Appendix A for a fractured, porous limestone with values $Z_{NV}^{dry} = 0.02 \ GPa^{-1}$ and $C_f = 0.44 \ GPa^{-1}$. The unit of $Z_{NV}$ in the X-axis is $GPa^{-1}$.](image)
\[ \gamma \equiv \frac{V_{S1}^2 - V_{S2}^2}{2V_{S2}^2}, \tag{6} \]

where \( V_{S1} \) and \( V_{S2} \) are given by
\[ V_{S1}^2 = \frac{\mu}{\rho}, \tag{7} \]
and
\[ V_{S2}^2 = \frac{\mu}{\rho} \left( \frac{c_{33}c_{55} - c_{35}^2}{c_{33} + c_{55} + \sqrt{(c_{33} - c_{55})^2 + 4c_{35}^2}} \right) \tag{8} \]
and \( \mu \) is the shear modulus of the background isotropic rock.

Using equation (2), we calculate the compliance matrix of the brine saturated rock. We then invert it to obtain the stiffness matrix and use equations (6) - (8) to compare the shear-wave splitting parameter of the dry and saturated rock. From equations (7) and (8) it is clear that the velocity of the \( S_1 \)-wave will not change with saturation since it depends only on the shear modulus of the isotropic background rock. However, the velocity of the \( S_2 \)-wave will change due to its dependence on the \( c_{33}, c_{55} \) stiffness components.

Figure 4 shows the dry-rock splitting parameter and the absolute and relative changes after saturation as a function of the compliance \( Z_{NV}^{dry} \). The calculations show that the velocity of the \( S_2 \)-wave is larger for the brine saturated rock than for the dry rock. Therefore, since the \( S_1 \) velocity is the same for the dry and saturated case, the shear-wave splitting parameter is smaller for the brine saturated rock and larger for the dry rock (Figure 4b).

The combination of dry fracture compliances that seems more reasonable according to the range of values reported by Nakagawa et al. (1999), is the one represented by the diamonds \( (Z_{NV}^{dry} \leq Z_{NV}^{dry}, Z_{NV}^{dry} \approx Z_{NV}^{dry}) \). Taking this as a representative example of what can be expected from experimental data, we see that the relative changes in \( \gamma \) can be as large as 10% of the dry value (Figure 4c). However, the larger percent changes are due in part to the simultaneous decrease of \( \gamma^{dry} \) with increasing values of \( (\gamma^{dry} - \gamma^{sat}) \).

We have done these calculations for an extreme case of fluid substitution from a dry rock to a 100% brine saturated rock. The dry case, for all practical purposes, models the rock saturated with air at ambient conditions \( (C_f \rightarrow \infty) \), and brine is probably the stiffest fluid of interest found in a natural rock \( (C_f \rightarrow 0) \). In field experiments changes in the fluid compressibility will always be smaller and we should expect that the changes in shear-wave splitting should be correspondingly smaller.

From the modeling we conclude that, in a rock with micro-corrugated fractures in an isotropic background, the vertical shear-wave splitting should increase if a low-compressibility fluid is replaced with a high-

\[ 100(V_{dry} - V_{sall}^{' dry}) / V_{dry} \tag{c} \]

Figure 4. Difference between dry and saturated splitting parameter as a function of \( Z_{NV}^{dry} \) for several values of the normal compliance \( Z_N \) of the dry rock. (a) Shear-wave splitting for the dry rock. (b) Absolute change in shear-wave splitting between the dry and saturated rock. (c) Percent change with saturation for each of the points in plots (a) and (b). Calculations are done for a porous quartz matrix with a modeled "isotropic" porosity of 10% and a fixed value of \( Z_{NV}^{dry} = 0.02 \) GPa\(^{-1}\). The unit of \( Z_{NV}^{dry} \) in the X-axis is GPa\(^{-1}\).
compressibility fluid (e.g. gas at low differential pressures that displaces oil or water). This result is qualitatively consistent with the findings of Guest et al. (1998) in which the splitting parameter increased due to a decrease in the $S_v$-wave velocity in areas believed to be saturated with highly compressible gas. This observation was done from interpretation of a single multicomponent survey in which the increase in $\gamma$ was very well correlated with the known gas cap of the reservoir.

From time-lapse multicomponent data, Duranti et al. (1999) have also reported changes in $\gamma$ that have been attributed to varying pore fluid compressibility during a $CO_2$ injection process. However, they observe changes on both $S_1$- and $S_2$-waves that result in a decrease of shear-wave splitting in $CO_2$ saturated regions instead of the increase shown in Figure 4b. The fact that they measure changes with saturation in the $S_1$-wave may indicate a more complex fluid substitution process than the one we have considered with the micro-corrugated fracture model.

**One Set of Dipping, Rotationally Invariant Cracks in a VTI Background**

Grechka and Tsvankin (2001) proved that a single set of dipping, rotationally invariant fractures embedded in a VTI background results in a monoclinic rock with vertical symmetry plane. Figure 5 illustrates the model in which the rotationally invariant cracks, originally with their normals in the $X_1$ direction, are rotated by the angle $\theta$ about the $X_2$ axis. After the rotation, the crack planes no longer contain the symmetry axis of the background and the effective medium becomes monoclinic.

Schoenberg and Sayers (1995) have shown that the excess fracture compliance for a set of rotationally invariant fractures with their normals in the $X_1$ direction can be written as

$$S_f^{\chi_1} = \begin{pmatrix}
Z_N & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z_T & 0 \\
0 & 0 & 0 & 0 & Z_T \\
0 & 0 & 0 & 0 & 0
\end{pmatrix},$$  \hspace{1cm} (9)

where $Z_N$ and $Z_T$ are the normal and tangential fracture compliances, respectively. To obtain the expression for dipping fractures, matrix (9) is rotated by the angle $\theta$ around the $X_2$ axis according to the Bond transformation represented by $S_f' = N(\theta)S_f^{\chi_1}N^T(\theta)$. After the rotation of the fractures, the effective compliance of the monoclinic rock can be written as $S_{\text{rock}} = S_{VTI} + S_f'$, where $S_f'$ has the following non-zero terms:

$$S_f' = \begin{pmatrix}
S_{11} & S_{13} & 0 & S_{15} & 0 \\
0 & 0 & 0 & 0 & 0 \\
S_{13} & 0 & S_{33} & 0 & S_{35} \\
0 & 0 & 0 & 0 & 0 \\
S_{15} & 0 & S_{35} & 0 & S_{55} \\
0 & 0 & 0 & 0 & 0 \\
S_{46} & 0 & S_{66}
\end{pmatrix}. \hspace{1cm} (10)$$

Equation (10), shows that the effective compliance of the monoclinic rock has non-zero elements in the off-diagonal blocks defined in Figure 1a. Therefore, according to the arguments given above, the shear compliances will be sensitive to fluid saturation. Since this model is also monoclinic with a vertical symmetry axis We use equations (6) · (8) to calculate the shear-wave splitting for the dry and brine saturated cases.

To model the dry rock, we use the VTI parameters measured on 18% porosity Berea sandstone for the background (Sarkar and Kranz, personal communication) and values of $Z_N$ and $Z_T$ that are consistent with Hudson’s theory of penny-shaped cracks. The parameters of the background rock are: $\varepsilon = 0.07, \delta = 0.04, \gamma = 0.09, V_p = 2.3 \text{ Km/s}$ and $V_s = 1.62 \text{ Km/s}$. For the crack compliances, we assume $Z_N \approx Z_T$ with $0 \leq Z_N \leq 0.02$.

Figure 6 shows that the changes in $\gamma$ with saturation are larger with increasing dip angle of the cracks. If the dip is zero, the effective symmetry of the rock becomes orthorhombic and, since the off-diagonal block elements are zero, there is no change in $\gamma$ with saturation. Also, when $Z_N$ and $Z_T \to 0$ (limit of no cracks) the medium has the VTI symmetry of the background, and $\gamma = 0$. Note that the changes are larger when the the normal
compliance of the cracks increases, which indicates that larger S-wave sensitivity to fluids should be expected with increasing crack density.

The results indicate that for this sandstone model the change in $\gamma$ can be as large as 15% when the cracks are dipping at 45°. However, in general we should expect that these changes will depend on the strength of the anisotropy of the background rock. For this model we arrive at the same conclusion as the one obtained for the micro-corrugated fractures: the splitting parameter increases with the compressibility of the saturating fluid.

**Figure 6.** Change of the splitting parameter with saturation as a function of the dip of the fractures for different values of the normal compliance of the dry fractures. (a) $\gamma$ for dry (thin lines) and saturated (thick lines) cases. Note that for all values of the normal compliance $\gamma^{dry} > \gamma^{sat}$. (b) Percent change in $\gamma$ with saturation as a function of dip angle of cracks. $Z_N$ is expressed in GPa$^{-1}$.

**Figure 7.** Two sets of vertical fractures with the normals making the angles $\phi_1$ and $\phi_2$ with the $X_1$ direction. The angle between fracture sets is $\psi = \phi_2 - \phi_1$.

**Two Vertical Sets of Non-Orthogonal Penny-Shaped Cracks in an Isotropic Background**

The compliance matrix of a fractured rock composed of two sets of non-orthogonal, vertical fractures can be obtained in the same way as before by summing the compliances of the background and the fracture systems. Bakulin et al. (2000) show that the effective symmetry of the medium is monoclinic with a horizontal symmetry plane and depends on eight fracture parameters. These parameters are: the background velocities $V_P$ and $V_S$, the compliances $Z_N$ and $Z_T$ of each fracture set and the angle between the sets $\psi$ (Figure 7).

Gredka et al. (2000) showed that the most convenient coordinate system to study this type of media is the one defined by the polarization directions of vertically traveling $S_I$- and $S_2$-waves. In this coordinate system, the monoclinic rock is characterized by nine anisotropic parameters defined in the same way as those for orthorhombic media plus three parameters that control the rotation of the $P$, $S_I$- and $S_2$-wave NMO ellipses with respect to the coordinate axes. Therefore, a "diagnostic" feature of this type of fractured rocks is that the direction of polarization of the fast shear waves does not coincide with either axis of the NMO ellipses of $P$, $S_I$- and $S_2$-waves.

For symmetries in which the off-diagonal block of the stiffness (and compliance) matrix is zero (HTI, VTI and orthorhombic) the semi-major axis of the NMO ellipses coincides with the polarization direction of the fast...
shear waves at vertical incidence. Non-zero elements in the off-diagonal block are responsible for the three extra parameters that are related to the three rotation angles \( \beta_r, \beta_s, \) and \( \beta_3 \) between semi-major axis of the NMO ellipses and the fast shear-wave polarization vector. From the discussion above it is clear that these rotation angles, which depend on terms in the off-diagonal stiffness (and compliance) blocks, should be sensitive to pore fluids.

The model is constructed by embedding two sets of penny-shaped cracks in the isotropic background. Then the rotation angles of the ellipses are calculated as a function of the angle between sets, crack density ratio and “isotropic” porosity. The calculations are performed for porous quartz, calcite and dolomite backgrounds.

Figure 8 shows the rotation angles of the NMO ellipses as a function of the angle between the fracture sets \( (\psi) \) for different lithologies, and its change when the rock is saturated with brine. In this case the “isotropic” porosity \( (\phi_p) \) is fixed at 15% and the crack density ratio of the two sets is \( \frac{e_1}{e_2} = \frac{1}{5} \) with the dominating set’s crack density \( e_1 = 0.09 \). In Figure 8a, when the angle between fracture sets is zero, the model becomes HTI and the angles \( \beta_1 \) between the \( S_1 \) polarization direction and the semi-major axes of the NMO ellipses vanish. If the angle between sets is 90°, the model is orthorhombic and the \( \beta_1 \)'s again go to zero.

From Figure 8a it is clear that the largest change with saturation occurs for the orientation of the \( S_1 \)-wave ellipse, while \( \beta_r \) and \( \beta_3 \) remain almost unchanged. Therefore, Figure 8b shows the change in the \( S_1 \) mode alone; note that for the “softer”, more compressible quartz matrix, the changes are larger than those for the stiffer dolomite.

Figure 9 shows the results for fixed “isotropic” porosity \( (\phi_p = 0.15) \), fixed angle between crack sets \( (\psi = 55^\circ) \) and varying crack ratio \( \frac{e_1}{e_2} \). In one extreme case, \( \frac{e_1}{e_2} \to 0 \), the crack density \( e_1 \) is dominant and the medium becomes HTI with all \( \beta_1 \)'s equal to zero. In the other extreme case, \( \frac{e_1}{e_2} \to 1 \), both fracture sets have the same crack density and the medium becomes orthorhombic with all \( \beta_1 \)'s equal to zero (Figure 9a).

The maximum value of the rotation angles and their largest change with saturation seems to occur when the dominant fracture set has approximately twice the crack density of the weaker set. Again, the largest change with saturation occurs for the \( S_1 \)-wave NMO ellipse (Figure 9a) and for the more compressible mineral material (Figure 9b).

In Figure 10, the change of the \( \beta_1 \)'s with saturation is computed as a function of the “isotropic” porosity \( (\phi_p) \) of the background. The computation is done for a fixed angle between fracture sets \( (\psi = 55^\circ) \) and a fixed crack-density ratio \( \frac{e_1}{e_2} = \frac{1}{5} \) with \( e_1 = 0.09 \).

Figure 10b shows that when \( \phi_p \) decreases, the change of \( \beta_{S_1} \) with saturation decreases slightly and then increases as \( \phi_p \to 0.01 \). The increase at low values of \( \phi_p \) seems counter-intuitive since we would expect to see smaller changes as the pore space available for fluid substitution decreases. However, this effect is due to the transition from a fluid substitution process dominated by the “isotropic” porosity to a process dominated by the crack porosity.

Bakulin et al. (2000) have shown that \( \beta_r, \beta_{S_1}, \) and \( \beta_3 \) depend on the normal compliances \( (Z_N \)'s) of both sets of penny-shaped cracks. Therefore, large changes in the normal compliances should produce large changes in the \( \beta \)'s.
In Appendix B equation (2) is used to obtain an expression for the change of \( Z_N \) with saturation. With this expression we prove that as \( \phi_p \) goes to zero (leaving only the crack porosity, \( \phi_c \)), the combination of parameters governing the change in \( Z_N \) is \( \frac{\phi_c}{K_f Z_N^{2\phi_p}} \), where \( K_f \) is the bulk modulus of the fluid. If this ratio is small compared to unity the changes in \( Z_N \) are large. For reasonable values of the crack porosity, normal compliance and fluid bulk modulus this combination of parameters may be considerably smaller than one. For example, if \( K_f = 2.25 \text{ GPa} \), \( \phi_c = 10^{-3} \) and \( Z_N^{2\phi_p} = 0.02 \), then \( \frac{\phi_c}{K_f Z_N^{2\phi_p}} = 0.022 \). However, an important caveat is that this calculation applies to the changes we would expect from a dry rock to a 100% brine saturated rock. More realistic fluid substitution processes will not occur between such extremes and the changes could be smaller.

**Discussion and Conclusions**

Under conditions of equilibrated pore pressures, we have shown that VTI, HTI, and non-tiled orthorhombic symmetry rocks have shear compliances \( (S_{44}, S_{55} \text{ and } S_{66}) \) that are not dependent on fluid saturation. However, fractured rocks with monoclinic or lower symmetry have fluid-sensitive shear compliances. This conclusion is obvious when we rewrite Brown and Koning’s equation in two-index notation. Changes in the shear components only occur if off-diagonal elements of the compliance matrix in rows 4, 5 and 6 are non-zero.

We have studied three different models of fractured
rocks with monoclinic symmetry. Two of the monoclinic models have a vertical symmetry plane and they present vertical shear-wave splitting that is dependent on fluid saturation. The monoclinic model with a horizontal symmetry plane does not exhibit fluid dependent splitting but the orientation of the S-wave NMO ellipses changes with saturation.

The two models with vertical symmetry planes contained a single set of parallel micro-corrugated fractures and a set of dipping cracks in a VTI background, respectively. Both models support the conclusion that the splitting of vertically traveling S-waves increases with the compressibility of the saturating fluid. We may expect to observe these variations after gas injection into water or oil saturated sections of the reservoir, or after water encroachment due to production in a zone originally saturated with gas.

The monoclinic rock with a horizontal symmetry plane contained two sets of non-orthogonal penny-shaped cracks in an isotropic background. For this model, except for density effects, vertically traveling shear waves are not sensitive to saturation changes. However, the NMO ellipses of the three pure modes \( P, S_1 \) and \( S_2 \), in particular \( S_1 \), are sensitive to the pore-fluid content. Analysis for different lithologies indicates that larger changes in the orientation of the \( S_1 \)-wave NMO ellipse with saturation should be expected for fractured rocks with a softer background matrix (e.g., quartz instead of dolomite).

An important point is whether it is possible to measure on surface seismic data the changes with saturation in splitting parameter that have been modeled in this paper. Even though the relative changes in splitting parameter can be as large as 15% in the monoclinic models with vertical symmetry plane, the absolute changes are small. Given the quality of surface seismic data it would be challenging to measure with certainty variations from 0.100 to 0.115 in \( \gamma \). However, Guest et al. (1998) report that in the Natih field “the shear-wave splitting in gas-filled fractures is approximately 50% higher than in brine-filled fractures”, which translates in variations from 0.10 to 0.15 in \( \gamma \).

The reason for the small variations in the splitting parameter presented in this paper, is that we have considered a fluid substitution that occurs throughout the entire pore space including all the “isotropic” and fracture porosity. If the background “isotropic” porosity is small, most of the fluid substitution will occur in the fractures and the variations in splitting parameter will be larger (i.e. \( \Delta \phi_f \rightarrow 0 \) in equation (2)). In equations (A8), (A9) and (B3) we have proven that if \( \phi_f = 0 \) the magnitude of the changes in the fracture compliances is determined by \( \frac{\phi_f}{K_f Z_N^2} \). If this combination of parameters is small compared to unity larger changes with saturation can be expected in the fracture compliances and in the shear-splitting parameter. Using reasonable values of the crack porosity, normal compliance and the bulk modulus of water (i.e. \( \phi = 10^{-3}, Z_N^2 = 0.02 \), and \( K_f = 2.25 \text{ GPa} \) \( \frac{\phi_f}{K_f Z_N^2} = 0.022 \), which is considerably smaller than unity.

Except for density effects, few cases of shear-wave velocity dependence on saturation have been reported in the literature. This is probably due to the small number of multicomponent, time-lapse experiments that have been acquired over fractured reservoirs. Since, not all the theories and assumptions presented in this paper have been tested it is important to design experiments on fractured rocks where the fluid dependence of shear-waves with saturation can be studied.

Due to the uncertainties in the fracture parameters of natural rocks it is convenient to conduct experiments on synthetic rocks for which all the parameters are well known. Rathore et al. (1994) manufactured a synthetic sandstone with epoxy-cemented sand in which metallic discs of known shape were embedded in successive layers. The discs were then leached out chemically leaving cracks of known geometry that generated a rock with transversely isotropic symmetry.

The same technique could be applied to build a monoclinic rock with two sets of non-orthogonal penny-shaped cracks. In this case the metal discs could be placed at a fixed angle in thin slabs of synthetic sandstone. Then the rock could be built by compressing several slabs together.

Another alternative is to follow the model of tilted rotationally invariant cracks in a VTI background proposed by Grechka and Tsvering (2000). First, the rock could be built following Rathore’s method to obtain one set of cracks. Then, the rock would be stressed with the maximum compressive direction at an angle from the crack normals. This will generate a second set of stress-induced cracks and make the symmetry of the rock monoclinic. However, the disadvantage of this method is that the shape and distribution of the cracks will not be known.

Once the sample is built, velocity measurements can be done for propagation parallel to the symmetry plane and out of the symmetry plane. Out-of-plane velocity measurements should give NMO ellipses whose orientation depend on saturation. Velocity measurements of shear-waves propagating in the plane of symmetry should be fluid dependent.
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APPENDIX A: Fluid sensitivity of the compliances $Z_V$ and $Z_{NV}$ for the micro-corrugated fractures.

The compliance of the rock with one system of dry micro-corrugated fractures can be written as the sum of the background isotropic rock compliance and the excess fracture system compliance matrix:

$$ S_{\text{rock}} = \begin{pmatrix} Z_N + \frac{1}{E} & -\frac{1}{E} & -\frac{1}{E} & 0 & Z_{NV} & 0 \\ -\frac{1}{E} & \frac{1}{E} & -\frac{1}{E} & 0 & 0 & 0 \\ -\frac{1}{E} & -\frac{1}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu} & 0 & 0 \\ Z_{NV} & 0 & 0 & 0 & Z_V + \frac{1}{\nu} & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_H + \frac{1}{\nu} \end{pmatrix}, \quad (A1) $$

where $E$, $\mu$ and $\nu$ are the Young’s modulus, shear modulus and Poisson’s ratio of the background rock, respectively. $Z_N$ is the normal fracture compliance, $Z_V$ and $Z_H$ are tangential compliances, and $Z_{NV}$ is the compliance element responsible for the normal-to-vertical coupling.

According to equation (2), the difference between the values of $S_{15}$ for the dry and saturated rock is:

$$ S_{15}^{dry} - S_{15}^{sat} = \frac{(V_1^{dry} - V_1^{sat})(V_5^{dry} - V_5^{sat})}{(C_{dry} - C_0) + (C_f - C_0) \phi_f}, \quad (A2) $$

where $V_1$ and $V_5$ are defined in equation (2).

From the compliance matrix (A1) it is clear that $V_5^{dry} = Z_{NV}^{dry}$ and $V_1^{dry} - V_1^{sat} = Z_{NV}^{dry} + \frac{1}{\mu} (\frac{1}{K_{dry}} - \frac{1}{K_0})$, where $K_{dry}$ and $K_0$ are the background dry-rock and mineral bulk moduli. Since the mineral material is assumed to be isotropic, $V_5^{dry} = 0$. Finally, since the fracture compliances are added to the background isotropic compliance, the difference between the dry and mineral compressibility is $C_{dry} - C_0 = Z_{NV}^{dry} + (\frac{1}{K_f} - \frac{1}{K_0})$, where $K_f$ is the fluid bulk modulus. From these relations and equation (A2) we find the difference between the dry and saturated values of the compliance term $Z_{NV}$:

$$ Z_{NV}^{dry} - Z_{NV}^{sat} = \frac{\left[Z_{NV}^{dry} + \frac{1}{\mu} (\frac{1}{K_{dry}} - \frac{1}{K_0})\right] Z_{NV}^{dry}}{Z_{NV}^{dry} + \left(\frac{1}{K_{dry}} - \frac{1}{K_0}\right) (\phi_f + \phi_p)}, \quad (A3) $$

where the total porosity ($\phi_t$) of the rock is written as the sum of the “crack” and “isotropic” porosities $\phi_t = \phi_c + \phi_p$.

The difference between the dry compressibility ($\frac{1}{K_{dry}}$) of the background rock and the compressibility of the mineral ($\frac{1}{K_0}$) can be written as (Mavko et al., 1999):

$$ \frac{1}{K_{dry}} - \frac{1}{K_0} = \frac{\phi_p}{K_0} \frac{1}{K_f}, \quad (A4) $$

where $K_f$ is the isotropic pore space compressibility. If the isotropic pore space can be modeled as a dilute collection of spherical pores ($\phi_p < 0.1$),

$$ \frac{1}{K_{dry}} - \frac{1}{K_0} = \frac{3(1-\nu)}{2(1-2\nu)} \frac{F_0}{K_0}, \quad (A5) $$
Substituting equations (A4) and (A5) into equation (A3), we obtain the change with saturation of the compliance $Z_{NV}$ for a fractured rock with an isotropic background composed of a dilute collection of spherical pores:

$$Z_{NV}^{dr} - Z_{NV}^{sat} =$$

$$\frac{1 + \frac{1}{3} K_0 \left\{ \phi_p \frac{\phi_p}{Z_{NV}^{dr}} \right\}}{1 + \frac{1}{\phi_c} \frac{1}{Z_{NV}^{dr}} + \left( \frac{1}{K_f} - \frac{1}{K_c} \right) \left\{ \phi_p \frac{\phi_p}{Z_{NV}^{dr}} \right\}}, \quad (A6)$$

In a similar way change in the compliance $Z_V$ with saturation is calculated by solving for the $S_{05}$ component in Brown and Koning’s equation (2). For a background rock composed of spherical pores,

$$Z_{NV}^{dr} - Z_{NV}^{sat} =$$

$$\frac{Z_{NV}^{dr}}{Z_{NV}^{sat}}^2 \left[ 1 + \frac{1}{\phi_c} \frac{1}{Z_{NV}^{dr}} + \left( \frac{1}{K_f} - \frac{1}{K_c} \right) \left\{ \phi_p \frac{\phi_p}{Z_{NV}^{dr}} \right\} \right] \quad (A7)$$

Note that in the limit in which $\phi_p \to 0$, equations (A6) and (A7) become:

$$Z_{NV}^{dr} - Z_{NV}^{sat} = \frac{1}{Z_{NV}^{dr}} + \left( \frac{1}{K_f} - \frac{1}{K_c} \right) \frac{\phi_p}{Z_{NV}^{dr}} \quad (A8)$$

and

$$Z_{NV}^{dr} - Z_{NV}^{sat} = \frac{Z_{NV}^{dr}}{Z_{NV}^{sat}}^2 \left[ 1 + \left( \frac{1}{K_f} - \frac{1}{K_c} \right) \frac{\phi_p}{Z_{NV}^{dr}} \right]. \quad (A9)$$

Since the fluid compressibility is much larger than the mineral compressibility, in this limit the change with saturation is largely dependent on the value of $\frac{\phi_p}{K_f Z_{NV}^{dr}}$. If this combination of parameters is small, changes in $Z_V$ are large.

APPENDIX B: Dependence of the compliance $Z_N$ on saturation for negligible “isotropic” porosity

For two sets of non-orthogonal fractures in an isotropic, porous background, the rotation angles of the NMO ellipses are a function of the normal fracture compliance ($Z_N$) of each fracture set. If the changes in $Z_N$ with saturation are large then the NMO ellipses’ rotation angles are strongly dependent on saturation as well. Here, we prove that in the limit of vanishing background “isotropic” porosity ($\phi_p$), the parameter $\frac{\phi_p}{K_f Z_{NV}^{dr}}$ determines how significantly $Z_N$ changes with saturation ($\phi_c$ is the crack porosity and $K_f$ is the fluid’s bulk modulus).

According to equation (2), the difference between the values of the component $S_{11}$ for the dry and saturated rocks is:

$$S_{11}^{dry} - S_{11}^{sat} = \frac{(V_{11}^{dry} - V_{11}^{sat})^2}{(C_{dry} - C_{0}) + (C_f - C_{0})\phi_p}, \quad (B1)$$

Since the fracture compliances are added to the isotropic background compliance, $V_{11}^{dry} - V_{0}^i = Z_{NV}^{dry} + \frac{1}{3} (\frac{1}{K_{dry}} - \frac{1}{K_{0}})$, where $K_{dry}$ and $K_0$ are the background dry rock and mineral moduli. Similarly, the difference between the dry and mineral compressibility is $C_{dry} - C_0 = Z_{NV}^{dry} + (\frac{1}{K_f} - \frac{1}{K_c})$. Using these relations and noting that $S_{11} = Z_N + \frac{1}{K_{sat}}$, where $K$ is the bulk modulus of the background rock, equation (B1) can be represented as:

$$Z_{NV}^{dry} - Z_{NV}^{sat} =$$

$$\frac{1}{Z_{NV}^{dry}} + \left( \frac{1}{K_{dry}} - \frac{1}{K_{0}} \right) + \left( \frac{1}{K_f} - \frac{1}{K_c} \right) \left( \phi_c + \phi_p \right)$$

$$- \frac{1}{9} \left\{ \frac{1}{K_{dry}} - \frac{1}{K_{sat}} \right\}, \quad (B2)$$

where the total porosity is expressed as the sum of background “isotropic” porosity ($\phi_p$) and fracture porosity ($\phi_c$).

When the “isotropic” porosity of the background rock $\phi_p$ approaches zero, the compressibility of the dry background rock approaches the compressibility of the mineral ($\{ \frac{1}{K_{dry}} - \frac{1}{K_0} \} \to 0$). Also, since $\phi_p \to 0$, there is no difference between the dry and saturated background compressibility ($\{ \frac{1}{K_{dry}} - \frac{1}{K_{sat}} \} \to 0$). Therefore, in the limit $\phi_p \to 0$, equation (B2) becomes

$$Z_{NV}^{dry} - Z_{NV}^{sat} = \frac{1}{Z_{NV}^{dry}} + \frac{\phi_c}{K_f Z_{NV}^{dry}}. \quad (B3)$$

Since for most fluids $\frac{1}{K_f} \gg \frac{1}{K_0}$, in this limit the magnitude of change in $Z_N$ is determined by $\frac{\phi_c}{K_f Z_{NV}^{dry}}$. If this combination of parameters is small, changes in $Z_N$ are large.