Characterization of dipping fractures in transversely isotropic background

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ABSTRACT

Although it is often believed that fractures in the earth have predominantly vertical orientation, existing geological and geophysical data indicate that certain mechanisms, such as strong oblique stresses, might cause the presence of dipping fractures in the subsurface. Here we examine the effective model produced by a single system of dipping rotationally invariant fractures embedded in a transversely isotropic (VTI) background rock. This model is a special case of monoclinic media with the vertical symmetry plane.

Multicomponent seismic reflection data acquired over such a medium possess several distinct features that allow one to characterize this fracture model using P- and PS-waves reflected from a horizontal interface. For example, the polarization vector of vertically propagating fast converted- or shear-wave usually points in the direction of the fracture strike and, therefore, can be used to obtain the azimuth of fractures. An independent estimate of the fracture azimuth can be made based on the measurements of azimuthally varying normal-moveout (NMO) velocities from a horizontal reflector because, due to the symmetry of the effective medium, the directions of semi-axes of the pure-mode NMO ellipses coincide with the dip- and strike-planes of the fractures. We prove the feasibility of a complete fracture-characterization procedure (i.e., estimating the velocities and Thomsen anisotropic parameters in VTI host, and the compliances and dip angle of the fracture set) using the vertical and NMO velocities of P- and two split S-waves (or converted waves) reflected from a horizontal interface.

Key words: linear slip theory, fracture characterization, dipping fractures, VTI background

Introduction

Characterization of naturally fractured reservoirs using seismic data is a topic of significant exploration interest. Although there exists an extensive list of papers aimed at detection of fractures and estimating their orientation (many relevant references can be found in the special section of Geophysics on P-wave anisotropy edited by Tsivankin and Lynn (1999)), a clear need remains for more quantitative methods of inverting seismic data for fracture parameters. An attempt to fill in this gap was made by Bakulin et al. (2000a, b, c), who examined such commonly measured seismic signatures as the normal-moveout (NMO) velocity and amplitude variation with offset and azimuth (AVO) in a number of effective fracture models. Essentially, Bakulin et al. (2000a, b, c) concluded that seismic data, while being unable to constrain microstructural parameters (e.g., shape of fractures, crack density, equant porosity, and type of infill) individually, might provide useful information about the excess fracture compliances and the orientation of fracture sets. Since the excess compliances are inherent parameters of the linear slip theory of Schoenberg (1980, 1983), it is natural that this theory is used here to build a link between the properties of fractures and seismic signatures.

Since the number of fracture parameters can be arbitrarily large in the presence of multiple fracture sets, whereas the number of parameters of effective media is
limited to 21 for the most general triclinic anisotropy, the procedure of fracture characterization has to become ambiguous at some level of microstructural complexity. Moreover, the fracture parameters may appear in the effective stiffness coefficients in certain combinations, producing another type of ambiguity, such as that when two orthogonal fracture sets exist in vertically transversely isotropic (VTI) host rock (Bakulin et al., 2001). Thus, one’s ability of unique fracture characterization based on seismic data depends on the types of fractures and the background. Therefore, a reasonable question to ask is whether or not there exist systems of fractures, in addition to those described by Bakulin et al. (2000a, b, c), that can be unambiguously constrained by seismic data. Our paper answers this question.

Here, we extend the first part of paper by Bakulin et al. (2000b), which deals with one set of vertical fractures in a VTI background rock, by allowing the fractures to be dipping. In contrast to the effective model of Bakulin et al. (2000b), which has orthorhombic symmetry, the medium produced by the dipping fractures is monoclinic. Still, as long as the fractures do not deviate by as much as $25^\circ - 30^\circ$ from the vertical, which is geologically reasonable, the behavior of seismic signatures resembles that for the vertical fractures. In particular, both the polarization vector of vertically propagating fast shear-wave and the semi-major axis of the $P$-wave NMO ellipse point in the strike direction of the fracture set. We devise a fracture-characterization procedure that uses the vertical velocities of $P$- and two split $S$-waves and the NMO ellipses of $P$ and shear (or converted) modes from a horizontal reflector to estimate all background and fracture parameters, including the fracture dip. Numerical tests show satisfactory accuracy and stability of the inverted quantities.

**Effective monoclinic medium**

Following the linear slip theory developed by Schoenberg (1980, 1983), the compliance matrix $s$ of an effective medium containing fractures is equal to the sum of the background compliance $s_b$ and the fracture compliance $s_f$:

$$s = s_b + s_f.$$  

(1)

Here $s$, $s_b$, and $s_f$ are $6 \times 6$ symmetric nonnegative-definite matrices. By definition, the compliance matrices are the inverse of the corresponding stiffness matrices. In particular, the background compliance $s_b$ is

$$s_b^{-1} = c_b,$$  

(2)

where $c_b$ is the stiffness matrix of the VTI host rock

$$c_b = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{46} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}.$$  

(3)

and $c_{12} = c_{11} - 2c_{66}$. The matrix $s_f^{-1}$ of the excess compliance for a set of vertical rotationally invariant fractures with normal in the $x_1$-direction is written as (Schoenberg and Sayers, 1995)

$$s_f^{-1} = \begin{pmatrix} K_N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_T & 0 & 0 \\ 0 & 0 & 0 & 0 & K_T & 0 \\ 0 & 0 & 0 & 0 & 0 & K_T \end{pmatrix},$$  

(4)

where $K_N$ and $K_T$ are the normal and tangential (shear) fracture compliances. To obtain the matrix for dipping fractures, we rotate the matrix $s_f^{-1}$ by the angle $\theta$ around the $x_2$-axis, which is the strike direction of the fracture set. Such a rotation is accomplished according to the so-called Bond transformation

$$s_f \equiv s_f^\theta = N(\theta) s_f^{-1} N^T(\theta),$$  

(5)

where the $6 \times 6$ matrix $N$ is explicitly written in Winterstein (1990), and $N^T$ denotes the transposed matrix.

Substituting equations (2)–(5) into equation (1), we obtain the effective stiffness matrix in the form

$$c = s^{-1} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{33} & 0 & 0 & 0 \\ c_{13} & c_{33} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{46} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}. $$  

(6)

The matrix $c$ describes a monoclinic medium with vertical symmetry plane that coincides with the dip plane of the fracture set. Such a symmetry of the effective medium could be easily predicted from the outset based on the fact that both the VTI host and the fractures have the same vertical symmetry plane.

Although the exact expressions for the stiffness coefficients (6) are lengthy, their useful approximations can be obtained assuming that anisotropy of host VTI rock is weak so that its Thomsen (1986) anisotropic coefficients satisfy the inequalities

$$|\epsilon_1| \ll 1, \quad |\delta_0| \ll 1, \quad \text{and} \quad |\gamma_1| \ll 1.$$  

(7)

In addition, if small crack density is assumed, the fracture weaknesses (Schoenberg and Hellbäg, 1997)
\[ \Delta_N = \frac{K_N c_{11b}}{1 + K_N c_{11b}} \quad \text{and} \quad \Delta_T = \frac{K_T c_{44b}}{1 + K_T c_{44b}} \]  

also become small,

\[ \Delta_N \ll 1, \quad \Delta_T \ll 1, \]  

which makes it possible to obtain concise approximations for \( c_{ij} \). Those expressions, fully linearized in \( \varepsilon_\delta, \delta_\gamma, \gamma_\delta, \Delta_N, \) and \( \Delta_T \), are given in Appendix A. They explicitly show, for example, that the effective medium becomes orthorhombic [examined by Schoenberg and Helbig (1997) and Bakulin et al. (2000b)] when the fractures are vertical, i.e., when the rotation angle \( \theta = 0 \) and, therefore, \( c_{15} = c_{25} = c_{46} = 0 \).

It is important to note that thirteen stiffness coefficients* of the effective matrix \( c \) [equation (6)] are constructed out of only eight independent quantities (five parameters of the VTI host, two fracture weaknesses, and dip of the fracture set). Thus, the elements of the effective stiffness matrix are not independent; five relationships exist among them. Although those relationships are not apparent even in the limit of weak anisotropy and small crack density [equations (A4)–(A16)], the fact of their existence is important in determining the minimum number of seismic signatures which are needed for unique characterization of such a fractured medium.

**Analysis of weak-anisotropy approximations**

**Phase velocities of body waves at vertical incidence**

The approximations (A4)–(A16) for the stiffness coefficients obtained in Appendix A allow us to draw several useful conclusions about the features of waves propagating in the vertical direction. Let us analyze the elements of the Christoffel matrix

\[ G_{ij} = c_{ijkl} n_j n_k - \rho V^2 \delta_{il}. \]  

Here \( c_{ijkl} \) are the components of the elastic stiffness tensor represented by matrix (6), \( n \) is the unit vector normal to the wavefront, \( \rho \) is the density, \( V \) is the phase velocity, and \( \delta_{il} \) is the symbolic Kronecker delta. The summation from 1 to 3 over all repeating indexes is assumed in equation (10).

Using the structure of the effective stiffness tensor [equation (6)] and the fact that \( n = [0, 0, 1] \) at the vertical incidence, we can write the Christoffel matrix in the form

\[ G = \begin{pmatrix} c_{55} - \rho V^2 & 0 & c_{35} \\ 0 & c_{44} - \rho V^2 & 0 \\ c_{35} & 0 & c_{33} - \rho V^2 \end{pmatrix}. \]  

Equation (11) indicates that there exists a vertically propagating pure shear wave whose polarization vector points in the direction \( [0, 1, 0] \), i.e., in the strike direction of the dipping fractures. The phase velocity of this wave is

\[ V_{S_1} = \sqrt{\frac{c_{44}}{\rho}}. \]  

Both the conclusion about the polarization direction and equation (12) for the phase velocity of the wave \( S_1 \) are exact because they are based on the structure of the effective stiffness matrix rather than on the linearized expressions for its elements given in Appendix A. The approximation for the velocity \( V_{S_1} \) obtained using equation (A13) is

\[ V_{S_1} \approx V_{S_{0b}} \left( 1 - \frac{\Delta_T}{2} \sin^2 \theta \right), \]  

where \( V_{S_{0b}} \) is the vertical \( S \)-wave velocity in the VTI host.

The squared phase velocities of the other two vertically propagating waves, the \( P \)- and \( S_\perp \)-waves polarized in the dip direction of the fractures, can be found by solving quadratic equation

\[ \det \begin{pmatrix} c_{55} - \rho V^2 & c_{35} \\ c_{35} & c_{33} - \rho V^2 \end{pmatrix} = 0, \]  

which follows from (11). Noting that the coefficient \( c_{35} \) is already linear in the fracture weaknesses \( \Delta_N \) and \( \Delta_T \) [equation (A12)] allows us to obtain the weak-anisotropy approximations for the phase velocities \( V_P \) and \( V_{S_\perp} \) directly from equations (A11) and (A15)

\[ V_P \approx \sqrt{\frac{c_{55}}{\rho}} \approx V_{P_{0b}} \left[ 1 - \frac{\Delta_N}{2} \left( 1 - 2 \frac{g_b}{\Delta_T} \left( \frac{3 \Delta_T}{2} + \frac{g_b}{4} \Delta_T + \Delta_N \right) \cos 2\theta \right) + \frac{g_b}{4} \left( \Delta_T - \Delta_N \right) \cos 4\theta \right]. \]  

and

\[ V_{S_\perp} \approx \sqrt{\frac{c_{55}}{\rho}} \approx V_{S_{0b}} \left[ 1 - \frac{1}{4} \left( g_b \Delta_N + \Delta_T \right) + \frac{1}{4} \left( g_b \Delta_N - \Delta_T \right) \cos 4\theta \right]. \]  

Here \( V_{P_{0b}} \) is the vertical \( P \)-wave velocity, and

\[ g_b = \frac{V_{S_{0b}}}{V_{P_{0b}}} \]  

is the ratio of the squared vertical velocities in the VTI background medium.

In the special case of vertical fractures (\( \theta = 0 \)), the velocities \( V_{S_1} \) and \( V_{S_\perp} \) given by equations (13) and (16) reduce to the expressions,
obtained by Bakulin et al. (2000b). In this case, as follows from equations (18) and the inequality 0 ≤ ΔT ≤ 1, the vertically propagating shear-wave polarized parallel to the fractures always has a greater phase velocity than that of the S-wave with polarization perpendicular to the fractures. This fact is emphasized by the notation V_{S1} and V_{S2}, where

\[ V_{S1} \geq V_{S2}, \]

(19)

and \( S_1 \) and \( S_2 \) denote the fast and slow shear-waves, respectively.

It is instructive to analyze approximations (13) and (16) for dipping fractures to determine whether or not the inequality

\[ V_{S1} \geq V_{S2}, \]

(20)

holds when \( \theta \neq 0 \). As is clear from the continuous dependences of \( V_{S1} \) and \( V_{S2} \) on \( \theta \), inequality (20) is satisfied within some range of \( \theta \geq 0 \). To get an estimate of the width of this range, we substitute equations (13), (16) into inequality (20) and rewrite it in the form

\[ \Delta_T \left( 4 \sin^4 \theta - 5 \sin^2 \theta + 1 \right) \geq -g_6 \Delta_N \sin^2 2\theta. \]  

(21)

The right-hand side of this inequality is always nonpositive. It reaches its maximum, which is equal to zero, when \( \Delta_N = 0 \), i.e., for fluid-filled fractures (Schoenberg and Sayers, 1995). This yields the low bound for the fracture angle

\[ \theta = 30^\circ. \]

(22)

When the cracks are partially or fully fluid-saturated, the normal weakness \( \Delta_N \) becomes positive which makes the right-hand side of inequality (21) negative and, correspondingly, increases the bound for \( \theta \). For dry fractures we apply the Schoenberg-Sayers (1995) criterion \( K_N = K_T \) or \( \Delta_T \approx g_6 \Delta_N \) [see equations (8) and (17)] and conclude that any \( \theta \) from 0° to 90° satisfies inequality (21).

Since the above conclusion, as well as the bound (22), are based on the assumption of weak anisotropy and small crack density, it is useful to check them using the exact equations. The shear-wave velocities in Figure 1, computed for a moderately anisotropic VTI background and a relatively large crack density \( c \approx \Delta_T/2 = 0.1 \), show that approximation (22) has a high accuracy (Figure 1a), whereas the statement that \( V_{S1} > V_{S2} \) for all angles \( \theta \) for dry fractures is less accurate (Figure 1b). Another useful observation that can be made from comparison of Figures 1a and 1b is that, in accordance with equation (13), the exact velocity \( V_{S1} \) is insensitive to the fluid content of the fractures.

Existing data about in situ stresses indicate that the magnitude of horizontal or oblique stress, which may cause fractures to be dipping, is usually smaller than that of the lithostatic pressure. This implies that, in practice, dipping fractures will probably be close to vertical. Therefore, we should normally expect that the angle \( \theta \) has some relatively small value, and, consequently, that the fast vertically propagating shear-wave is polarized in the strike plane of the fracture set.

**Pure-mode NMO velocities from a horizontal reflector**

Let us suppose that we can perform shear-wave polarization analysis of split shear-waves (e.g., Alford,

**Figure 1.** Exact phase velocities of shear-waves polarized in the dip (circles) and strike (triangles) plane of dipping fractures. The background parameters are \( V_{P0b} = 2 \) km/s, \( V_{S0b} = 1 \) km/s, \( \epsilon_b = 0.3 \), \( \delta_b = 0.2 \), \( \gamma_b = 0.4 \). The fracture weaknesses are \( \Delta_T = 0.2 \), \( \Delta_N = 0 \) (a) and \( \Delta_T = 0.2 \), \( \Delta_N = 0.8 \) (b).
1986), which allows us to estimate the fracture azimuth and measure the vertical velocities given by the approximations (13), (15), (16). Here we show that supplementing those measurements with the normal-moveout velocities from a horizontal interface makes it possible to fully characterize the fractured medium, i.e., obtain all parameters of the VTI host and the fracture weaknesses.

As shown by Grechka and Tiwari (1998), the azimuthal variation of pure-mode normal-moveout (NMO) velocities \( V_{\text{NMO}}(\alpha) \) is always an ellipse as long as the reflection traveltime increases with the offset in common midpoint (CMP) geometry:

\[
V_{\text{NMO}}^2(\alpha) = W_{11} \cos^2 \alpha + 2W_{12} \sin \alpha \cos \alpha + W_{22} \sin^2 \alpha. \tag{23}
\]

The elements of the \( 2 \times 2 \) symmetric matrix \( W \) are expressed through the derivatives of the horizontal components of the slowness vector \( p = [p_1, p_2, q] \) with respect to the source or receiver coordinates. If the medium above the reflector is homogeneous, the matrix \( W \) is given by (Grechka et al., 1999)

\[
W = \begin{pmatrix} p_1 q_1 + p_2 q_2 - q & q_{22} - q_{12} \\ q_{11} q_{22} - q_{12}^2 & -q_{12} & q_{11} \end{pmatrix}, \tag{24}
\]

where \( q \equiv q(p_1, p_2) \) denotes the vertical component of the slowness vector \( q_i \equiv \partial q/\partial p_i \), and \( q_{ij} \equiv \partial^2 q/\partial p_i \partial p_j \). The horizontal slowness components \( (p_1 \) and \( p_2 \) \) and the derivatives in equation (24) are evaluated for the zero-offset ray.

For a horizontal reflector, the zero-offset slowness vector is vertical \( p = [0, 0, q] \), and, therefore, equation (24) reduces to

\[
W = \begin{pmatrix} -q & q_{22} - q_{12} \\ q_{11} q_{22} - q_{12}^2 & -q_{12} & q_{11} \end{pmatrix}. \tag{25}
\]

Further simplification can be done by noting that the dip plane of fractures is the vertical symmetry plane of our effective monoclinic medium. Therefore, the semi-axes of three pure-mode NMO ellipses will be aligned with the dip and strike directions of the fracture set. Selecting the dip direction as the \( x_1 \)-axis (which can be easily done because the fracture azimuth can be determined independently from the shear-wave polarization) makes the derivative \( q_{12} \) vanish. Thus, the matrix (25) becomes diagonal:

\[
W = \begin{pmatrix} -q/q_{11} & 0 \\ 0 & -q/q_{22} \end{pmatrix}. \tag{26}
\]

The latter allows us to define the NMO velocities in the dip- and strike-directions:

\[
V_{\text{NMO}}^{\text{Dip}} = \sqrt{-q/q_{11}} \quad \text{and} \quad V_{\text{NMO}}^{\text{Str}} = \sqrt{-q/q_{22}}. \tag{27}
\]

Next, we again assume weak anisotropy and small crack density [equations (7) and (9)] and derive linearized expressions for \( V_{\text{Dip}}^{\text{NMO}} \) and \( V_{\text{Str}}^{\text{NMO}} \) similar to those obtained for the phase velocities. This yields

\[
V_{\text{Dip}}^{\text{NMO}} \approx V_{\text{P0s}} \left[ 1 + \frac{\Delta b + \Delta N}{2} - \frac{3 g_b^2}{4} \right],
\]

\[
-\frac{g_b}{4} (\Delta_T - \Delta N) g_b (1 - g_b) \cos 2\theta,
\]

\[
-\frac{7 g_b}{4} (\Delta_T - g_b \Delta_N) \cos 4\theta \right], \tag{28}
\]

\[
V_{\text{Str}}^{\text{NMO}} \approx V_{\text{S0s}} \left[ 1 + \frac{\Delta b + \Delta N}{2} - \frac{3 g_b^2}{4} \right],
\]

\[
-\frac{3 g_b}{4} (\Delta_T + g_b (\Delta_T - g_b \Delta_N) \cos 2\theta,
\]

\[
-\frac{g_b}{4} (\Delta_T - g_b \Delta_N) \cos 4\theta \right], \tag{29}
\]

Examining equations (28)–(33) reveals that the differences

\[
\chi_\text{P} = V_{\text{P, NMO}}^{\text{Dip}} - V_{\text{P, NMO}}^{\text{Str}} \approx 2 g_b V_{\text{P0s}} \cos^2 \theta \left[ 2 \Delta_T - \Delta_N (1 + g_b) + 3 (\Delta_T - g_b \Delta_N) \cos 2\theta \right], \tag{34}
\]

\[
\chi_\text{S} = V_{\text{S, NMO}}^{\text{Dip}} - V_{\text{S, NMO}}^{\text{Str}} \approx V_{\text{S0s}} \sin^2 \theta \left[ g_b \Delta_N - \frac{\Delta_T}{2} \right] + (\Delta_T - g_b \Delta_N) \cos 2\theta \right], \tag{35}
\]

\[
\chi_\text{S} = V_{\text{S, NMO}}^{\text{Dip}} - V_{\text{S, NMO}}^{\text{Str}} \approx V_{\text{S0s}} \left[ -\Delta_T - g_b \Delta_N + 7 (\Delta_T - g_b \Delta_N) \cos 4\theta \right] \tag{36}
\]

do not depend on the background anisotropic coefficients.
\[ c_0, \delta_1, \text{ and } \gamma_0. \] This result is not surprising. It should be expected based on the "addition rule" of the background and fracture-induced anisotropy (Balakin et al., 2000b). Since the pure-mode NMO ellipses from horizontal reflectors degenerate into circles in a VT1 host rock, the contribution of background anisotropy can be eliminated by subtracting the NMO velocities measured at different azimuths in effective fractured media.

It is interesting to note that, as follows from equation (34), \( \chi_r < 0 \) if \( \theta < 24^\circ \) for fluid-filled cracks \( (\Delta_N = 0) \) and \( \chi_r \) is always negative when the fractures are dry \( (\Delta_T \approx g_0 \Delta_N) \). Thus, the lengths of semi-axes of the P-wave NMO ellipse give us another criterion (in addition to the shear-wave polarization) for estimating azimuth of the fracture set.

Let us now investigate the feasibility of obtaining parameters of fractures based on the expressions (13), (15), (16) for the vertical velocities and on equations (34)–(36). Those relations can be treated as six constraints for five unknowns: two background velocities \( V_{P0} \) and \( V_{S0} \), two fracture weaknesses \( \Delta_N \) and \( \Delta_T \), and the fracture dip \( \phi = 90^\circ - \theta \). Although it is possible to prove that all the unknowns can be resolved individually from those approximate equations, we show this in the next section using exact expressions for the effective stiffness coefficients and velocities.

**Fracture characterization**

Here, we discuss parameter estimation for a single horizontal fractured layer. Let us suppose that the vertical velocities of the P- and two split S-waves were measured in a borehole. The most practical way of obtaining shear-wave NMO velocities is to use 3-D multiaxiality P and PS (PS1 and PS2) reflection data. Then, applying the methodology of Grechka and Tsvankin (2001), we reconstruct the traveltimes of pure S-waves. Note that the absence of the horizontal symmetry plane in our model makes the converted-wave reflection traveltimes asymmetric with respect to the zero offset, such as is usually observed for dipping reflectors (e.g., Tsvankin and Grechka, 2000). Figure 2 illustrates a typical PS2 (the wave S2 is polarized in the dip plane of the fractures) moving along a line in the dip direction of dry fractures. The PS-wave moveout asymmetry, however, does not prevent the algorithm of Grechka and Tsvankin (2000) from obtaining pure S-wave traveltimes, which are symmetric (i.e., reciprocal) in the common-midpoint geometry.

Next, we apply 3-D velocity analysis (Grechka and Tsvankin, 1999), which results in pure-mode NMO ellipses (Figure 3), and use them, along with the vertical velocities, as data for stacking-velocity tomography. This technique, described by Grechka et al. (2001) for transversely isotropic media, allows one to estimate anisotropic parameters of the subsurface and the positions and shapes of reflectors. Here we use it for fracture characterization.

Figure 4 shows the inversion results for the model described in Figure 2. To verify the stability of inversion, we added Gaussian noise with standard deviations of 0.5% and 2% to the vertical and NMO velocities, respectively, and repeated parameter estimation 100 times.
Characterization of dipping fractures

Figure 4. Inversion of fracture and background parameters. Dots represent the correct values (see Figure 2). The error bars correspond to the 95% confidence intervals in the estimated quantities.

Figure 5. Same as Figure 4 but for fluid-filled fractures ($\Delta_N = 0$). The values of parameters $V_{p0b}$, $V_{S0b}$, and $\theta$ (not shown in this Figure) are same as those in Figure 2. The 95% confidence intervals for them are 2.0%, 0.5%, and 1.7°, respectively.

for different realizations of the noise. As Figure 4 demonstrates, the confidence intervals for the background anisotropic coefficients and the tangential weakness $\Delta_T$ are about 0.02, which indicates that the inversion is stable. The error bar for the normal weakness $\Delta_N$ is higher, being equal to 0.06. The reason for this reduced accuracy can be understood analyzing the weak-anisotropy approximations discussed in the previous section. As equations (13), (15), (16), and (28)–(33) show, most of the terms containing the weakness $\Delta_N$ have the prefactor $g_b$, which is about 0.2 in our model. Therefore, the sensitivity of velocities to $\Delta_N$ is lower compared to that with respect to other anisotropic parameters, and, consequently, the errors in the estimated $\Delta_N$ values are greater. In these numerical tests we also obtained the vertical velocities $V_{p0b}$, $V_{S0b}$ in the background and the fracture angle $\theta$ (not shown in Figure 4). The confidence intervals for those quantities are 2.8%, 0.5%, and 3.6°, respectively.

Similar inversion results with somewhat smaller error bars are shown in Figure 5 for fluid-saturated cracks. This time, we observe non-physical negative values of the normal weakness $\Delta_N$ because random errors added to the data may produce the velocities that do not correspond to any fractured media.

Conclusions

It is known that several models containing vertical fracture sets can be fully characterized using seismic data (Bakulin et al., 2000a, b, c). Since the fractures in the subsurface may be dipping because of, for instance, the presence of strong oblique stresses, it is important to find out whether or not seismic signatures can be uniquely inverted for the parameters of those fractures. To the best of our knowledge, here we described the first model with dipping fractures that can be unambiguously constrained based on seismic data. We examined media produced by one set of rotationally invariant dipping fractures embedded in a VT background rock, which leads to effective monoclinic models with vertical symmetry plane in the dip direction of the fractures.

Analyzing the weak-anisotropy approximations for the effective stiffness coefficients, we found that seismic signatures in the resulting specific form of effective monoclinic media resemble those in the higher-symmetry
orthorhombic models produced by vertical fractures in a VTI host rock (Bakulin et al., 2000b). In particular, if the dipping fractures are sufficiently close to vertical, the polarization vectors of vertically propagating fast shear-waves point in the direction of fracture strike in both media. A similar observation has been made regarding the azimuths of semi-major axes of P-wave NMO ellipses measured from horizontal reflectors. Again, their azimuths coincide with the fracture strike in both media.

Although the effective media we discussed here are monoclinic, they are described by only eight independent parameters \( (V_{P00}, V_{S00}, \epsilon, \delta, \gamma_0) \) of the VTI background, the normal and tangential fracture weaknesses \( \Delta_N, \Delta_T \), and the fracture angle \( \theta \) compared to twelve in general monoclinic models. This reduction in the number of independent medium parameters has important implications for the parameter estimation. We showed that all eight quantities that characterize the fractures and the host rock can be obtained from the vertical velocities of \( P \) - and two split \( S \)-waves. From the pure-mode NMO ellipses corresponding to horizontal reflectors. The numerical examples presented here indicate that the estimates produced by such an inversion can be accurate and stable.

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APPENDIX A:

Weak-anisotropy approximation for elastic stiffness coefficients of effective media

Here we give approximate expressions for the elements of the effective stiffness matrix (6) assuming that anisotropy of the VTI background is weak and that the crack density is small. For this purpose, it is convenient to use Thomsen (1986) parameterization of VTI media in terms of the vertical \( P \)- and \( S \)-wave velocities \( V_{P00}, V_{S00} \) and dimensionless anisotropic coefficients \( \epsilon_0, \delta_0, \) and \( \gamma_0 \). Then, the assumption of weak anisotropy means that
\[ |c_6| \ll 1, \quad |\delta_6| \ll 1, \quad \text{and} \quad |\gamma_6| \ll 1. \]  
\( \text{(A1)} \)

Similarly, the fracture set can be described in terms of the dimensionless weaknesses (Schoenberg and Helbig, 1997)

\[
\Delta_N \equiv \frac{K_N c_{11b}}{1 + K_N c_{11b}} \quad \text{and} \quad \Delta_T \equiv \frac{K_T c_{44b}}{1 + K_T c_{44b}} 
\]

\( \text{(A2)} \)

which vary from 0 to 1, and

\[
\Delta_N \ll 1, \quad \Delta_T \ll 1 
\]

\( \text{(A3)} \)

when the crack density is small.

Assuming that the anisotropic coefficients and the fractures weaknesses have the same order, we linearize the expressions (1)–(5) in those quantities and obtain the following stiffness coefficients (6) of the effective monoclinic medium:

\[
c_{11} = \rho V^2_{P0b} \left[ 1 + 2 c_6 - \Delta_N \left( 1 - 2 g_6 + \frac{3 g_6^2}{2} \right) - \frac{g_6}{2} \Delta_T + 2 \Delta_N g_6 (1 - g_6) \cos 2\theta + \frac{g_6}{2} (\Delta_T - g_6 \Delta_N) \cos 4\theta \right] 
\]

\( \text{(A4)} \)

\[
c_{12} = \rho V^2_{P0b} \left[ 1 - 2 g_6 + 2 c_6 - 4 g_6 \gamma_6 - \Delta_N \left( 1 - 3 g_6 + 2 g_6^2 \right) + \Delta_N g_6 (2 g_6 - 1) \cos 2\theta \right] 
\]

\( \text{(A5)} \)

\[
c_{13} = \rho V^2_{P0b} \left[ 1 - 2 g_6 + \delta_6 - \Delta_N \left( 1 - 2 g_6 + \frac{g_6^2}{2} \right) + \frac{g_6}{2} \Delta_T + \frac{g_6}{2} (g_6 \Delta_N - \Delta_T) \cos 4\theta \right] 
\]

\( \text{(A6)} \)

\[
c_{15} = \rho V^2_{S0b} \left[ \Delta_N (1 - g_6) + (g_6 \Delta_N - \Delta_T) \cos 2\theta \right] \sin 2\theta 
\]

\( \text{(A7)} \)

\[
c_{22} = \rho V^2_{P0b} \left[ 1 + 2 c_6 - \Delta_N (1 - 2 g_6)^2 \right] 
\]

\( \text{(A8)} \)

\[
c_{23} = \rho V^2_{P0b} \left[ 1 - 2 g_6 + \delta_6 - \Delta_N (1 - 2 g_6) (1 - g_6) + \Delta_N g_6 (1 - 2 g_6) \cos 2\theta \right] 
\]

\( \text{(A9)} \)

\[
c_{25} = \rho V^2_{S0b} \Delta_N (1 - 2 g_6) \sin 2\theta 
\]

\( \text{(A10)} \)

\[
c_{33} = \rho V^2_{P0b} \left[ 1 - \Delta_N \left( 1 - 2 g_6 + \frac{3 g_6^2}{2} \right) - \frac{g_6}{2} \Delta_T + 2 \Delta_N g_6 (g_6 - 1) \cos 2\theta + \frac{g_6}{2} (\Delta_T - g_6 \Delta_N) \cos 4\theta \right] 
\]

\( \text{(A11)} \)

\[
c_{35} = \rho V^2_{S0b} \left[ \Delta_N (g_6 - 1) + (\Delta_T - g_6 \Delta_N) \cos 2\theta \right] \sin 2\theta 
\]

\( \text{(A12)} \)

\[
c_{44} = \rho V^2_{S0b} \left( 1 - \Delta_T \sin^2 \theta \right) \sin 2\theta 
\]

\( \text{(A13)} \)

\[
c_{46} = \rho V^2_{S0b} \Delta_T \sin \theta \cos \theta 
\]

\( \text{(A14)} \)

\[
c_{55} = \rho V^2_{S0b} \left[ 1 - \frac{1}{2} (g_6 \Delta_N + \Delta_T) + \frac{1}{2} (g_6 \Delta_N - \Delta_T) \cos 4\theta \right] 
\]

\( \text{(A15)} \)

\[
c_{66} = \rho V^2_{S0b} (1 + 2 \gamma_6 - \Delta_T \cos^2 \theta) 
\]

\( \text{(A16)} \)

Here \( \rho \) is the medium density,

\[
g_6 = \frac{V^2_{S0b}}{V^2_{P0b}} 
\]

\( \text{(A17)} \)

is the ratio of the squared vertical velocities in the background, \( \theta \) is the angle of the fracture planes with the vertical (the dip of the fracture set is \( \phi = \pi/2 - \theta \)), and the azimuth of the fractures is zero. The stiffness coefficients \( c_{ij} \) not listed in equations (A4)–(A16) are equal to zero.