Multicomponent stacking-velocity tomography for transversely isotropic media

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ABSTRACT
Accurate estimation of the velocity field is the most difficult step in imaging of seismic data from anisotropic media. Here, the velocity-analysis problem is examined for the most common anisotropic model of sedimentary formations—transverse isotropy (TI) with arbitrary orientation of the symmetry axis. We show that supplementing wide-azimuth reflected PP data with mode-converted (PS) waves yields more stable estimates of the anisotropic coefficients and, in many cases, helps to constrain the depth scale of the model.

An important processing step preceding the inversion is reconstruction of the traveltimes of the pure SS-waves from those of the PP- and PS-waves based on the technique recently developed by Grechka and Tsvankin. This procedure allows us to replace PS-wave moveout, which is generally asymmetric with respect to zero offset, with the symmetric (hyperbolic on short spreads) moveout of the pure SS reflections. Then, generalizing the algorithm previously suggested for PP data, we develop a joint tomographic inversion of the normal-moveout (NMO) ellipses and zero-offset traveltimes of PP- and SS-waves.

Application of the method to wide-azimuth PP and PS reflections from a dipping interface beneath a homogeneous TI layer shows that for a range of reflector dips and tilt angles of the symmetry axis it is possible to build the anisotropic velocity field in the depth domain. We also extend our procedure to layered TI media with curved interfaces and study its stability in the presence of noise and heterogeneity.

Key words: converted waves, TI media, parameter estimation

Introduction
A number of case studies involving multicomponent land and offshore data demonstrated that mode (P-to-S) conversions can supplement or even replace pure-mode reflections in such applications as imaging beneath gas clouds (e.g., Granli et al. 1999; Thomsen, 1999) and characterization of fractured reservoirs (e.g., Pérez et al. 1999). Processing of PS-waves, however, is complicated by the strong influence of seismic anisotropy on their signatures. For example, the velocity anisotropy of SV- and PSV-waves in TI media is mostly controlled by the coefficient

$$\sigma \equiv \left( \frac{V_{p0}}{V_{s0}} \right)^2 (\epsilon - \delta),$$

which is typically much larger than the Thomsen (1986) parameters $\epsilon$ and $\delta$ governing P-wave data ($V_{p0}$ and $V_{s0}$ are the vertical P- and S-wave velocities, respectively). Mis-ties between PP and PS sections produced by conventional isotropic imaging methods (e.g., Nolte et al., 1999) indicate the need for joint anisotropic velocity analysis of PP and PS reflection events.

As shown by Tsvankin and Grechka (2000a,b) for TI media with a vertical symmetry axis (VTI), wide-azimuth reflection traveltimes of PP- and PSV-waves from a single mildly dipping reflector are sufficient for estimating all relevant parameters ($V_{p0}$, $V_{s0}$, $\epsilon$ and $\delta$). If the reflector is horizontal, the joint inversion of PP and PSV moveout data is nonunique, even if uncommonly long offsets are available (Grechka and Tsvankin, 2001b). It should be emphasized that the vertical velocity and reflector depth are difficult to determine using PP moveout alone. Le Stunff et al. (1999) and Grechka
et al. (2000a,b) showed that depth-domain velocity analysis of PP reflections in VTI media is feasible for only a limited subset of models with dipping or curved intermediate boundaries.

Here we extend our previous results on the inversion of PP and PS data by introducing the methodology of anisotropic multicomponent stacking-velocity tomography and applying it to TI media with an arbitrary tilt of the symmetry axis. Rather than working with PS data directly, we combine them with PP data to compute the traveltimes of the pure SS (SV or SH for TI media) reflections from the same interface using the algorithm of Grechka and Tsvankin (2001a). The reconstruction of SS traveltimes is entirely data-driven and does not require knowledge of the velocity model. This procedure makes it possible to avoid inherent problems of PS-wave velocity analysis caused by the asymmetry of SS moveout with respect to zero offset on CMP (common midpoint) and CCP (common-conversion-point) gathers, conversion-point dispersal and polarity reversals.

In contrast to the more complicated moveout of mode conversions, reflection traveltimes of pure SS-waves is symmetric with respect to zero offset and, for moderate offset-to-depth ratios, can be described by the NMO ellipse (Grechka and Tsvankin, 1998). Hence, the theory of the NMO ellipses and NMO-velocity surfaces (Grechka et al. 1999, Grechka and Tsvankin, 1999b) is directly applicable to SS-wave moveout. In particular, Grechka and Tsvankin (1999b) showed that pure-mode NMO ellipses in heterogeneous arbitrary anisotropic media can be built in a computationally efficient way by tracing just one (zero-offset) ray for each reflection event. This modeling technique was used by Grechka et al. (2000a,b) to develop tomographic-style inversion of PP-wave NMO ellipses in VTI media composed of homogeneous layers separated by planar or curved interfaces.

The methodology of Grechka et al. (2000a,b) is generalized here for the combination of conventional-spread PP and SS data. The tomographic algorithm operates with the NMO ellipses, zero-offset traveltimes and reflection slopes (measured on zero-offset time sections) of PP-waves and the reconstructed SS-waves. We examine a wide range of homogeneous TI models with a tilted symmetry axis (including horizontal transverse isotropy, or HTI) and establish the conditions needed for stable parameter estimation. Then the method is applied to layered TI models to estimate the internal medium parameters and the shapes of reflecting interfaces.

**Methodology of stacking-velocity tomography**

The goal of the tomographic algorithm introduced here is to estimate the anisotropic subsurface model using wide-azimuth measurements of stacking (moveout) velocities of PP- and SS-waves on moderate-length CMP spreads (i.e., spreads close to the reflector depth). Therefore, this approach can be classified as anisotropic multicomponent stacking-velocity tomography. Although limiting the input data to stacking velocities excludes the far-offset information from analysis, it has important advantages over conventional tomography.

First, azimuthally-varying moveout velocity, described by the NMO ellipse, can be computed by tracing only one zero-offset ray per common midpoint and per reflector, which makes anisotropic tomography computationally feasible (Grechka and Tsvankin, 1999b; Grechka et al., 2000a,b). Second, even for lower-symmetry systems NMO ellipses can be described by semi-analytic expressions providing valuable insight into the parameter combinations constrained by a certain set of input data (e.g., Grechka and Tsvankin, 1999c). Third, stacking-velocity tomography can be performed locally on the horizontal scale of a single CMP gather, and the velocity field can be estimated separately for relatively small blocks containing several adjacent common midpoints. Within each block, the layers can be treated as homogeneous, and the interfaces can be approximated by simple smooth surfaces, such as low-order polynomials. Then global smoothing can be applied to build the laterally varying anisotropic velocity field and reflecting interfaces.

We implemented the multicomponent tomographic procedure for TI media composed of homogeneous layers separated by plane or smooth curved interfaces. The algorithm includes the following main steps:

1. Picking PP and PS traveltimes from pre-stack 3D data volumes and identifying the events reflected from the same interface.


3. Azimuthal velocity analysis to reconstruct the NMO ellipses of the PP- and SS-waves (Grechka and Tsvankin, 1999a).

4. Inversion of the NMO ellipses, zero-offset traveltimes, and reflection slopes for the interval anisotropic parameters by extending to multicomponent data the approach of Grechka et al. (2000a,b).

The data vector used in the inversion for an N-layered TI medium is given by

\[ d(Q, Y, n) \equiv \{ \tau_Q(Y, n), p_Q(Y, n), W_Q(Y, n) \}, \]  

where \( Q = PP \) or \( SS \) is the mode type, \( Y = [Y_1, Y_2] \) is the CMP coordinate, \( n = 1, 2, ..., N \) is the reflector number, \( \tau_Q \) is the zero-offset traveltime, \( p_Q \) is the reflection slope on zero-offset time sections, and \( W \) are the 2x2
matrices (Grechka and Tsvankin, 1998) describing the NMO ellipses.

Our goal is to find the model vector \( \mathbf{m} \) which contains the interval medium parameters and the coefficients of the polynomials used to describe the model interfaces. For TI media with an unknown tilt of the symmetry axis, the inversion of \( PP \) and \( SVSV \)-waves can be used to estimate six interval parameters — the symmetry-direction \( P \)- and \( S \)-wave velocities \( V_{P0} \) and \( V_{S0} \), anisotropic coefficients \( \epsilon \) and \( \delta \), and two angles responsible for the symmetry-axis orientation.

In general, the parameter-estimation algorithm is organized in the same way as that introduced for \( PP \)-waves by Grechka et al. (2000a,b). For a given set of trial interval anisotropic parameters, the zero-offset traveltimes \( \tau_Q \) and the reflection slopes \( p_Q \) are used to compute the one-way zero-offset rays for all reflection events. Then the interfaces for the trial model are reconstructed by fitting 2-D polynomials to the termination points of the zero-offset rays. Finally, the interval parameters are obtained by minimizing the following objective function:

\[
\mathcal{F}(\mathbf{m}) \equiv \sum_{Q,y,n} || W_Q^{\text{exp}}(Y, n, m) - W_Q^{\text{meas}}(Y, n) ||^2.
\]

The norms in the function (3) contain the differences between the computed and measured NMO ellipses \( W \) for all modes and all reflectors at each CMP location.

### Inversion of \( PP \) and \( SS \) data for a homogeneous TI medium

Consider the model of a single homogeneous TI layer with a planar lower boundary (horizontal or dipping) and arbitrary orientation of the symmetry axis. The problem addressed here is whether or not wide-azimuth reflection traveltimes of \( PP \)- and \( SS \)-waves (i.e., \( SV \) reflections reconstructed from \( PP \) and \( PSS \) data) can be inverted for the symmetry-direction velocities \( V_{P0} \) and \( V_{S0} \), the parameters \( \epsilon \) and \( \delta \), and the axis orientation. It is convenient to study the feasibility of parameter estimation by applying the weak-anisotropy approximation to the NMO ellipses and zero-offset traveltimes.

The derivations have to be performed for \( P \)-waves only, because any kinematic signature of \( SV \)-waves for weak transverse isotropy can be obtained from the corresponding \( P \)-wave signature by making the following substitutions: \( V_{P0} \rightarrow V_{S0} \), \( \delta \rightarrow \sigma \), and \( \epsilon \rightarrow 0 \) (Tsvankin, 2001; see Table 1). A similar substitution rule for \( SH \)-waves is \( V_{P0} \rightarrow V_{S0} \), \( \delta \rightarrow \gamma \), and \( \epsilon \rightarrow \gamma \). However, \( SH \)-wave anisotropy is elliptical, and most kinematic signatures can be obtained in closed form without applying the weak-anisotropy approximation.

### VTI layer

<table>
<thead>
<tr>
<th>( P )</th>
<th>( SV )</th>
<th>( SH )</th>
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<tbody>
<tr>
<td>Kinematic parameters</td>
<td></td>
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<tr>
<td>( V_{P0} )</td>
<td>( V_{S0} )</td>
<td>( V_{S0} )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0</td>
<td>( \gamma )</td>
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<tr>
<td>( \delta )</td>
<td>( \sigma )</td>
<td>( \gamma )</td>
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<tr>
<td>Time-processing parameters (VTI)</td>
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<tr>
<td>( V_{\text{nmo},P} )</td>
<td>( V_{\text{nmo},SV} )</td>
<td>( V_{\text{nmo},SH} )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>( -\sigma )</td>
<td>( 0 )</td>
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</table>

**Table 1.** Correspondence between the parameters responsible for the kinematic signatures of \( P \)-, \( SV \)-, and \( SH \)-waves in weakly anisotropic TI media.

The substitution rules for different modes hold (in the weak-anisotropy limit) for the processing parameters as well. Alkhalifah and Tsvankin (1995) showed that \( P \)-wave time processing in VTI media with a laterally homogeneous overburden above the target reflector is governed by the zero-dip \( P \)-wave NMO velocity

\[
V_{nmo,P}^2 = V_{P0}^2 (1 + 2\delta)
\]

(4)

and the anellipticity coefficient

\[
\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}.
\]

(5)

Note that this result is valid for any strength of velocity anisotropy. Time processing of \( SV \)-waves for weakly anisotropic VTI media is then controlled by the zero-dip \( SV \)-wave NMO velocity

\[
V_{nmo,SV}^2 = V_{S0}^2 (1 + 2\sigma)
\]

(6)

and the parameter \( -\sigma / (1 + 2\sigma) \) [equation (1)] that plays the role of \( \eta \). Time processing of elliptically anisotropic \( SH \)-waves requires just the NMO velocity

\[
V_{nmo,SH}^2 = V_{S0}^2 (1 + 2\gamma)
\]

(7)

because the quantity corresponding to \( \eta \) goes to zero. This well-known result stems from the fact that isotropic time-processing algorithms are entirely valid for elliptical anisotropy (Dellinger and Muir, 1988; Alkhalifah and Tsvankin, 1995).

Suppose the data include multi-azimuth (3-D) traveltimes of \( PP \)- and \( PSS \)-reflections from a planar dipping interface below a homogeneous VTI layer. Without losing generality, the dip plane of the reflector is assumed to coincide with the coordinate plane \( [x_1, x_2] \). Applying the methodology of Grechka and Tsvankin (2001a), we reconstruct the traveltimes of the pure \( SS \) (\( SVSV \) reflections and use azimuthal moveout analysis (Grechka and Tsvankin, 1999a) to obtain the NMO ellipses of both \( PP \)- and \( SS \)-waves.
Since the model is symmetric with respect to the dip plane, the axes of the NMO ellipses have to be aligned within the dip and the strike directions (Grechka and Tsvarkin, 1998). The linearized approximations for the semi-axes of the \( P \)-wave NMO ellipse are given by (Alkhalifah and Tsvarkin, 1995; Grechka and Tsvarkin, 1998)

\[
V_{nmo,p}^{2} = \frac{V_{nmo,p}^{2}}{1 - y_{p}} \times \left[ 1 + \frac{2 \eta y_{p}}{1 - y_{p}} (6 - 9 y_{p} + 4 y_{p}^{2}) \right]
\]

and

\[
V_{nmo,p,strike}^{2}(p_{SV,1}) = V_{nmo,p}^{2} [1 + 2 \eta y_{p} (2 - y_{p})],
\]

where \( y_{p} \equiv p_{P,1}^{2} V_{nmo,p}^{2} \) and \( p_{P,1} \) is the horizontal slowness component of the \( P \)-wave zero-offset ray (or the dip component of the reflection slope); the strike component \( p_{P,2} = 0 \). Equations (8) and (9) can be inverted for the zero-dip NMO velocity \( V_{nmo,p} \) and the anisotropic coefficient \( \eta \), if the reflector dip (expressed through the slope \( p_{P,1} \)) is not too small. The inversion of the \( PP \)-wave NMO ellipse using the exact equations is discussed by Grechka and Tsvarkin (1998), who find that the dip should exceed 25° to ensure stable estimation of \( \eta \).

The weak-anisotropy approximations for the dip and strike components of the \( SV \)-wave NMO velocity can be obtained directly from equations (8) and (9) using the conversion rule from Table 1:

\[
V_{nmo,p}^{2} = \frac{V_{nmo,SV}^{2}}{1 - y_{SV}} \times \left[ 1 - \frac{2 \eta y_{SV}}{1 - y_{SV}} (6 - 9 y_{SV} + 4 y_{SV}^{2}) \right],
\]

\[
V_{nmo,SV,strike}^{2}(p_{SV,1}) = V_{nmo,SV}^{2} [1 - 2 \sigma y_{SV} (2 - y_{SV})].
\]

Here \( y_{SV} \equiv p_{SV,1}^{2} V_{nmo,SV}^{2} \) and \( p_{SV,1} \) is the horizontal slowness component of the \( SS \)-wave zero-offset ray; the strike component \( p_{SV,2} = 0 \). Similarly to the parameter estimation for \( PP \)-waves described above, equations (10) and (11) can be inverted for \( V_{nmo,SV} \) and \( \sigma \). Furthermore, substituting \( V_{nmo,SV} \) and \( \sigma \) into equation (6) yields the shear-wave vertical velocity \( V_{50} \). Then the zero-offset traveltimes and the reflection slope of the \( SS \) reflection can be used to reconstruct the depth and dip of the reflector.

Next, we demonstrate that adding this information to the traveltimes of \( PP \)-waves is sufficient for estimating the vertical velocity \( V_{P0} \) and the anisotropic coefficients \( \epsilon \) and \( \delta \). The equation of the planar reflecting interface can be written in the form

\[
b \cdot (x - r_{P}) = 0,
\]

where \( b \) is the unit vector normal to the reflector, and \( r_{P} \) is the coordinate of the \( PP \)-wave zero-offset reflection point in a Cartesian coordinate system \( x \) with the origin at the common midpoint. Similarly, for the zero-offset \( SS \) ray reflected from the point \( r_{SV} \) we have

\[
b \cdot (x - r_{SV}) = 0.
\]

Combining equations (12) and (13) yields

\[
b \cdot r_{P} = b \cdot r_{SV}.
\]

The vector \( b \) can be replaced in equation (14) by the normalized (so that the magnitude is equal to unity) slowness vectors of the zero-offset \( PP \) and \( SS \) rays because the slowness (or phase-velocity) vectors of pure-mode reflections at zero offset are orthogonal to the reflector. Also, the coordinates \( r_{P} \) and \( r_{SV} \) can be expressed through the group-velocity vectors of the \( PP \)-and \( SS \)-waves. Further linearization of equation (14) in the anisotropic parameters leads to

\[
\tau_{P} V_{nmo,p}^{2} \left[ 1 - \delta \left( 1 - p_{P,1}^{2} V_{nmo,p}^{2} \right) + \eta p_{P,1}^{4} V_{nmo,p}^{4} \right]
= \tau_{SV} V_{nmo,SV}^{2} \left[ 1 - \sigma \left( 1 - p_{SV,1}^{2} V_{nmo,SV}^{2} \right) + \eta p_{SV,1}^{4} V_{nmo,SV}^{4} \right],
\]

where \( \tau_{P} \) and \( \tau_{SV} \) are the zero-offset traveltimes of the \( PP \)-and \( SS \)-arrivals. Note that the quantities on the left- and right-hand sides of equation (15) comply with the conversion rule in Table 1. The result equivalent to equation (15) can be obtained by using Snell’s law for the zero-offset \( P \)- and \( SV \)-rays instead of equations (12) and (13).

Although the parameters \( V_{nmo,p}, V_{nmo,p}^{2} \) and \( \eta \) and \( \sigma \), which can be obtained from the \( PP \)- and \( SS \)-wave NMO ellipses [equations (8)–(11)], are sufficient to find the vertical velocities \( V_{P0}, V_{50} \) and the coefficients \( \epsilon \) and \( \delta \), the inversion of the \( PP \)-wave NMO ellipse requires reflector dips of at least 25° (Alkhalifah and Tsvarkin, 1995; Grechka and Tsvarkin, 1998). (The NMO ellipse of the \( SS \)-wave is more sensitive to dips than that of the \( PP \)-wave because of relatively large values of \( \sigma \).) Suplementing equations (8)–(11) with equation (15) adds another constraint on the anisotropic parameters and helps to obtain an accurate result for smaller dips. Indeed, the numerical tests below confirm that dips as small as 15° are sufficient for stable estimation of the VTI parameters.

Figure 1 illustrates application of our methodology to noise-contaminated wide-azimuth \( PP \) and \( SS \) (\( SV \) and \( SV \)) traveltimes (the \( SS \) traveltimes are supposed to be reconstructed from the \( PP \) and \( SS \) data) generated for a homogeneous VTI layer with the lower boundary dipping at 15°. We applied nonlinear inversion (the
Gauss-Newton method) based on the exact equations for the NMO ellipses, zero-offset traveltimes and reflection slopes; the objective function is given in equation (3). The dots in Figure 1 mark the estimated VTI parameters for different realizations of the noise added to the input data. The standard deviations in the inverted parameters (1% for $V_{P0}$ and $V_{S0}$, 0.03 for $\epsilon$ and 0.02 for $\delta$) indicate that the noise does not get amplified by the parameter-estimation procedure, so the inversion is reasonably stable. Note that the estimated values of $\epsilon$ and $\delta$ cluster near the line of the correct parameter $\eta \approx \epsilon - \delta$ because the difference $\epsilon - \delta$ is well-constrained by both $PP$ and $SS$ traveltimes (Tsvankin and Grechka, 2000a).

The only parameter of VTI media that cannot be obtained from $PP$ and $SV$-$SV$ data is $\gamma$ — the anisotropic coefficient responsible for the elliptical anisotropy of $SH$-waves. Tsvankin and Grechka (2000b) showed that $\gamma$ can be determined from converted $PSH$-waves, which are generated for all azimuthal directions outside the dip plane. The methodology introduced here can be applied to estimate $\gamma$ from the NMO ellipses of the pure $SH$-wave reflections reconstructed from the $PP$ and $PSH$ data. Thus, the combination of $P$, $PSV$- and $PSH$-wave reflection data makes it possible to estimate all five VTI parameters and build the VTI model in the depth domain.

**HTI layer**

Contreras et al. (1999) studied the inversion of wide-azimuth $PP$ data for HTI media and showed that the symmetry-direction velocity $V_{P0}$, the coefficients $\epsilon$ and $\delta$ (or $\epsilon^V$ and $\delta^V$, see Tsvankin, 1997), and the azimuth $\beta$ of the horizontal symmetry axis can be found from the $PP$-wave NMO ellipses from a horizontal and a dipping reflector. However, the need to use two different dips for each depth interval makes this algorithm difficult to implement in practice. Our approach is designed to estimate the HTI parameters using the NMO ellipses of $PP$- and $SS$($SV$-$SV$)-waves* from a single reflector that can be either horizontal or dipping.

The inversion for a **horizontal** HTI layer confirms the results of Tsvankin (1997) who pointed out that the combination of wide-azimuth $PP$- and $SS$-wave moveout data is sufficient for estimating the symmetry-direction velocities $V_{P0}$ and $V_{S0}$ and the parameters $\epsilon$ and $\delta$. For this model, the velocities $V_{P0}$ and $V_{S0}$ can be found directly from normal moveout because they are equal to the corresponding NMO velocities measured in the direction orthogonal to the symmetry axis (i.e., in the isotropy plane). Typical results of inverting noise-contaminated $PP$ and $SS$ traveltimes for the parameters of a horizontal HTI layer are shown in Figure 2. The standard deviations in all estimated parameters, including the azimuth $\beta$ of the symmetry axis not shown on the plot, are quite small (the deviation in $\beta$ is 0.6°).

To examine the inversion for **dipping** interfaces, we adapted for **$SS$-waves** (see the substitution rule in Table 1) the weak-anisotropy approximations for $PP$-wave NMO ellipses given by Contreras et al. (1999). These results are similar to the ones discussed in the previous section for VTI media and, therefore, are not given here. Both the theoretical analysis and the inversion based on the exact equations (see Figure 3) prove that the parameter estimation remains stable for the whole range of dips from 0° to 90°.

* Note that by the “$SV$-wave” in both HTI and TTI media we mean the mode polarized in the plane formed by the slowness vector and the symmetry axis. If the symmetry axis is horizontal or tilted, this plane is no longer vertical, but we still prefer to keep the notation commonly used for VTI media.
The dots of the estimated parameter values in Figure 3 form smaller clouds than those in Figure 1, which indicates that the inversion of dipping events for HTI media is more stable compared to that for VTI media. Obviously, for a horizontal VTI layer the inversion for \( V_{P0} \), \( \varepsilon \) and \( \delta \) cannot be performed at all. We also noticed that the inversion algorithm for HTI media converges much more rapidly towards the correct model than it does for VTI media.

**TTI layer**

The parameter-estimation problem for transverse isotropy with a tilted symmetry axis (TTI media) includes only one additional unknown compared to the HTI case – the tilt \( \nu \). This, however, makes the inversion substantially more ill-posed than that for HTI media because the pure-mode NMO ellipses are nonlinear functions of \( \nu \), even for weak anisotropy. Grechka and Tsvankin (2000) found a nonlinear dependence on the tilt in the weak-anisotropy approximations for the PP-wave NMO ellipse in TTI media, and adaptation of their equations for shear modes using Table 1 leads to the same result for both SV- and SH-waves. Therefore, the misfit function for the NMO ellipses that has to be minimized in the nonlinear inversion may have local minima. The multimodal nature of the misfit (objective) function usually requires performing several inversions starting from different points in the model space.

The schematic summary of our numerical results in Figure 4 illustrates the influence of the tilt of the symmetry axis and reflector dip on the stability of parameter estimation in TTI media. When the tilt is small, the properties of TTI media are similar to those for vertical transverse isotropy, and the inversion cannot be carried out for small reflector dips. With increasing tilt, the TTI model approaches HTI, and the parameter estimation generally becomes more stable. However, to achieve the accuracy comparable to that for HTI media, it is necessary to fix the tilt of the symmetry axis at the correct value (in HTI media, the tilt is known to be equal to 90°).

As expected, there is a relatively broad intermediate area of moderate tilts and dips where the parameter estimation is theoretically possible but relatively unstable. Such an example is shown in Figure 5 where the tilt of the symmetry axis is 20° and the reflector dip is 30°. As in the previous examples, we inverted the NMO ellipses, zero-offset traveltimes and reflection slopes of the PP- and SS-waves contaminated with Gaussian noise. Although the obtained parameters scatter around the correct values, the standard deviations (4% in \( V_{P0} \) and \( V_{S0} \), 0.05 in \( \varepsilon \), 0.04 in \( \delta \), 1.5° in \( \beta \), and 0.8° in \( \nu \)) are larger than those for VTI or HTI media and the same dip.

The stability of the inversion for models may be increased by adding SH-wave NMO ellipses and zero-offset traveltimes to the input data. The traveltimes of pure SH reflections can be obtained using PSH converted waves generated for source-receiver azimuths outside of the vertical symmetry plane(s) of the model (see the discussion above).

Thus, the inversion of multicomponent (PP and PS), multiazimuth reflection data for the parameters of homogeneous TI media is unique for a broad range of reflector dips and tilts of the symmetry axis. The highest stability is observed for near-horizontal orientations of the symmetry axis, while for a vertical and tilted symmetry axis the inversion generally becomes more stable with increasing dip.

**Parameter estimation for layered TI media**

Here we describe application of the multicomponent
Figure 5. Results of the inversion (dots) of PP and SS traveltimes data for a dipping TTI layer (the dip is 30°). The data were contaminated by noise with the same standard deviations as those in Figure 1. The correct layer parameters are marked by the crosses. The velocities in the symmetry-axis direction $V_{p0}$ and $V_{S0}$ are in km/s, the tilt $n$ and azimuth $\beta$ of the symmetry axis are in degrees ($\beta$ is measured with respect to the dip plane).

Numerical examples

The input data from Figure 6 were distorted by Gaussian noise with the standard deviation of 2% for the NMO velocities and 1% for the zero-offset traveltimes and reflection slopes. The inversion results for 100 realizations of the input data in Figure 7 indicate that the noise does not get amplified by the parameter-estimation procedure. The standard deviations in the inverted parameters are less than 1% for $V_{p0}$ and $V_{S0}$ (not shown), and about 0.01 for $\epsilon$ and $\delta$.

Note that neither of the interfaces in the model from Figure 8 has steeply dipping segments. The high sensitivity of the SS-wave NMO ellipse to reflector dip and the addition of the ratio of the zero-offset PP and SS traveltimes [equation (15)] ensures the stability of the joint inversion of wide-azimuth PP and SS data for mild dips as small as 15°. Including the NMO ellipses of the SH-waves may help to constrain the anisotropic coefficient $\gamma$ and build a complete VTI model in depth.

Figures 8 and 9 show the tomographic inversion of multicomponent data for a more complicated model that includes VTI, HTI and TTI layers. Despite the larger error bars for deeper horizons, the overall stability of the algorithm is satisfactory. A general increase in errors with depth, caused by the relatively small contribution of
the deeper layers to the reflection traveltimes from their lower boundaries, is typical for all Dix-type algorithms. Also, although we do not differentiate Dix-type formulae explicitly to obtain the interval anisotropic coefficients, errors in the parameters of the upper layers still propagate into the inversion results for the deeper part of the section.

Influence of errors in the symmetry type

In the examples discussed above it was assumed that the type of anisotropy (i.e., anisotropic symmetry) in each layer was known in advance. Since it is not necessarily the case in practice, it is instructive to examine the inversion of error-free data using an intentionally incorrect anisotropic symmetry in one of the layers.

We specified a TI model composed of two VTI layers on top of an HTI layer (Figure 10) with the interval parameters listed in the top row of Table 2. Then the tomographic parameter estimation was performed under the erroneous assumption that the bottom (HTI) layer has VTI symmetry (the second row in Table 2). As expected, the inversion produced seriously distorted values of the symmetry-direction velocities $V_{70.3}$ and $V_{50.3}$ and the anisotropic coefficients $\epsilon_3$ and $\delta_3$. It is interesting that the parameters of the two upper layers are also inaccurate because the error in the bottom layer gets distributed throughout the whole model to minimize the objective function (3). Hence, errors can propagate not only downward (accumulate with depth) but also upward, albeit with a substantially smaller amplification.

Another implication of this observation is that it might be preferable to perform tomographic inversion in a layer-stripping mode, starting with estimation of the parameters of the subsurface layer using the most shallow $PP$ and $PS$ reflections. Then, fixing the obtained values, we can determine the interval parameters of the second layer by inverting the traveltimes from its bottom, and the parameter-estimation procedure continues downward. In addition to eliminating upward error propagation from the deeper layers, the stripping approach is computationally efficient because only a few unknowns need to be found at each stage (i.e., for each layer). The main shortcoming of layer stripping is implicit reliance on the assumption that the reflections from the bottom of the layer contain full information about the layer parameters. Since it is not always the case for $PP$-waves (Le Stunff et al., 1999; Grechka et al., 2000a,b), one can expect that for some models the layer-stripping method may create ambiguity in the joint inversion of $PP$ and $PS$ data.

In the second test, the top (VTI) layer was assumed to have HTI symmetry. Then, in addition to the expected significant errors in the parameters of the top layer, we also obtained distorted parameters in the deeper layers (see the third row in Table 2). This test underscores the importance of choosing the right type of anisotropy in the overburden, because any errors in the shallow part of the section will propagate through the whole model.

An alternative to assuming a specific type of anisotropy is to adopt the more general tilted TI (or even orthorhombic) model from the outset of the inversion. The correct type of anisotropy can then be identified from the determined orientation of the symmetry axis.
Table 2. Correct parameters of the model composed of two VTI layers on top of an HTI layer (top row) and the inversion results based on erroneous assumptions about the symmetry in one of the layers.

![Figure 10. Zero-offset rays of the PP- and SS(SV SV)-waves for a model composed of two VTI layers on top of an HTI layer. The interval parameters are listed in the top row of Table 2.](image)

(for TTI media) or relationships between the estimated anisotropic coefficients (for orthorhombic media). However, according to the above numerical results, the need to determine the orientation of the symmetry axis often reduces the stability of the algorithm.

**Influence of heterogeneity**

Vertical and lateral velocity variations cause the most difficult problems in isotropic reflection tomography. For example, certain types of vertical velocity variations can never be resolved from the reflection traveltimes, no matter how the inversion is performed (e.g., Goldin, 1986). Given the complexity of this problem for anisotropic media, the discussion here is limited to a numerical example illustrating the errors in the estimated anisotropic parameters caused by heterogeneity.

Let us examine the tomographic inversion for the VTI overburden for the model from Table 2 (Figure 10) using only the reflections from the second interface. The effective parameters of the overburden change both vertically (since it actually consists of two layers) and laterally because the first interface is not horizontal. Although the vertical variation of the VTI parameters cannot be resolved without including the reflections the first interface, we can try to obtain estimates of the lateral variations of their effective values by performing the inversion for a range of CMP coordinates $Y_1$ (Figure 10).

The data $d(Q, Y, n)$ [equation (2)] were generated for $Q = P, SV, n = 2$ and CMP coordinates $Y_1 = [-0.6, -0.4, \ldots, 0.8, 1.0]$ km, $Y_2 = [-0.2, 0.2]$ km. In each inversion, we used four adjacent common midpoints (that form the corners of a rectangle) and assigned the estimated anisotropic coefficients to the center of the rectangle. Treating the overburden as a single homogeneous VTI layer yields the parameters displayed in Figure 11. Clearly, all estimated quantities vary laterally because the variations in the dip and depth of the intermediate interface make the overburden laterally heterogeneous. It is interesting that while the effective vertical velocities $V_p$ and $V_S$ (Figure 11a, b) can be regarded as certain averages of the interval velocities, the best-fit anisotropic coefficients $\epsilon$ and $\delta$ often lie outside the range of the corresponding interval coefficients (Figure 11c, d).

This result, which seems to be puzzling, can be explained by the fact that we operate with a variety of different averages in the stacking-velocity tomography. The "average," or effective parameters produced by the tomographic inversion process are not necessarily bounded by the minimum and maximum interval values. As an example, in Appendix A we show that the effective anisotropic coefficient $\delta$ derived from the PP-wave NMO velocity for a stack of plane homogeneous VTI layers can often exceed the maximum interval coefficient $\delta_n$. In the special case of isotropy, when all $\delta_n = 0$, the effective $\delta$ above vertically heterogeneous media is always positive (Grechka and Tsvankin, 2001c). The model in
in the effective properties which cannot be obtained by the straightforward arithmetic averaging of the corresponding interval (local) properties.

**Discussion and conclusions**

We introduced a multicomponent tomographic algorithm designed to invert wide-azimuth reflection travel-times for the interval parameters of transversely isotropic media. The method operates with reflection moveout of PP-waves and converted PS-waves, so it can be applied in multicomponent ocean-bottom surveys. PS data, however, are not used directly in the velocity-analysis procedure. Instead, they are combined with PP-wave moveout to reconstruct the reflection traveltimes of SS-waves using the model-independent kinematic algorithm of Grechka and Tsvankin (2001a).

The SS traveltime, in contrast to the more complicated moveout of converted waves, is symmetric with respect to zero offset and (on conventional-length spreads) can be described by the NMO velocity. Azimuthal semblance analysis of PP and SS traveltimes on CMP gathers produces the NMO ellipses (i.e., azimuthally varying stacking velocities) and zero-offset traveltimes, which serve as the input to the tomographic inversion. Although this “stacking-velocity tomography” of PP and SS data excludes the far-offset information (i.e., nonhyperbolic moveout) from analysis, it has several advantages over conventional reflection tomography.

The first advantage, which is critically important in anisotropic media, is related to computational efficiency. Since the NMO ellipse (and, therefore, the multiazimuth, multioffset hyperbolic moveout as a whole) can be computed by tracing only one zero-offset ray for each reflection event at a given CMP location, the number of rays to be generated in forward modeling is reduced by at least an order of magnitude, which makes anisotropic traveltime tomography computationally feasible. Second, semi-analytic expressions for the NMO ellipse that can be derived even for arbitrary anisotropy (Grechka and Tsvankin, 1999b; Grechka et al., 1999) help to identify the parameters (or the parameter combinations) constrained in the inversion of NMO (stacking) velocities. Third, restricting the range of source-receiver offsets reduces the influence of lateral heterogeneity on reflection traveltimes, and the velocity field can be estimated separately for homogeneous blocks of relatively small lateral extent.

Here, the multicomponent tomography was implemented for a stack of transversely isotropic layers separated by smooth interfaces. The detailed analysis for a homogeneous TI medium and numerical testing for layered models show that for a range of reflector dips and tilt angles of the symmetry axis the combination of PP

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**Figure 11.** Effective parameters (dots) of the VTI overburden estimated using the reflections from the second interface in Figure 10. The correct parameters of the VTI layers that make up the overburden are marked by the dashed lines. The coordinate Y₁ (in km) is defined in the text.
and \textit{PSV} (or \textit{SVS}V) data can be used to build anisotropic models for \textit{depth} processing. The most notable exception is horizontally layered VTI media, where even long-spread (nonhyperbolic) moveout of \textit{PP}- and \textit{PSV}-waves does not constrain the vertical velocities (Greczka and Tsvankin, 2001b). On the other hand, for HTI media the ray tracing procedure is quite stable for both horizontal and dipping reflectors. It should be emphasized that the parameter-estimation results can be compromised by assuming the wrong anisotropic symmetry in one of the layers (e.g., VTI instead of HTI). In principle, such errors can be avoided by using the most general TTI model in the inversion, but the need to estimate the tilt often reduces the stability of the algorithm.

For a restricted class of layered VTI models with dipping or irregular interfaces \textit{PP} reflection data alone can be used to determine the depth scale of the model and parameters \(\epsilon\) and \(\delta\) (Greczka et al., 2000a,b). This inversion, however, breaks down if the difference \(\epsilon - \delta\) is small, and the anisotropy is close to elliptical. As follows from our results, combining \textit{PP}- and \textit{PSV}-waves resolves this ambiguity (in the presence of reflector dip). Extending the argument of Dellaire and Muir (1988), we can state that since the velocity function of \textit{SV}-waves in elliptical media is purely isotropic, it does not allow the “stretching” of the model in the vertical direction that causes the depth uncertainty for \textit{PP}-waves.

Some common features of geologic formations, such as small-scale velocity heterogeneity, cannot be incorporated into our models because of the limited spatial and amplitude resolution of seismic data. Those features, however, do influence the reflection traveltimes and can significantly alter the values of the estimated parameters. We demonstrated that if heterogeneity is not properly accounted for, the inverted effective parameters providing the best fit to the input data may lie outside the range determined by the corresponding minimum and maximum interval (local) values. Therefore, the anisotropic parameters obtained from the tomographic inversion may bear a significant imprint of the adopted subsurface model.

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APPENDIX A: Effective parameter $\delta$ for layered VTI media

As an example of the relationship between the effective and interval anisotropic parameters, consider the effective $P$-wave NMO velocity $\langle V_{\text{anno}, p} \rangle$ for a stack of horizontally homogeneous VTI layers. (Here $\langle a \rangle$ denotes the effective value of the parameter $a$.) The effective value $\langle V_{\text{anno}, p} \rangle$ can be found using the Dix (1955) averaging of the interval NMO velocities

$$\langle \tau \rangle (V_{\text{anno}, p})^2 = \sum_n \tau_n V_{\text{anno}, p, n}^2,$$  \hspace{1cm} (A1)

where $\tau_n$ are the interval one-way zero-offset traveltimes, $V_{\text{anno}, p, n}$ are the interval zero-dip NMO velocities, and

$$\langle \tau \rangle = \sum_n \tau_n$$  \hspace{1cm} (A2)

is the total (effective) zero-offset traveltime. $V_{\text{anno}, p, n}$ are expressed through the interval vertical velocities $V_{p, n}$ and anisotropic coefficients $\delta_n$ in equation (4):

$$V_{\text{anno}, p, n}^2 = V_{p, n}^2 (1 + 2 \delta_n).$$  \hspace{1cm} (A3)

The products of the interval vertical velocities $V_{p, n}$ and zero-offset traveltimes $\tau_n$ yield the layer thicknesses

$$h_n = V_{p, n} \tau_n.$$  \hspace{1cm} (A4)

The effective vertical velocity can be written as

$$\langle V_{p} \rangle = \frac{\langle h \rangle}{\langle \tau \rangle} = \frac{\sum_n h_n}{\sum_n \tau_n} = \frac{\sum_n h_n}{\sum_n V_{p, n} \tau_n}.$$  \hspace{1cm} (A5)

Using equations (A4) and (A5), we find

$$\left[ \frac{1}{\langle V_{p} \rangle} \right]^{-1} = \langle V_{p} \rangle = \frac{\langle h \rangle}{\sum_n h_n V_{p, n}} = \left[ \frac{1}{\langle h \rangle} \sum_n h_n V_{p, n} \right]^{-1}.$$  \hspace{1cm} (A6)

Equation (A6) shows that the effective vertical velocity $\langle V_{p} \rangle$ is the harmonic average of the interval vertical velocities $V_{p, n}$.

Next, we introduce the effective anisotropic coefficient $\langle \delta \rangle$ defined as in equation (A3),

$$\langle V_{\text{anno}, p} \rangle^2 = \langle V_{p} \rangle^2 (1 + 2 \langle \delta \rangle).$$  \hspace{1cm} (A7)

Our goal is to find the relationship between the effective $\langle \delta \rangle$ and the interval coefficients $\delta_n$. Substituting equations (A3), (A4), (A5), and (A7) into the Dix formula (A1) leads to

$$\langle V_{p} \rangle (1 + 2 \langle \delta \rangle) = \frac{1}{\langle h \rangle} \sum_n h_n V_{p, n} (1 + 2 \delta_n),$$  \hspace{1cm} (A8)

or

$$1 + 2 \langle \delta \rangle = \frac{\frac{1}{\langle h \rangle} \sum_n h_n V_{p, n}}{\langle V_{p} \rangle} = \frac{\frac{1}{\langle h \rangle} \sum_n h_n V_{p, n} \delta_n}{\langle V_{p} \rangle} + 2.$$  \hspace{1cm} (A9)

Note that equation (A9) is exact. The first term on the right-hand side is the ratio of the arithmetic and harmonic averages of the vertical velocities, which is always
greater than or equal to unity. Therefore, \( \langle \delta \rangle \) satisfies the following inequality:

\[
\langle \delta \rangle \geq \frac{1}{\langle V_{P0} \rangle} \sum h_n V_{P0,n} \delta_n .
\] (A10)

In particular, if \( \delta_n \) is constant throughout the section,

\[
\delta_1 = \delta_2 = \ldots = \delta_n = \ldots = \delta ,
\] (A11)

inequality (A10) yields

\[
\langle \delta \rangle \geq \delta .
\] (A12)

The equality \( \langle \delta \rangle = \delta \) is reached only if the interval velocities \( V_{P0,n} \) are equal, which means that the stack of the layers degenerates into a homogeneous medium. Hence, the effective \( \langle \delta \rangle \) always overestimates the interval values of this parameter.

In the special case of isotropy \( \langle \delta \rangle = 0 \),

\[
\langle \delta \rangle \geq 0 .
\] (A13)

According to inequality (A10), isotropic layering creates an effective VTI medium with a positive parameter \( \delta \). For a more detailed discussion of this model, see Grechka and Tsvankin (2001c).

The results of this appendix may help to explain the well-known discrepancy between the laboratory measurements on shale cores, which give both positive and negative values of \( \delta \) (Thomsen, 1986; Vernik and Lin, 1997), and predominantly positive \( \delta \) values obtained from surface seismic measurements (e.g., Alkhalifah et al., 1996).

**Two VTI layers**

Here, we present estimates of the effective parameter \( \langle \delta \rangle \) for a simple model composed of two horizontal VTI layers with equal thicknesses \( h_1 = h_2 \). In this case, equation (A9) reduces to

\[
1 + 2 \langle \delta \rangle = \frac{V_{P0,1} + V_{P0,2}}{4 V_{P0,1} V_{P0,2}} \left[ V_{P0,1} + V_{P0,2} \right] + 2 \left( V_{P0,1} \delta_1 + V_{P0,2} \delta_2 \right) .
\] (A14)

Introducing the ratio of the vertical velocities

\[
v = \frac{V_{P0,2}}{V_{P0,1}}
\] (A15)

we rewrite equation (A14) as

\[
1 + 2 \langle \delta \rangle = \frac{1 + v}{4 v} \left[ 1 + v + 2 \left( \delta_1 + v \delta_2 \right) \right] .
\] (A16)

Equation (A16) allows us to express the effective \( \langle \delta \rangle \) through the velocity ratio \( v \):

\[
\langle \delta \rangle = \frac{(1 - v)^2}{8 v} + \frac{1 + v}{4 v} \left( \delta_1 + v \delta_2 \right) .
\] (A17)