Quartic moveout coefficient: 3-D description and application to tilted TI media
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Summary
Nonhyperbolic (long-spread) moveout provides essential information for a number of seismic inversion/processing
applications, particularly for parameter estimation in anisotropic media. Here, we present a 3-D analytic
expression for the quartic moveout coefficient \( A_4 \) which is responsible for the magnitude of nonhyperbolic moveout
of pure (non-converted) modes. Our result takes into account reflection-point dispersal on irregular interfaces
and is valid for arbitrarily anisotropic, heterogeneous media.

This general equation is used to study azimuthally varying
nonhyperbolic moveout of P-waves in a dipping trans-
versely isotropic (TTI) layer with an arbitrary tilt \( \nu \) of the
symmetry axis. The weak-anisotropy approximation for
\( A_4 \) is proportional to the anellipticity coefficient \( \eta \approx c - \delta \)
and does not depend on the individual values of the
Thomsen parameters \( \epsilon \) and \( \delta \). The azimuthal variation
of the quartic coefficient is governed by the tilt \( \nu \) and
reflector dip \( \phi \) and has a much more complicated character
than the NMO ellipse. If the symmetry axis is orthogonal
to the reflector, which is typical for thrust-and-fold belts,
the dip-line quartic coefficient rapidly decreases with \( \phi \),
while the strike-line \( A_4 \) for any dip is defined by the well-
known expression for a horizontal VTI (TI with a vertical
symmetry axis) layer. Therefore, the magnitude of nonhy-
perbolic moveout for this medium (and, typically, for other
TI media as well) is largest near the reflector strike. The
high sensitivity of the quartic moveout coefficient to the
parameter \( \eta \) and tilt \( \nu \) can be exploited in the anisotropic
inversion of wide-azimuth, long-spread P-wave data.

Introduction
The influence of heterogeneity or anisotropy causes de-
viations from hyperbolic reflection behavior which
sometimes cannot be ignored even for offsets-to-depth
ratios close to unity. A detailed overview of nonhyper-
bolic moveout analysis in anisotropic media can be found
in Tsukanik (2001). Most existing algorithms operating
with nonhyperbolic moveout are based on the equation
of Tsukanik and Thomsen (1994) that can be applied
to media with any symmetry if the quartic moveout
coefficient \( A_4 \) and the normal-moveout (NMO) velocity
\( V_{nmo} \) are known. For example, Al-Dajani and Tsukanik
(1998) derived the coefficient \( A_4 \) for a TI medium with a
horizontal symmetry axis (HTI) and extended the
Tsukanik-Thomsen equation to horizontally layered
HTI models. A more general approach to the analytic
description of \( A_4 \) that accounts for reflection-point dispersal at dipping or curved interfaces was developed
by Fomel and Grechka (2001).

Here, we introduce a 3-D expression for the quartic move-
out coefficient and use it to describe nonhyperbolic move-
out of P-waves for TI media with an arbitrary orientation
of the symmetry axis. Models with the symmetry axis
tilted away from the vertical (TTI, or tilted TI media)
are typical for fold-and-thrust belts such as the Canadian
Foot Hills and for sediments near the flanks of salt domes
(e.g., Isaac and Lawton, 1999; Tsukanik, 2001).

Nonhyperbolic moveout equation
To describe reflection traveltimes of pure (non-converted)
modes for the whole offset range used in seismic explora-
tion, Tsukanik and Thomsen (1994) suggested the following equation:

\[
\begin{align*}
\tau^2(X) &= \frac{X^2}{V_{nmo}^2} + \frac{A_4 X^4}{1 + AX^2},
\end{align*}
\]

where \( t_0 \) is the zero-offset traveltime, \( X \) is the source-
receiver offset,

\[
A_4 = \frac{1}{2} \frac{d}{d(X^2)} \left( \frac{d(\tau^2)}{d(X^2)} \right)_{X=0}, \quad \text{and} \quad A = \frac{A_4}{V_{hor}^2 - V_{nmo}^2},
\]

\( V_{hor} \) is the horizontal group velocity. The first two terms
in equation (1) describe the hyperbolic part of the move-
out curve, while the quartic coefficient \( A_4 \) is primarily
responsible for nonhyperbolic moveout. The coefficient \( A \)
makes the equation convergent at infinitely large offsets.

Equation (1) was originally developed for VTI media,
but its generic form makes it suitable for azimuthally
anisotropic models as well (Al-Dajani and Tsukanik,
1998). The effective \( V_{nmo} \) in heterogeneous, anisotropic
media can be found using the formalism for NMO ellipses
developed by Grechka et al. (1999). Therefore, the key
issue in applying equation (1) to a particular model is to
obtain the corresponding quartic moveout coefficient \( A_4 \).
The dependence of \( A_4 \) on the medium parameters also
provides useful insight into the properties of nonhyper-
bolic moveout.

General expression for the quartic moveout coefficient
Using the so-called normal-incidence-point (NIP) the-
orem (e.g., Fomel and Grechka, 2001), we derived
the quartic moveout coefficient \( A_4 \) for an arbitrar-
ily anisotropic, heterogeneous medium overlaying an
irregular reflector (Figure 1):

\[
A_4(L) = \frac{\tau_0}{48} \frac{\partial^2 \tau_0}{\partial y_p \partial y_m \partial y_n \partial y_n} L_p L_h L_m L_n.
\]
**Nonhyperbolic moveout in TTI media**

![Diagram of nonhyperbolic moveout in TTI media]

Fig. 1: Reflection traveltimes from an irregular interface are recorded in a multi-azimuth CMP gather over an arbitrarily anisotropic, heterogeneous medium. The derivation of the quartic moveout coefficient $A_4$ accounts for reflection-point dispersal.

\[
- \frac{\tau_0}{16} \frac{\partial^3 \tau_0}{\partial \xi \partial \eta \partial \phi} \left( \frac{\partial \tau_0}{\partial \xi \partial \eta} \right)^{-1} \frac{\partial^2 \tau_0}{\partial x \partial y} L_h L_m L_n + \frac{1}{16 \tau_0^2 V_{z_{\text{mono}}}(L)}.
\]

Here $y$ defines the CMP location, $x$ corresponds to the zero-offset reflection point, $\tau_0$ is the one-way zero-offset traveltimes, $L$ is a unit vector parallel to the CMP line and $V_{z_{\text{mono}}}(L)$ is the NMO velocity on the line $L$.

For relatively simple models, the traveltimes $\tau_0$ can be expressed explicitly as a function of $y$ and $x$, and $A_4$ can be evaluated in closed form (see below). If the medium is laterally heterogeneous and/or has a low anisotropic symmetry, all derivatives in equation (2) can be computed during the tracing of the zero-offset ray.

**Coefficient $A_4$ in a homogeneous TTI layer**

Here, we apply equation (2) to analysis of $P$-wave nonhyperbolic moveout in a homogeneous TTI layer above a plane dipping reflector. The symmetry axis is assumed to be confined to the dip plane of the reflector, which is the quite common in practice. Hyperbolic reflection moveout and the dependence of NMO velocity on the anisotropic parameters for this model are discussed in Tsvankin (1997, 2001). Following Tsvankin (1997), we parameterize the medium by the symmetry-direction velocities of $P$-waves ($V_{p0}$) and $S$-waves ($V_{s0}$) and Thomsen’s anisotropic coefficients $\epsilon$, $\delta$ and $\gamma$ specified with respect to the symmetry axis. The tilt $\nu$ of the symmetry axis is considered positive if the axis points towards the reflector (i.e., if the symmetry axis and the reflector normal deviate from the vertical in the same direction).

**Weak-anisotropy approximation for $A_4$**

To gain analytic insight into the dependence of $A_4$ on the model parameters, it is convenient to employ the weak-anisotropy approximation and linearize equation (2) in $\epsilon$ and $\delta$:

\[
A_4^{\text{TTI}} = \frac{-2\eta}{\tau_{P0}} V_{P0}^2 \left[ F(\alpha, \phi, \nu) + C \right],
\]

where $\eta \equiv (1 + 2\delta)/(1 + \epsilon - \delta) \approx \epsilon - \delta$ is the anellipticity parameter responsible for $P$-wave time-domain signatures in VTI media (Alkhalifi and Tsvankin, 1995). $C = 9/64$ is a constant, $\tau_{P0}$ is the two-way zero-offset $P$-wave traveltimes, $\alpha$ is the azimuth of the CMP line measured from the dip plane, $\phi$ is the reflector dip and $F$ is a rather complicated function of the angles $\alpha$, $\phi$ and $\nu$, which is analyzed for special cases below. Clearly, regardless of the tilt of the symmetry axis and reflector dip, the $P$-wave coefficient $A_4$ for weak transverse isotropy is controlled by a single anisotropic parameter $\eta$. If the medium is elliptical ($\eta = 0$), $A_4$ vanishes and reflector moveout becomes purely hyperbolic; this result remains valid for any strength of the anisotropy.

Figure 2 shows that the linearized equation (3) is sufficiently close to the exact quartic coefficient for relatively small values of the anisotropic parameters. As demonstrated by Tsvankin and Thomsen (1994), the weak-anisotropy approximation may readily lose its accuracy with increasing parameters $\epsilon$ and $\delta$. However, equation (3) can still be used for qualitative analysis of nonhyperbolic moveout in tilted TTI media. Note that equation (3) and all weak-anisotropy results below can be adapted for $SV$-waves by making the following substitutions (Tsvankin, 2001): $V_{P0} \rightarrow V_{S0}$, $\delta \rightarrow \sigma$, and $\epsilon \rightarrow 0$, where $\sigma \equiv (V_{P0}/V_{S0})^2(\epsilon - \delta)$.

**Dip and strike components of $A_4$**

Since the dip plane of the reflector contains the symmetry
Nonhyperbolic moveout in TTI media

axis of the overburden, it represents a vertical symmetry plane for the whole model. Therefore, the dip and strike directions of the reflector determine “the principal axes” of the azimuthally-varying coefficient $A_4$. For the dip line of the reflector ($\alpha = 0^\circ$), equation (3) yields

$$A_{4, \text{ dip}}^{\text{TTI}} = -\frac{2\eta}{\mu_0 V_{P0}^4} \cos^3 \phi \cos(4\nu - 3\phi).$$

(4)

Since the dip-line quartic coefficient is proportional to $\cos^3 \phi$, $A_{4, \text{ dip}}^{\text{TTI}}$ has a decreasing trend with dip. According to equation (4), nonhyperbolic moveout on the dip line vanishes if $(3\phi - 4\nu) = n\pi/2$ ($n = \pm 1, \pm 3, \pm 5, ...$). For a fixed reflector dip, $\cos(3\phi - 4\nu)$ goes to zero for two different values of the tilt $\nu$ between $0^\circ$ and $90^\circ$.

For purposes of anisotropic parameter estimation, it is more convenient to rewrite equation (4) as a function of the horizontal component $p$ of the slowness vector associated with the zero-offset ray (i.e., through the ray parameter responsible for reflection time slopes). It can be shown that both $A_{4, \text{ dip}}^{\text{TTI}}(p)$ and the dip-line NMO velocity $V_{\text{dip}}^{\text{TTI}}(p)$ are controlled by the NMO velocity from a horizontal reflector $V_{\text{dip}}^{\text{TTT}}(0)$ and the combinations $[\eta \cos 4\nu]$ and $[\eta \sin 4\nu]$. In principle, it may be possible to estimate those three parameter combinations, if the dip-line NMO velocity and the coefficient $A_4$ are measured for two different dips.

The coefficient $A_4$ on the strike line ($\alpha = 90^\circ$) has the form

$$A_{4, \text{ strike}}^{\text{TTI}} = -\frac{2\eta}{\mu_0 V_{P0}^4} \cos^4 (\phi - \nu).$$

(5)

While both the dip and strike components of $A_4$ are proportional to $\eta$, their dependencies on reflector dip $\phi$ and the symmetry-axis tilt $\nu$ are entirely different. $A_{4, \text{ strike}}^{\text{TTI}}$ goes to zero only if the symmetry axis is perpendicular to the reflector normal (i.e., the symmetry axis is confined to the reflecting plane). For example, if the reflector is vertical ($\phi = 90^\circ$), the strike-line quartic coefficient vanishes for VTI media ($\nu = 0^\circ$). Indeed, for such a model reflected rays are confined to the horizontal (isotropy) plane where velocity is independent of angle.

Azimuthal dependence of $A_4$

Unlike NMO velocity that has a simple elliptical azimuthal dependence, the variation of the quartic moveout coefficient with azimuth has a much more complicated character. The function $A_4(\alpha)$ may have multiple zeros whose positions strongly depend on both dip $\phi$ and tilt $\nu$. Figure 3 displays a polar plot with a typical azimuthal signature of $A_4$, with zeros at azimuths of $38^\circ$ and $142^\circ$. (The quartic coefficient and moveout signature as a whole have an azimuthal period of $180^\circ$.) The sign of $A_4$ changes from negative near the dip direction (i.e., for the horizontally oriented lobe) to positive for the lobe corresponding to $38^\circ < \alpha < 142^\circ$. On the whole, the magnitude of nonhyperbolic moveout for dipping reflectors is usually highest in the strike direction.

![Fig. 3: Azimuthally-varying coefficient $A_4$ for a TTI layer computed from equation (3). The tilt $\nu$ is $40^\circ$ and the reflector dip $\phi$ is $15^\circ$; the azimuth is measured from the dip plane marked by the arrow.](image)

Evidently, the azimuthal signature of the quartic coefficient can provide useful information for anisotropic parameter estimation. In particular, the azimuthal directions of the CMP lines with vanishing $A_4$ depend on certain combinations of $\nu$ and $\phi$ and can be used to constrain the orientation of the symmetry axis.

Symmetry axis orthogonal to the reflector

Because of the complicated structure of equation (3), below we focus on two special cases of practical importance. Models with the symmetry axis orthogonal to the reflector ($\phi = \nu$) are typical, for example, for dipping TTI shale layers in the Canadian Foothills (e.g., Isaac and Lawton, 1999). The azimuthal dependence of $A_4$ for $\phi = \nu$ has the form

$$A_{4, \phi = \nu}^{\text{TTI}}(\phi = \nu) = -\frac{2\eta}{\mu_0 V_{P0}^4} \cos^4 (\phi - \nu).$$

(6)

The quartic coefficient goes to zero when $|\cos 2\alpha| = 1/\sin \phi$, which is possible only on the dip line ($\alpha = 0^\circ$) of a vertical reflector ($\phi = 90^\circ$), which implies a horizontal symmetry axis). The dip and strike components of the quartic coefficient are given by

$$A_{4, \text{ dip}}^{\text{TTI}}(\phi = \nu) = -\frac{2\eta}{\mu_0 V_{P0}^4} \cos^3 \phi,$$

(7)

$$A_{4, \text{ strike}}^{\text{TTI}}(\phi = \nu) = -\frac{2\eta}{\mu_0 V_{P0}^4} \cos^4 \phi.$$  

(8)

Equation (8), which shows that the strike-line component of $A_4$ is independent of dip (or tilt), is well known for the special case of VTI media and a horizontal reflector, when $\phi = \nu = 0$ (Tsvankin and Thomsen, 1994). Whereas the strike-line component of $A_4$ does not change with dip, the dip-line component is proportional to $\cos^3 \phi$ [equation (7)]. Therefore, nonhyperbolic moveout for dipping reflectors rapidly decays away from the strike direction.
Nonhyperbolic moveout in TTI media

![Diagram showing moveout in TTI media](image)

Fig. 4: Azimuthally varying coefficient $A_4$ for a VTI layer computed from equation (9). Reflector dip is $30^\circ$; the dip direction is marked by the arrow.

Dipping reflector beneath a VTI layer

The linearized quartic coefficient in VTI media ($\nu = 0^\circ$) is expressed as

$$A_4^{VTI} = -\frac{2 \eta}{\rho_0^2} \frac{\cos^4 \phi}{V_4^2} \left(1 - 4 \sin^2 \phi \cos^2 \alpha\right). \quad (9)$$

For a horizontal reflector ($\phi = 0^\circ$), the model as a whole is azimuthally isotropic, and $A_4$ is determined by equation (8). A discussion of the exact (i.e., not limited to weak anisotropy) quartic moveout coefficient of both $P$- and $S$-waves in horizontally layered VTI media can be found in Tsvankin (2001). If the reflector is vertical ($\phi = 90^\circ$), $A_4$ vanishes regardless of the azimuth of the CMP line because reflected rays are confined to the horizontal isotropy plane where velocity is constant.

The coefficient $A_4$ for dipping reflectors goes to zero in azimuthal directions satisfying $|\cos \alpha| = 1/(2 \sin \phi)$. If the dip is equal to $30^\circ$, $A_4$ vanishes only for a single azimuth $\alpha = 0^\circ$ that corresponds to the dip plane (Figure 4); this analytic result is in good agreement with the numerical study of NMO velocity in Tsvankin (2001). For any dip between $30^\circ$ and $90^\circ$, $A_4 = 0$ in two different azimuthal directions. Finally, if the dip is smaller than $30^\circ$, the quartic coefficient is negative for any azimuth.

Discussion and conclusions

We have introduced an exact expression for the quartic moveout coefficient $A_4$ valid for arbitrarily anisotropic, heterogeneous media. Substitution of the quartic coefficient into the general moveout equation of Tsvankin and Thomsen (1994) yields a good approximation for nonhyperbolic moveout of $P$-waves in anisotropic media with realistic structural complexity. All quantities needed to calculate $A_4$ for any orientation of the CMP line can be obtained by tracing a single (zero-offset) ray. Computing the zero-offset ray is also sufficient to construct the NMO ellipse (Grechka et al., 1999), so our results make it possible to model long-spread moveout without multi-offset, multi-azimuth ray tracing.

The developed equation for $A_4$ provides valuable insight into $P$-wave nonhyperbolic moveout in TTI media. The azimuthal signature of the quartic coefficient (i.e., the azimuths of vanishing $A_4$ and the signs of $A_4$ in different azimuthal sectors) depends on the tilt $\nu$ of the symmetry axis and reflector dip $\phi$. For example, if $\nu = 0^\circ$ (VTI media) and reflector dip is mild ($\phi < 30^\circ$), $A_4$ is negative for all azimuths, and its magnitude increases away from the dip direction. For a $30^\circ$ dip, nonhyperbolic moveout in VTI media vanishes on the dip line; if the dip exceeds $30^\circ$, $A_4$ goes to zero in two different azimuths that do not coincide with either strike or dip directions. We also gave a detailed analysis of $A_4$ for the symmetry axis orthogonal to the reflector.

The dip components of both $V_{\text{omo}}$ and $A_4$ expressed through the ray parameter (i.e., through the reflection time slope) depend on the same three parameter combinations involving $\eta$, $\nu$, and the NMO velocity from a horizontal reflector. This result and the high sensitivity of the azimuthal signature of $A_4$ to the symmetry-axis orientation indicate that $P$-wave nonhyperbolic moveout may provide valuable information for velocity analysis in TTI media. Although the trade-off between $V_{\text{omo}}$ and $A_4$ makes quantitative estimates of the quartic coefficient relatively unstable (Tsvankin, 2001), the azimuthal variation of the sign of $A_4$ and the directions of vanishing or small nonhyperbolic moveout should be detectable from wide-azimuth reflection data.

References


