Velocity Analysis in the Presence of Amplitude Variation
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Summary
Conventional semblance velocity analysis (Taner and Koehler, 1969) is equivalent to modeling the prestack data with events that have hyperbolic moveout but no amplitude variation with offset (AVO). As a result of its assumption that amplitude is independent of offset, this method may not perform well for events with strong AVO, especially for events with polarity reversals, such as reflections from tops of some class 1 and class 2 sands. To account for AVO, the semblance method can be extended (Corcoran, 1989; Sarkar et al. 1999) by modeling the data with events that have both hyperbolic moveout and amplitude variation, expressed by AVO intercept and gradient (i.e., Shuey approximation). However, due to the extra degrees of freedom introduced by modeling AVO with the AVO-sensitive semblance, resolution of the estimated velocities tends to decrease. This is because the data can be modeled acceptably with a combination of incorrect velocities and incorrect AVO behavior, resulting in large values of AVO-sensitive semblance even at the wrong velocities. One solution, suggested by Sarkar (1999), was a hybrid method that combined the AVO-sensitive semblance and ordinary semblance methods through use of an arbitrary regularization parameter. Here, we also use the Shuey equation to describe the amplitude variation and, in addition, constrain the AVO parameters (intercept and gradient) be linearly related. With this constraint we are able to preserve velocity resolution, without the use of a regularization parameter, while improving the quality of velocity analysis in the presence of amplitude variation with offset.

Introduction
Corcoran (1989) and Sarkar et al. (1999) have shown that the semblance measure of Taner and Koehler (1969) is based on the implicit assumption that the wavelet does not vary with offset. Success with that measure lies in its model simplicity, which makes the procedure robust against noise. The semblance measure, evaluated at zero-offset time \( t_o \), is defined as

\[
S(v, t_o) = \frac{\sum_{x} \left( \sum_{y} D_v(t_1, x,y) \right)^2}{N \sum_{t_1} \sum_{x} D_v^2(t_1, x)},
\]

where \( D_v(t_1, x) = D(t_1, x) - \hat{D}(t_1, x) \), with \( \hat{D}(t_1, x) = \sqrt{t_1^2 + \frac{x^2}{a^2}} \); \( t_1 \) are zero-offset times within a window centered at \( t_o \); and \( N \) is the number of traces. \( D_v \) represents the moveout-corrected data at velocity \( v \) and \( D \) represents the data without NMO correction. Due to the assumption that amplitude is independent of offset, the measure is degraded in the presence of large amplitude variations. In the presence of polarity reversals, such as happens for some reflections from the tops of class 1 and class 2 sands, the method yields highly erroneous results.

One way to improve its ability to estimate velocities where amplitude has large variation with offset is to incorporate an amplitude-dependent function, such as that in Shuey (1985), in the semblance measure (Corcoran, 1989). This method successfully estimates velocities of seismic events that have, not only large AVO, but also polarity reversals. The increased parameterization, however, results in the loss of velocity precision, especially when amplitude does not vary substantially with offset (Sarkar et al., 1999). To improve the estimation of velocities for events with polarity reversals, Sarkar (1999) suggested solving the problem as a mixed-determined problem (Menke, 1984) that incorporates the use of a regularization term. That formalism showed how one could move from traditional semblance to an amplitude-dependent semblance by varying the regularization parameter. To implement that method successfully, whether or not amplitude variations are large, the user is required to choose a value for the regularization parameter — a task that is often difficult. Here, we describe a method that does not require a regularization term and preserves the good aspects of both traditional semblance and amplitude-dependent semblance, regardless of amplitude variation with offset.

AVO-sensitive semblance formalism
We define the generalized semblance as

\[
S(v, t_o) = 1 - \frac{\|M - D_v\|^2}{\|D_v\|^2},
\]

where, as before, \( v \) is a trial velocity and \( t_o \) is zero-offset time at the center of a semblance window. \( D_v = D_v(t_1, x) \) is the data after moveout correction with velocity \( v \), and \( M = M(t_1, x) \) is a suitably parameterized model of the moved-out data. The model parameters are obtained by minimizing \( \|M - D_v\|^2 \) or, equivalently, maximizing the generalized semblance. The vector norms include sums over all offsets and all zero-offset times in the semblance window.

The simplest parameterization for the model \( M \) in equation (2) is an offset-independent model; i.e., \( M(t_1, x) = A(t_1) \). Here, if \( N_t \) denotes the number of time samples in a semblance window, there are \( N_t \)
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parameters to be estimated for each semblance window. The optimal parameter values are simply \( A(t_1) = \frac{1}{p} \sum D_v(t_1, x) \), so, in this case, the generalized semblance [equation (2)] reduces to the traditional semblance [equation (1)].

To account for AVO, we choose a model parameterization based on the Shuey (1985) simplification of the Zoeppritz equation, i.e.,

\[
M(t_1, x) = A(t_1) + B(t_1) \sin^2 \theta_x, \tag{3}
\]

where \( \theta_x = \theta_x(t_1, v) \) is the angle of incidence at the reflector. This is the model used by both Corcoran (1989) and Sarkar et al. (1999). Here, the model contains \( 2N_t \) parameters for each semblance window, namely, all of the \( A(t_1) \) and \( B(t_1) \) coefficients. We shall refer to the resulting generalized semblance as the "AB semblance." Equation (3) models AVO behavior independently at each zero-offset time \( t_1 \). Unfortunately, this allows too much freedom to fit events using combinations of incorrect velocity and incorrect AVO, resulting in poor velocity resolution. To reduce the degrees of freedom, we assume that the semblance window contains just a single event or several events that all have identical AVO behavior; more specifically, we assume that the ratio \( K = \frac{B(t_1)}{A(t_1)} \) is constant throughout a semblance window. This leads to the model parameterization,

\[
M(t_1, x) = A(t_1)(1 + K \sin^2 \theta_x), \tag{4}
\]

where \( K \) is fixed within each semblance window. This model contains only \( N_t + 1 \) parameters for each semblance window. We refer to the resulting generalized semblance as "AK semblance."

Results

We applied the method and two alternative ones to the synthetic common-midpoint (CMP) gather shown in Figure 1, analyzing two reflections — one with almost no variation of amplitude with offset (event A) and the other with a polarity reversal (event B) — with the velocity analysis performed on a window centered on each event. The semblance curves obtained by the three methods are shown in Figures 2 and 3. For the event with no amplitude variation with offset, all three methods have the maximum semblance value near the correct stacking velocity (1685 m/s). Of the three methods traditional semblance, however, has the smallest width, indicative of its good velocity resolution, while AB semblance has the largest width, indicative of its poor velocity resolution. The AK semblance overlaps that of the traditional semblance near the correct velocity (where performance is most important), while it jumps to approximate the AB semblance for velocities far from the correct one. Thus, with little or no amplitude variation, the AK semblance measure matches the resolution and accuracy performance of traditional semblance without the need of a regularization parameter. Figure 3 shows the performance of the different methods in the presence of a polarity reversal. As expected, the traditional semblance method fails — in terms of velocity accuracy, resolution and standout of the peak. Both, the AB semblance and the AK semblance, however, peak near the correct stacking velocity, 1720 m/s. Hence, the AK semblance provides a method for estimating velocities of both events with polarity reversals and those without any amplitude variation.

Conclusions and Discussion

Polarity reversals are not common occurrences in CMP gathers, but when they do occur they are of special interest because they may indicate the presence of hydrocarbons. Hence, their detection may be crucial. We follow previous work based on Shuey's (1985) equation, which allows for amplitude variation with offset in the semblance measure. The method uses more fitting parameters than does traditional semblance, but fewer than that required in AB Semblance. In doing so, we
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![Figure 2](image1.png)

**Figure 2.** Semblance curves for event A (no amplitude variation). The correct stacking velocity (1685 m/s) is marked with an arrow.

![Figure 3](image2.png)

**Figure 3.** Semblance curves for event B (polarity reversal). The correct stacking velocity (1720 m/s) is marked with an arrow.

...have been able to preserve the good points of both traditional semblance and AB Semblance, which requires a regularization term. The pattern in Figure 2 is interesting: for an event with little amplitude variation with offset, it appears that AK semblance follows the traditional semblance near the correct velocity and then jumps to the AB semblance at velocities far from the correct one. This observation may be important for a possible extension of this work to improve semblance measures further.

Although both the AK and AB semblance measures can yield accurate velocities for events with large amplitude variation with offset and those with polarity reversals, neither is as robust as traditional semblance.

In particular, both are highly sensitive to the size and location of the semblance window used, and their performance degrades in the presence of overlapping events.

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**References**


