A platform for data mapping in scalar models of data acquisition

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Summary

Data mapping is a procedure for transforming data from a given input source/receiver configuration and macro earth model to a different output source/receiver configuration and background model. We propose, here, a "platform" for data mapping for scalar wavefields. This is a platform, only, because we do not carry out calculations in this paper that would specialize our formulas to any one of the possible data mapping options that we have in mind. We provide platform formulas in 3D and in 2.5D. For the latter, the platform equation has been checked by applying it to a Kirchhoff approximate representation of the upward scattered data from a single reflector and for an arbitrary source/receiver configuration. We find that the data mapping formalism maps this model data to Kirchhoff data in the output variables, with one exception. The method does not map the reflection coefficient. In this sense, this is a "true amplitude" formalism. We also propose a method for estimating the specular angle, both in the input configuration and in the output configuration.

Introduction

Data mapping is the term we use for the transformation of observed data from one prescribed source/receiver configuration and model of the propagating medium to another configuration and model. Our data mapping is a "true amplitude" process in the following sense. (i) Travel time geometrical spreading effects of the input configuration are transformed to those effects of the output configuration. (ii) Reflector curvature geometrical spreading effects of the input configuration are transformed to those effects of the output configuration. However, the reflection coefficient of the input configuration is preserved in the output data. The formalism also provides a mechanism for determining both the input and the output geometrical optics incidence angles of the reflection process, thereby providing a basis for amplitude versus angle (AVA) analysis.

The basic idea of these methods is to cascade an inversion formula with a modeling formula. The combined formula maps a given data set to another. The result is an integral over the variables of the input data set to produce an earth model combined with an integral over the coordinates of the earth model to produce the output data set. The operator is a function of the input and output parameters and the earth model variables, as well. The idea, then, is to carry out the integration over the last set of (earth modeling) variables, asymptotically, to obtain a weight that is a function of the input and output variables, only. This weight is then applied to the input data set to produce the output data set. It is this asymptotic analysis that can only be partially carried out in the absence of an explicit data mapping implementation; hence, the word, "platform," to characterize this point of departure from which to derive the mapping of specific implementations.

The forward model we use here is a volume integral form of the Kirchhoff approximation that derives from the introduction of the *singular function* of the reflecting surface into the standard surface integral form of the Kirchhoff approximation. The singular function is a Dirac delta function of normal distance from the reflector. We make a further approximation of the *obliquity factor* appearing in the Kirchhoff representation and we evaluate the reflection coefficient at specularity. These are the new ideas that we bring to the derivation of the basic mapping formalism as compared to earlier work by Tygel, et al [1998]. Once that is done, we have a straightforward inversion procedure from the volume Kirchhoff approximate forward model to an asymptotic inversion of observed data that produces that reflectivity function.

Some of the applications of data mapping that we have in mind are (i) offset continuation and TZO; (ii) transformation of common-offset data to common-shot data; (iii) "unconverting" mode-converted waves; (iv) velocity analysis; (v) wave equation datuming; (vi) mapping of swath data to zero azimuth; (vii) mapping of data from variable background propagation parameters to constant background parameters; (vii) combinations of the above. For example, consider the application of downward continuation of receivers (or sources)—a form of datuming. If we start with a prescribed shot gather with a given cable length of receivers, the range of validity of the downward continued data is of a smaller length. On the other hand, if we first create a single common shot data set from the full array of a common offset gather, this new data set effectively has a "cable length" equal to the length of the survey, typically, much longer than the cable length for each shot. Now, as we downward continue the receivers, the range of validity of the new data "shrinks" from an initial length equal to the survey length. One can expect that the data can be continued much deeper into the subsurface and maintain properly transformed geometrical spreading effects and traveltime corrections.

3D Data Mapping

In this section, we outline the derivation of the fundamental equation for space-frequency domain data mapping in 3D. We start from a volume integral form of the Kirchhoff approximation:

$$u_O(\xi_O,\omega_O) \sim i\omega_O F(\omega_O) \int a_O(\mathbf{x},\xi_O) \cdot \hat{\mathbf{n}}_R \cdot \nabla_x \tau_O(\mathbf{x},\xi_O) \cdot R(\mathbf{x},\mathbf{x}_s) z\delta(n_r)\, e^{i\omega_O \tau_O(\mathbf{x},\xi_O)} dV. \tag{1}$$

Here, the subscript O is used to denote output variables; later, input variables with subscript I will be introduced. The upward unit normal to the reflecting surface, S_R is denoted by $\hat{\mathbf{n}}_R$; $\delta(n_R)$, is the *singular function of the reflecting surface* S_R, with n_R measuring normal distance from any point on the reflector; $dV = dS_r dn_R$ is the differential volume element. The two component vector, ξ_O, parameterizes the source and receiver, $\mathbf{x}_s(\xi_O)$ and $\mathbf{x}_g(\xi_O)$, respectively; ω_O is the frequency. The phase and amplitude are given by

$$\begin{aligned} \tau_O(\mathbf{x},\xi_O) &= \tau(\mathbf{x},\mathbf{x}_s(\xi_O)) + \tau(\mathbf{x},\mathbf{x}_g(\xi_O)); \\ a_O(\mathbf{x},\xi_O) &= A(\mathbf{x},\mathbf{x}_s(\xi_O)) \cdot A(\mathbf{x},\mathbf{x}_g(\xi_O)), \end{aligned} \tag{2}$$

with the separate traveltimes and phases being solutions of appropriate eikonal and transport equations, respectively, with initial point, $\mathbf{x}_s$ or $\mathbf{x}_g$, and final point, $\mathbf{x}$. We allow for different propagation speeds in the eikonal equations (mode conversion) and transport equations. The amplitudes can also include products of transmission coefficients arising from interfaces above the surface, S_R.

We make two further approximations in this Kirchhoff representation, namely, we set

$$\hat{\mathbf{n}}_R \cdot \nabla_x \tau_O(\mathbf{x},\xi_O) = -|\nabla_x \tau_O(\mathbf{x},\xi_O)|, \tag{3}$$

and we replace the reflection coefficient by its stationary value. These approximations amount to replacing the functions by the values they assume at the stationary points of the phase. They are consistent with our intent of deriving an inversion for the reflectivity function, only; it acknowledges that we are emphasizing reflection data to detect reflecting surfaces. The product, $R\delta$, is just the *reflectivity function*, $\beta(\mathbf{x})$, of our inversion theory [Bleistein, et al., 1998, eq. 5.1.21], and we recast (1) as

$$u_O(\xi_O,\omega_O) \sim -i\omega_O F(\omega_O) \int a_O(\mathbf{x},\xi_O) |\nabla_x \tau_O(\mathbf{x},\xi_O)|\, \beta(\mathbf{x}) e^{i\omega_O \tau_O(\mathbf{x},\xi_O)} dV. \tag{4}$$

The derivation of an inversion formula will then follow along the lines used for inverting the volume integral forward model of Born-approximate data, but without the small perturbation constraints of that forward model; see Bleistein, et al [1998], for example. The reflectivity function is given by

$$\beta(\mathbf{x}) = \frac{1}{8\pi^3} \int d^2\xi_I \frac{|h(\mathbf{x},\xi_I)|}{a_I(\mathbf{x},\xi_I)|\nabla_x \tau_I(\mathbf{x},\xi_I)|} \cdot \int i\omega_I\, d\omega_I\, e^{-i\omega_I \tau_I(\mathbf{x},\xi_I)} u_I(\xi_I,\omega_I). \tag{5}$$

Here, the subscript I is used to denote input variables. The two component vector, ξ_I, parameterizes the source and receiver of the input data, $\mathbf{y}_s(\xi_I)$ and $\mathbf{y}_g(\xi_I)$, respectively; ω_I denotes the frequency of the input wave. The phase and amplitude are given by

$$\begin{aligned} \tau_I(\mathbf{x},\xi_I) &= \tau(\mathbf{x},\mathbf{y}_s(\xi_I)) + \tau(\mathbf{x},\mathbf{y}_g(\xi_I)); \\ a_I(\mathbf{x},\xi_I) &= A(\mathbf{x},\mathbf{y}_s(\xi_I)) \cdot A(\mathbf{x},\mathbf{y}_g(\xi_I)). \end{aligned} \tag{6}$$

The traveltimes and amplitudes are again solutions of the eikonal and transport equations, but now of the input physical model. Furthermore,

$$h(\mathbf{x},\xi_I) = \det \begin{bmatrix} \nabla_x \tau(\mathbf{x},\xi_I) \\ \dfrac{\partial}{\partial \xi_{I1}} \nabla_x \tau(\mathbf{x},\xi_I) \\ \dfrac{\partial}{\partial \xi_{I2}} \nabla_x \tau(\mathbf{x},\xi_I) \end{bmatrix} \tag{7}$$

is the Beylkin determinant.

Equation (5) is an inversion formula for β derived directly from a Kirchhoff-approximate forward model for a single reflector. Here, however, we have another objective, namely, the mapping of an input data set, with its source/receiver configuration and background parameters (macro-model) to an output data set, with its source/receiver configuration and background parameters; thus, the different subscripts, I and O on the variables. To this end, we substitute the representation (5) into (4) to obtain the following representation for the mapping of data from any input source/receiver configuration and background model to any output source/receiver configuration and background model:

$$u_O(\xi_O,\omega_O) \sim -\frac{i\omega_O}{8\pi^3} \int i\omega_I\, d\omega_I\, d^2\xi_I\, u_I(\xi_I,\omega_I) \cdot \int \frac{a_O(\mathbf{x},\xi_O)}{a_I(\mathbf{x},\xi_I)} \frac{|\nabla_x \tau_O(\mathbf{x},\xi_O)|}{|\nabla_x \tau_I(\mathbf{x},\xi_I)|} |h(\mathbf{x},\xi_I)| \cdot e^{[i\omega_O \tau_O(\mathbf{x},\xi_O) - i\omega_I \tau_I(\mathbf{x},\xi_I)]} dV. \tag{8}$$

The first line, here, depends on the input data, only. The integration over the interior volume in the second and third lines could be carried out for given ξ_I, ω_I, ξ_O, ω_O to obtain an operator kernel that is a function of these variables only. Indeed, we anticipate carrying out those integrations by analytical methods including asymptotic methods, such as multidimensional stationary phase, for each prescribed data mapping. Numerical integration is out of the question. There are $O(n^3)$ coordinates of integration, with $O(n^3)$ input variables and $O(n^3)$ output variables. Clearly, the n's are different, but this is still an intractably large set of variables.

The denominator of the second line of the operator contains factors that "undo" point source spreading and the obliquity of the input data in a model-consistent way. The numerator reintroduces these same factors in the output source/receiver configuration, now consistent with the background physcial model of the output. All of these variables will be evaluated at stationarity of the integrand. In the simplest of such evaluations, the Hessian—the determinant of the matrix of second derivatives—evaluated at the stationary point will correct for curvature effects from input to output configuration and model.

The Beylkin determinant h, plays a different role. As in the inversion integral, itself, (5), this is the factor that could not be predicted from migration arguments. According to Beylkin [1985], the inversion operator, (5), is asymptotically an inverse Fourier transform with respect to a wave vector, $\mathbf{k}$, although the integration is over variables, ξ_I and ω_I. The wave vector is locally defined for each $\mathbf{x}$ and ξ, ω as

$$\mathbf{k} = \omega_I \nabla_x \tau_I(\mathbf{x}, \xi_I).$$

Thus, in (5) and in (8) the factor of h arises through the identity

$$dk^3 = \left| \frac{\partial(\mathbf{k})}{\partial(\xi, \omega_I)} \right| d^2\xi_I d\omega_I = \omega_I^2 \left| h(\mathbf{x}, \xi_I) d^2\xi_I d\omega_I . \right|$$

If we use this result in (8) to rewrite it as an integral over the $\mathbf{k}$ and $\mathbf{x}$, the frequency dependence of the operator becomes ω_O/ω_I and has the same quotient symmetry and explanation as the spatial factors discussed above.

Determination of Incidence Angle

When the input data is dominated by isolated specular reflection returns, the output will be dominated by such returns, as well. Asymptotically, in this case, the cascade of integrals is dominated by stationary phase contributions where the isochrons of the input and output traveltimes are tangent and share the same normal direction. That direction is also normal to the reflector at the specular point. When there is no mode conversion, at this specular point

$$|\nabla_x \tau_I(\mathbf{x}, \xi_I)| = \frac{2\cos\theta_I}{c(\mathbf{x})}, \quad |\nabla_x \tau_O(\mathbf{x}, \xi_O)| = \frac{2\cos\theta_O}{c(\mathbf{x})},$$

where $2\theta_I$ and $2\theta_O$ are the opening angles between the incidence rays of the input and output source/receiver configurations at the point $\mathbf{x}$. Thus, we introduce two other data mapping operators,

$$\cos_I = -\frac{1}{8\pi^3} \int |\nabla_x \tau_I(\mathbf{x}, \xi_I)| (\cdots),$$

$$\cos_O = -\frac{1}{8\pi^3} \int |\nabla_x \tau_O(\mathbf{x}, \xi_O)| (\cdots).$$

Here, $(\cdots)$ denotes the three lines of (8) beyond the integral sign. Then, the ratios of outputs, $\cos_I/u_I(\xi_O, \omega_O)$, $\cos_O/u_O(\xi_O, \omega_O)$, will provide asymptotic estimates of $2\cos\theta_I/c$ and $2\cos\theta_O/c$, respectively. With the consequent estimate of these incidence angles, one could then contemplate an AVA analysis based on them.

2.5D Data Mapping

Two-and-one-half (2.5D) processing is derived from an appropriate *thought experiment.* We consider a medium in which the propagation speed and other medium parameters are independent of one transverse direction, say x_2. We further assume that the input data is gathered on lines of constant x_2, say, $x_2 = \xi_{I2}$. Finally, we must consider an output source/receiver configuration that is also confined to lines of constant value of this out-of-plane coordinate. In this case, we can conclude that the input data is independent of the out-of-plane variable, namely, ξ_{I2}, and we can proceed to carry out the integration in ξ_{I2} and in x_2 by the method of stationary phase.

Applying the method of stationary phase to the integral in (8) in both variables and taking account of the results stated above, leads to the following 2.5D analog of that earlier result:

$$\begin{aligned} u_O(\xi_O, \omega_O) \sim & \frac{\sqrt{|\omega_O|} e^{-i\pi \mathrm{sgn}(\omega_O)/4}}{4\pi^2} \\ & \cdot \int \sqrt{|\omega_I|} e^{i\pi \mathrm{sgn}(\omega_I)/4} d\omega_I d\xi_I \\ & \cdot \int u_I(\xi_I, \omega_I) \frac{a_O(\mathbf{x}, \xi_O)}{a_I(\mathbf{x}, \xi_I)} \frac{|\nabla_x \tau_O(\mathbf{x}, \xi_O)|}{|\nabla_x \tau_I(\mathbf{x}, \xi_I)|} \\ & \cdot \frac{\sqrt{\sigma_{Is} + \sigma_{Ig}}}{\sqrt{\sigma_{Os} + \sigma_{Og}}} \frac{\sqrt{\sigma_{Os}\sigma_{Og}}}{\sqrt{\sigma_{Is}\sigma_{Ig}}} |H(\mathbf{x}, \xi_I)| \\ & \cdot e^{[i\omega_O \tau_O(\mathbf{x}, \xi_O) - i\omega_I \tau_I(\mathbf{x}, \xi_I)]} d^2x. \end{aligned} \quad (9)$$

In this equation, we have rewritten ξ_I for ξ_{I1} and ξ_O for ξ_{O1}, since the second component is no longer in the representation, at all, and the former two component vectors are now scalars. Furthermore, σ_{Is} (σ_{Ig}) is a running ray parameter along the ray from the source (receiver) to the scattering point, $\mathbf{x}$. It is the natural parameter that arises to describe out-of-plane geometrical spreading in the 2.5D case. It is related to traveltime through the equation,

$$\frac{d\sigma}{d\tau} = c^2(\mathbf{x}), \tag{10}$$

with the subscripts, s, g, corresponding to source or receiver, while the subscripts, I, O, correspond to input or output. Also, the 3D Beylkin determinant, h, appearing in (8) has been replaced by a 2D Beylkin determinant,

$$H(\mathbf{x}, \xi_I) = \det \begin{bmatrix} \nabla_x \tau(\mathbf{x}, \xi_I) \\ \dfrac{\partial}{\partial \xi_I} \nabla_x \tau(\mathbf{x}, \xi_I) \end{bmatrix}. \tag{11}$$

In this last equation, the gradient is now a two component operator in (x_1, x_3) and ξ_I is now a scalar variable.

Equation (9) provides a platform for mapping data at any linear input source/receiver configuration and input medium parameters to a linear output source/-receiver configuration and output medium parameters. It is a 2.5D transformation, meaning that it accounts for out-of-plane geometrical spreading but assumes modeling parameters that do not depend on the out-of-plane variable. We view it as a "platform," only, because it still requires an integration of the *operator* over the interior scattering variables. As above, for each choice of source/receiver configuration and medium parameters of input and output, this integral should be carried out in advance, preferably, analytically, invoking asymptotic methods as appropriate. Then, for a given data set, one needs only to process the line of data by carrying out the integrals in the first line with the simplified weighting function obtained by the preprocessing analysis of the second and third lines. This is the usual form of TZO (NMO/DMO), for example.

Conclusions

We have presented platform formulas for data mapping of scalar wavefields in 3D and 2.5D. The formulas assume knowledge of a physical model for both the input and output data and prescribed input and output source/receiver configurations. They were derived by cascading an inversion formula with a modeling formula. This cascade is a single reflector formalism in the absence of multiple reflections and multi-pathing. In that sense, it is still a somewhat limited result, at the level of generality of Kirchhoff migration or inversion formulas. In the absence of a specific application, each formula includes a multifold integration over the physical model space that must be evaluated analytically for each example of data mapping in order to derive a computationally feasible formalism for implementation. Specialization to constant background produces the result in Bleistein, et al [1998], which is the true amplitude version of constant background NMO/DMO. This was a straightforward exercise that was not included in this paper. Specialization to constant background produces the result in Bleistein, et al [1998].

On the other hand, we can show that Kirchhoff approximate model data in an input configuration is mapped to Kirchhoff data in an output configuration for the 2.5D case. We can do this in great generality, without specifying any particular configuration transformation. From this result, we can conclude that the traveltime and geometrical spreading effects of the input model are properly mapped to their counterparts in the output model, while the reflection coefficient is not mapped.

In future papers, we will specialize the mapping formulas to achieve specific data mapping objectives.

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