$3D$ Multi-valued Travel Time and Amplitude Maps

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Summary

Ray-theoretic modeling requires accurate amplitude as well as phase both for forward modeling and Kirchhoff inversion, among other applications. For the case of linear sloth (slowness squared or inverse wavespeed squared), analytical solutions of the ray equations *do* exist, leading to a combined numerical analytical technique in tetrahedral models. The ray tracing with this method is relatively fast. However, the wavespeed model generated by this technique is not sufficiently smooth to produce accurate amplitudes. Recent attempts to further smooth the physical model defeat the advantage of speed of the algorithm because the smoothness conditions across the faces of the tetrahedra generate a coupled system of equations of a size proportional to the number of tetrahedra in the global physical model. In 3D, this is not practical. Thus, we conclude that a standard smoothed physical model on a Cartesian grid is likely to lead to a computer code of competitive cpu speed, when amplitude accuracy—dynamics—are of as much concern as travel time accuracy—kinematics.

In either case, we use a *wave front construction* technique, in which the size of triangular plates connecting three nearby rays on the isochron (surface of constant travel time) are used as an indicator of adequate density of rays. When the criteria for density of rays are violated, data at new points on the wavefront are interpolated into the family of rays and the wave front construction continues. In this manner, the method does not require excessive density of rays at small travel times in order to maintain adequate density of rays at larger travel times. The technique allows for multi-pathing (caustics) and for amplitude propagation along each of the branches of the wave front.

An application of the modeling technique is presented here; more will be presented in the talk.

Introduction

In this report, we address the problem of accurate and efficient determination of multi-valued $3D$ maps for amplitude as well as travel times or any other ray-related variables throughout the target zone from any shot and receiver position. The current interest in $3D$ seismic imaging has considerably increased the importance of ray tracing methods in wave field computations. Among seismic modeling methods, ray tracing methods provide a reasonable compromise between accuracy and computational efficiency. For the computation of travel time, various methods have been described. Among those, the finite-differencing (FD) method, i.e. FD-solvers of the eikonal equation, has recently become a popular method for calculation of "first arrival" travel times (Vidale, 1988). However, this method suffers from the disadvantages that it is restricted to the computation of first arrivals only and it produces unreliable amplitudes. Both are severe disadvantages for Kichhoff-type algorithms, such as the Bleistein/Cohen inversion (Bleistein *et al.*, 1987), where the calculation of amplitudes is necessary to determine the weighting factor in inhomogeneous media. Furthermore, in complex media, such as near salt domes and in sub-salt regions, later arrival travel times should be considered to obtain better image quality. Amplitudes can be used, among other things, to find most energetic arrivals.

Simultaneous computation of travel times and amplitudes is possible by dynamic ray tracing (DRT). It can be carried out either by numerical solution of ray tracing equations in general smooth grid-based models or by piecewise analytic solutions for certain simple velocity functions in tetrahedral models. The analytic ray tracing is usually performed for models in which either the velocity $v(\mathbf{x})$, or $1/v(\mathbf{x})$, or $1/v^2(\mathbf{x})$ is a linear function of Cartesian coordinates (Červený, 1987; Meng & Bleistein, 1997). However, this assumption leads to tetrahedral cells with artificial second-order discontinuities at their interfaces. As a consequence, this approach produces unreliable amplitude coefficients across the internal boundaries. To overcome this problem, tedious function smoothing at high cpu cost is necessary for dynamic computation. Körnig (1995) chose to smooth the sloth by making it quadratic in tetrahedra. Thus, the resulting amplitudes were stable and accurate. The analytic solutions of the ray equations for such a velocity function are determined by using Laplace transform. However, the problem of determining the cell constants in quadratic sloth leads to a huge matrix inversion problem, and is impractical in $3D$. Only $2D$ implementation of the travel time computation was carried out by Körnig (1995). We have concluded that the analytic approach in tetrahedral cells does not likely offer efficient algorithms in dynamic applications.

The wavefront construction technique (Vinje *et al.*, 1996) is applied to numerical DRT in $3D$ complex models for estimations of both travel times and amplitude coefficients. The numerical DRT is performed by shooting a fan of rays from the source and extrapolating travel times and amplitudes away from the rays into their nearby regions. The dynamic interpolation of new rays assures that the wavefront has sufficient ray density at each computational step. Linear interpolation of travel time with respect to the simulated wave fronts and linear interpolation of amplitude in terms of the tube cross sectional area are performed at grid points that fall into the sub-volume formed by every two successive wavefronts. A grid point can be passed by different sequences of wavefronts and, thereby, multi-valued arrivals can be detected and recorded. In this manner, accurate—perhaps multi-valued—travel times and amplitudes are available at all grid points of interest.

In the following sections, attention will be focused first on the possibility of applying analytic solutions in tetrahedral models for amplitude estimation. Thereafter, we address some important issues in numerical DRT such as interpolations of data at new rays and at grid points. We also propose a smooth gridded model representation for the purpose of computational efficiency. Finally, we show results of applying this method to different velocity models. These results demonstrate that gridded, smooth velocity models are good candidates for dynamic ray tracing.

Dynamic ray tracing

It is known that the realistic velocity field of interest is often rather complicated and usually does not allow a general analytic solution of the ray tracing system. However, analytic ray tracing plays an important role in wave field computation. This is due, in part, because the analytic solutions are valuable in the cell approach, in which the whole model is subdivided into a set of tetrahedral cells with simple velocity functions within cells. As we have mentioned before, the constraint of linear physical models does not provide enough smoothness for amplitude estimation. In this section, we discuss the analytic solutions for *quadratic sloth.*

The quadratic sloth distribution, denoted as q, is defined as a quadratic function in space,

$$q(x_1, x_2, x_3) = \frac{1}{v^2(x_1, x_2, x_3)} = A + 2B_i x_i + C_{ij} x_i x_j . (1)$$

The analytical solutions for this distribution were found by Körnig (1995) using the Laplace transform. In the Laplace domain, the ray coordinates, $X_i(s)$, are ratios of polynomials of sixth and seventh order, respectively, in s; this is the Laplace variable corresponding to σ, the ray tracing integral variable with $d\sigma = v^2 d\tau$. The expressions for the ray trajectories, $x_i(\sigma)$, can be obtained explicitly by inverse Laplace transform of $X_i(s)$ using partial fraction expansions. Depending on the distribution of eigenvalues of C_{ij}, the solutions of $x_i(\sigma)$ are generalized into seven different forms. For each case, the ray trajectories are in the general form

$$x_i(\sigma) = w_{ik} f_k(\sigma), \quad i = 1, 2, 3, \quad k = 1, 2, ..., 7. \qquad (2)$$

Here, the w_{ik}'s are the weighting factors, which are functions of x_{i0}, p_{i0}, B_j and C_{ij} with x_{i0} and p_{i0} being the initial position and slowness components; $f_k(\sigma)$ are the basis functions corresponding to the inverse transform of the partial fraction expansions. One of them is unity, the others are either low-order polynomials in σ, or trigonometric, or hyperbolic function.

Now, we propose an alternative to Körnig's (1995) approach to calculate the amplitudes along rays. Notice that the standard DRT system produces the geometrical spreading factors along rays, which are the determinants of transform matrix from ray coordinates $(\gamma_1, \gamma_2, \gamma_3)$, to the general Cartesian coordinates (x_1, x_2, x_3),

$$J = \det \left[\frac{\partial(x_1, x_2, x_3)}{\partial(\gamma_1, \gamma_2, \gamma_3)} \right]. \qquad (3)$$

The ray Jacobian J is an important factor in computation of the ray amplitude by the relationship $A(\mathbf{x}, \mathbf{x_0}) = const/\sqrt{|J|}$ (Bleistein, 1984). Therefore, if we choose $\gamma_1 = p_{10}, \gamma_2 = p_{20}$ and $\gamma_3 = \sigma$ in (3), the ray Jacobian J can be calculated explicitly from (2) for all the seven cases, which is less computationally costly than solving the eight integral equations of DRT system in small time step.

This approach of making quadratic sloth assumption in tetrahedral models has eliminated the smoothing procedure across the internal interfaces. However, the constants A, B_i and C_{ij} in (1) have to be determined in advance from the given physical model. The assumption of quadratic sloth is equivalent to the continuities of both q and its gradient across the internal cell faces. Therefore, the 10 cell constants in one tetrahedron cannot be determined by the velocity values at its four apices only, but also depend on the values in the neighboring cells. Such a model design problem for all the tetrahedral cells leads to a huge inverse problem. If the whole model is divided into N cells, the size of the coupled system of equations is proportional to $10N$, making the computation very time consuming. Furthermore, this inverse system is not always solvable, or has solutions in a least squares sense, at best. This is impractical in $3D$ and considerably limits the applicability of this approach. Furthermore, the difficulty and inefficiency of determining the cell constants exists in all extensions to quadratic physical models. Therefore, we conclude that the analytic ray tracing in tetrahedral models is not likely to give us an efficient module for dynamics, although it has its applications in travel time calculations.

Wavefront construction on smooth gridded models

Here and below, we are going to focus on numerically solving ray tracing equations on smooth gridded models.

We will apply the technique of wavefront construction (WFC) to both kinematics and dynamics.

In the WFC method, the wavefront(WF) is represented by triangular plates that connect every three neighboring ray endpoints on the WF. The nearby rays in $3D$ are then defined and organized by such a triangular mesh consisting of the internal ordering of connecting endpoints in each triangle and adjacent triangle(s) to each of its side. The processes of checking, interpolating of new rays and estimating of grid point parameters are all performed within such a triangular network. Rays are added and the original triangle is subdivided into new triangles when certain criteria, restricting the size of the triangular plates, are violated. In this manner, the wavefront always has sufficient ray density without *a priori* estimation of the number of rays needed. For complex $3D$ velocity models, the wavefront surface may be very complicated, folding in on itself at some parts, for example; however, no tears or holes in the interior of the surface are allowed. In this sense, the wavefront is complete. On the other hand, a grid point can be passed by different sequences of wave fronts; multi-valued arrivals can then be estimated and distinguished by their initial take-off angles.

The most attractive advantage of the WFC method is that it is more efficient than the conventional ray tracing method. In addition, WFC gives better ray coverage, especially in areas of large geometrical spreading where conventional ray tracing may give no arrivals. Furthermore, compared to FD-solvers, the WFC method is not restricted to the calculation of only first arrivals. Amplitude and other ray theoretical quantities are also available. Thus, it meets the requirements for accurate modeling of amplitude as well as phase, a requirement for inversion as opposed to migration.

Ray interpolation

The wavefront construction method is largely dependent on the procedure of interpolation of the wavefront at each step. New ray endpoints must be added along the simulated wavefront and must have the propagation direction that the ray would have had if it had been shoot from the source. This section addresses an algorithm for this procedure.

Fig 1-a illustrates the interpolation of a single new ray between a pair of exiting rays. The two ray endpoints and their propagation directions expand to two straight lines in $3D$. An "approximate" center is defined as the midpoint of the line segment that connects the two straight lines at their points of shortest distance. This approximate center, along with the two rayends at the old WF, form a fan and a circular curve connecting the two rayend points. The new ray position is then found along the dividing direction from the approximate center, and at its intersection with the circular curve. Other parameters along the new ray are interpolated linearly.

Grid interpolation

Another interpolation procedure in DRT is the estimation of ray data at the output grid points. We perform the grid interpolation within ray tubes, which are prism-shaped bodies bounded by three rays and the triangles that connect them on the two WFs. First the grid points falling into (or close to) each cell are found. Then, the travel time can be estimated at each grid point in a similar way as in the interpolation procedure for new rays(see Fig 1-b). The approximate center is determined by the three rays with ray endpoints on any of the two WFs. The distances from each grid point to the approximate center and to the simulated wavefront are calculated. The travel time at the grid point is then recorded as $t_0 + d/v$, with t_0 the time at the wavefront, d the distance of the grid point away from the wavefront, and v the velocity at the grid point. For the amplitude estimation at the grid point, since the ray Jacobian is proportional to the cross sectional area of the ray tube, we interpolate the ray Jacobian linearly with respect to the triangle areas on the two wavefronts.

Model representation

The smoothness of the velocity model is critical to the calculation of amplitudes. The integration of the DRT system requires continuity of the velocity field up to the second derivatives. Many ray tracing procedures involve a type of spline interpolation for the evaluation of velocities at arbitrary points. Spline interpolation, however, is a time consuming procedure. Here, we combine the techniques of smoothing the discontinuous velocities and defining the velocity model on a fine grid (about three or four grid points per shortest significant model wavelength). Velocities as well as their first and second derivatives at all grid points are pre-calculated by finite differences of second order. For the evaluation velocities and their derivatives at arbitrary points we use linear interpolation. For the smooth models defined on fine grids, the difference between this linear representation and a spline representation of the model is negligible.

Examples

The first example provides a test for the accuracy of this modeling technique. Figure 2 shows the $3D$ wave field from a velocity distribution with constant gradient of squared slowness being $(0, 0, -0.2)s^2/km^3$. In such a medium, both travel time and amplitude field can be

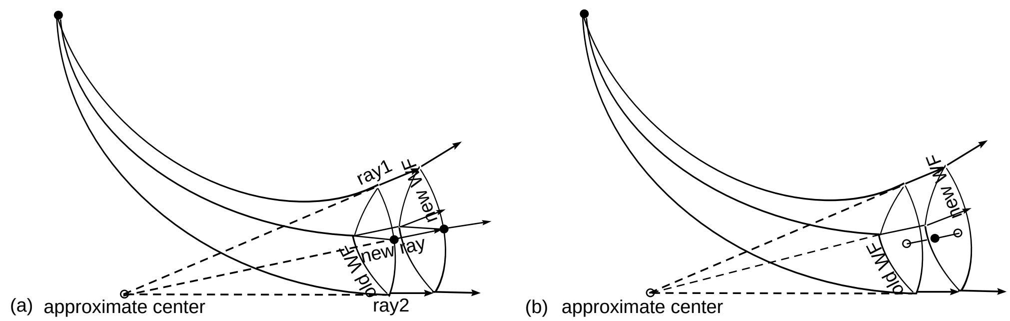

Figure 1. (a) The new ray position is found on the circular cure formed by the two existing ray endpoints on the WF and an "approximate" center. (b) Ray data at the grid are estimated with respected to the two simulated WFs.

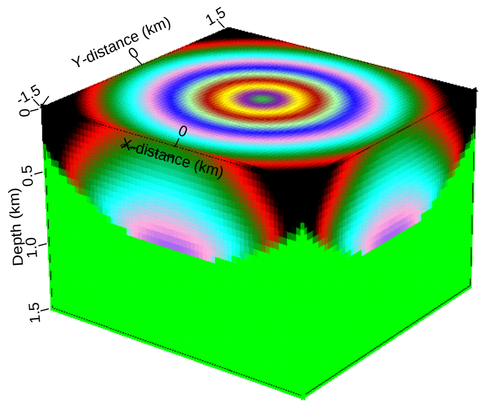

Figure 2. $3D$ wave field of a linear sloth model using WF construction method.

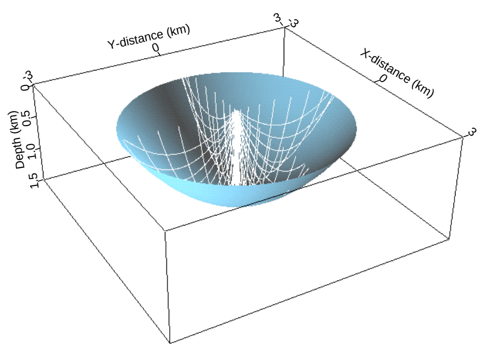

Figure 3. The grey surface is the caustic surface of the ray equations for this model.

expressed exactly by analytical solutions for comparison purpose. The grey part in Fig 2 is the shadow zone. The relative differences of computed travel times and the analytic ones are less than 0.1% through the whole interest area, while the differences between computed and analytical amplitudes are no more than 1%. In Figure 3, the grey surface is the caustic surface of the ray equations. It is the envelope of the parabolic rays.

Demonstrations of this algorithm in more complicated models and examples of amplitudes in models with interfaces will be included in the oral presentation of this work.

Conclusions

We have demonstrated that numerically solving ray tracing equations on a smooth gridded model provides fast and accurate forward modeling for both amplitudes and travel times. The alternative tetrahedron-based analytic approach is not ideal for the amplitude calculation, due to the difficulties in obtaining accurate amplitude across the internal faces.

The WFC procedure based on proper interpolation of new rays makes the dynamic ray tracing more efficient and results in a dense and consistent ray coverage throughout the model, even in areas of large geometrical spreading. When accurate amplitudes are required, we believe that this is a competitive method for development of ray theoretic wavefields.

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