Seismic Data Mapping

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Summary

This work describes Seismic Data Mapping (SDM), its definition, properties, applications, limitations and goals.

A variety of problems such as offset–continuation, azimuthal–continuation (AMO), layer replacement and datuming can be cast as special cases of SDM. In general, this process is a cascade of prestack migration and inverse migration (demigration). We find a simplification of the operator that requires less computational effort than either the individual prestack migration and demigration processes that generated it.

Introduction

The cascading of prestack migration and modeling operators is a valuable tool in addressing different problems in seismic data processing. Rocca (1985), for example, proposed that dip moveout (DMO) can be seen as a cascade of migration followed by a zero–offset modeling operator. Jorden (1987) developed a concise mathematical formulation of Rocca's idea. Biondi and Chemingui (1994) defined a process called azimuthal moveout (AMO) that can be seen as the cascade of a prestack migration operator that acts on data with a given offset and azimuth, followed by a forward modeling operator that reconstructs the data at different offset and azimuth. Following Tygel et al. (1996), we modify the above cascaded operator by substituting for an inverse migration (demigration operator) the modeling operator and define this process as seismic data mapping (SDM). Demigration maps imaged data into modeled data for a given source/receiver configuration geometry and given model parameters. The amplitude on the demigrated data should account for the proper geometrical spreading due to a point source and reflector curvature, and it should preserve the reflection coefficient embedded in the imaged data. We call this amplitude treatment "true amplitude." Some of the applications of the SDM operator are offset–continuation, AMO, datuming and layer replacement.

Basic elements of SDM

To define SDM more precisely, we first introduce both the prestack migration operator and the demigration operator for data from an acoustic medium.

Prestack migration

Equation(1) shows the prestack migration operator

$$\beta(\boldsymbol{x}) \sim -\mathrm{Re}\,\frac{1}{4\pi^2}\int d\boldsymbol{\xi}\,\frac{h_B(\boldsymbol{x},\boldsymbol{\xi})}{a(\boldsymbol{x},\boldsymbol{\xi})\,|\nabla_x\phi(\boldsymbol{x},\boldsymbol{\xi})|} \times \left.\frac{\partial}{\partial t}D(\boldsymbol{\xi},t)*\Delta(t)\right|_{t=\phi(\boldsymbol{x},\boldsymbol{\xi})}. \tag{1}$$

(Jaramillo, 1998). Here Δ represents an analytic (i.e., one–sided) delta distribution, so its convolution with the data $D(\boldsymbol{\xi},t)$ produces analytic traces (complex traces whose imaginary part is the Hilbert transform of the real part); h_B represents the Beylkin determinant (Bleistein, 1987), $a(\boldsymbol{x},\boldsymbol{\xi})$ is the product of the WKBJ amplitudes from source and receiver to the scatter (output) point $\boldsymbol{x}$, and ϕ is the total traveltime from source to $\boldsymbol{x}$ and to receiver, so that $\nabla_x\phi$ is the slowness vector for the combined source/receiver pair. It should be clear then that the reflectivity function β is an imaging function that maps data into images in the space of models. Bleistein (1987) showed that asymptotically (for high frequencies) this reflectivity function represents traces that locate reflectors in their correct position and weight them with the proper oblique (source/receiver) reflection coefficient. The two–dimensional parametric vector $\boldsymbol{\xi}$ uniquely defines the source/receiver locations within a subset (e.g., common–shot or common–offset) of the seismic data. The prestack migration operator (1) is an integral over the recording surface prescribed by $\boldsymbol{\xi}$. This summation is performed over times defined by the equation $t=\phi(\boldsymbol{x},\boldsymbol{\xi})$ for a fixed output $\boldsymbol{x}$ point. The surface $(\boldsymbol{\xi},\phi(\boldsymbol{x},\boldsymbol{\xi}))$ is a diffraction corresponding to $\boldsymbol{x}$. From here originates the name diffraction stack (Tygel *et al.*, 1996).

Demigration

Jaramillo (1998) derived a demigration formula from the Kirchhoff approximation. This formula, shown in equation (2) is used to compute the output data $D(\tilde{\boldsymbol{\xi}},\tilde{t})$

$$D(\boldsymbol{\xi},t) \sim \mathrm{Re}\int d\Sigma_I\,\frac{a(\boldsymbol{x},\boldsymbol{\xi})}{|\nabla_x\phi|} \times \left.\frac{\partial\beta(\boldsymbol{x})}{\partial n}\right|_{t=\phi(\boldsymbol{x},\boldsymbol{\xi})}*\Delta(t). \tag{2}$$

from the reflectivity function $\beta(\boldsymbol{x})$. This operator is also a surface integral, this time over the surface defined by the implicit equation $t=\phi(\boldsymbol{x},\boldsymbol{\xi})$ for a fixed time t and source/receiver combination parameterized with $\boldsymbol{\xi}$. This surface is called an isochron; hence the demigration operator here is called an isochron stack (Tygel *et al.*, 1996). The product of WKBJ amplitudes $a(\boldsymbol{x},\boldsymbol{\xi})$, as well as the total traveltime ϕ (and $\nabla_x\phi$), are computed for each point of the isochron. Asymptotically, this isochron stack creates seismic reflections at the appropriate modeling locations and introduces the correct geometrical spreading for point source and reflector curvature. The oblique reflection coefficient embedded in the reflectivity data is preserved, consistent with what we call "true amplitude" processing.

The SDM chained operator

We are now ready to give the precise mathematical definition of SDM. By inserting equation (1) into equation (2), we create the SDM operator. This operator takes data $D(\boldsymbol{\xi}, \boldsymbol{t})$ from a given source/receiver configuration and model parameters into data $D(\tilde{\boldsymbol{\xi}}, \tilde{t})$ for a different source/receiver configuration and model parameters. It is understood here that the reflector boundaries remain in the same place for both operators.

Given that demigration is the (asymptotic) inverse of prestack migration, we find that the composition (cascading) of two SDM operators is another SDM operator. Likewise, the inverse of an SDM operator is an SDM operator and the identity* function is an SDM operator (resulting from cascading prestack migration and demigration without change in the model). These properties show that space of SDM operators has the algebraic structure of a group.

One limitation of SDM, as defined here, is that it is expensive and therefore is not of practical computational interest. We will show techniques that shorten and simplify the computations of SDM. The following simple example illustrates one of those shortcuts.

Let us assume that the input data set corresponds to a common–offset section produced by a homogeneous isotropic horizontal layer with reflector depth $d = 2$ km, semi–offset $h = 1$ km and velocity $v_1 = 2$ km/s. Then compute the reflectivity function (1) for these data and, after that, apply the demigration operator (2) to the resulting image. However, for the demigration step, let us change the velocity field to $v_2 = 4$ km/s and the offset to $h = 0$ km. We would obtain a data set with the new source/receiver configuration and model parameters, but still with the oblique reflection coefficient corresponding to the input model. Figure 1 illustrates this experiment. A shortcut of this process can be accomplished by convolving the data with the wavelet

$$A_0\,\delta_B(t - t_0), \tag{3}$$

where A_0 undoes the point focusing and de–focusing associated with the source and reflector curvature for the input model, and introduces the influence of the point source and reflector curvature for the output model. Time t_0 in equation (3), given by

$$t_0 = 2\,\sqrt{d^2 + h^2}/v_1 - 2\,d/v_2, \tag{4}$$

reverses the kinematics of the input model (the normal–moveout correction) and generates the kinematics of the output model (the $2\,d/v_2$ term). While the shortcut here certainly would be effective, this example is valid for illustration purposes only; as mentioned above.

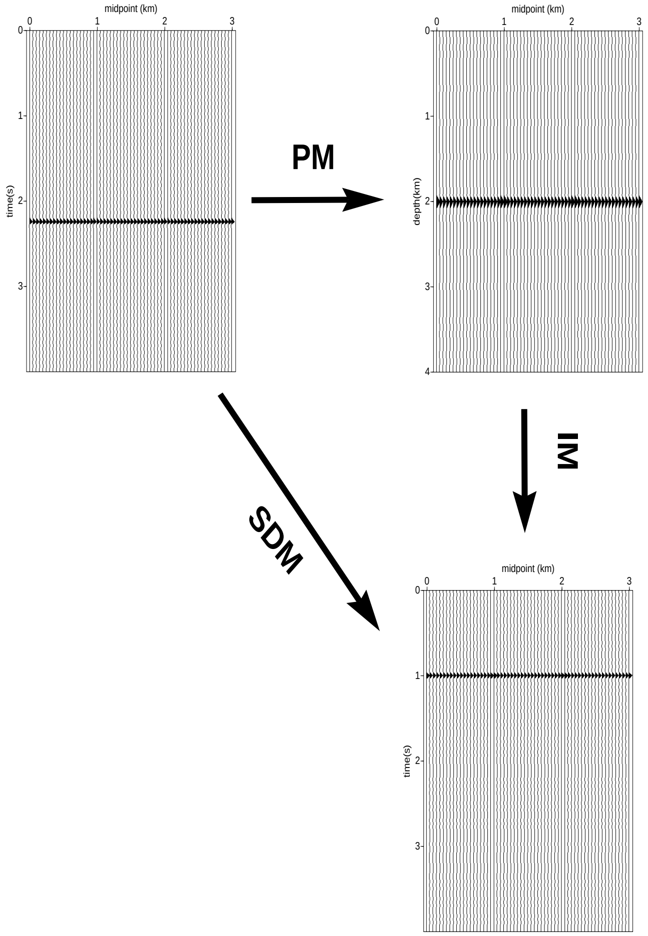

Figure 1. SDM is the cascade of a prestack migration operator (PM) with a demigration operator (IM). We desire shortcuts for this process.

For more general problems, the shortcut that we propose is the analytic evaluation of the SDM integral along isochrons. In some cases this evaluation can be made possible by using the method of stationary phase as in the following expression resulting form evaluation of the chain SDM integral:

$$D(\tilde{\boldsymbol{\xi}}, \tilde{t}) \sim \mathrm{Re} \int d\boldsymbol{\xi}\; w_{SDM}(\tilde{\boldsymbol{\xi}}, \tilde{t})\;\; \frac{\partial}{\partial t} D(\boldsymbol{\xi}, t) * \triangle(t)\Big|_{t=\phi}, \tag{5}$$

where $\phi = \phi_{SDM}(\boldsymbol{\xi}, \tilde{\boldsymbol{\xi}}, \tilde{t})$. This shortcut of the SDM

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* The identity distribution is a delta function with zero lag. Due to bandlimiting, we associate the identity with a sinc function.

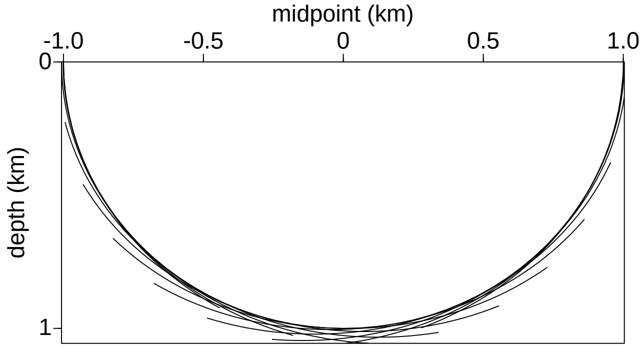

Figure 2. The inline output isochron for a TZO experiment in a constant–velocity medium is a circle (bold line). The inline input isochrons (thin lines) for the same experiment are ellipses. The stationary phase condition requires that those ellipses be tangent to the output isochron. The square root of the positive difference of curvatures between output and input isochrons is reciprocal to the amplitude weight (6)

operator is a surface integral of the time derivative of the analytic data, weighted by an amplitude factor $w_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}},\tilde{t})$. This operator is a shortcut in two ways. It is a surface (two–fold) integral instead of the four–fold integral representation of SDM proposed above. Also, the surface of integration is smaller than either the diffraction surface or the isochron surfaces of the prestack migration and demigration operators, respectively. We define *stacking surface* as the surface of points $(\boldsymbol{\xi},\phi_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}}))$. The time $t=\phi_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}})$ is found by performing a ray tracing to the isochron for the output model using the input source/receiver configuration.

Jaramillo (1998) derived two forms for the weight $w_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}},\tilde{t})$. The first form is for the difference of Gaussian curvatures between the output model isochron (defined uniquely by the parameters $\tilde{t}$ and $\tilde{\boldsymbol{\xi}}$ and the output velocity field) and the input isochron (defined uniquely by the parameters t and $\boldsymbol{\xi}$ and the input velocity field). This weight is given by

$$\begin{aligned} w_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}},\tilde{t}) &= -\frac{1}{2\pi}\,\frac{\tilde{a}(\boldsymbol{x},\tilde{\boldsymbol{\xi}})}{a(\boldsymbol{x},\boldsymbol{\xi})} \\ &\times \frac{h_B(\boldsymbol{x},\boldsymbol{\xi})}{|\nabla_x\phi(\boldsymbol{x},\boldsymbol{\xi})|^2\sqrt{|K|}}\,e^{\frac{i\pi}{2}(1+\mathrm{sgn}(L_{ij})/2)}. \end{aligned} \tag{6}$$

Figure 2 shows the inline portion of the output isochron (the surface of integration for the demigration operator) and some of the input isochrons corresponding to the input model, for a transformation–to–zero–offset (TZO) experiment. The input modeling is a finite–offset section for a medium with constant velocity v, and the output modeling corresponds to a zero–offset section for the same velocity v. Jaramillo (1998) shows that the stationary phase condition implies that each input isochron should be tangent to the output isochron at the stationary phase point. The *Gaussian curvature* between the difference of the output isochron and each of the input isochrons is represented by the factor K. L_{ij} is the *second fundamental tensor* for the difference between the output isochron and each input isochron (Laugwitz, 1965). The number $\mathrm{sgn}(L_{ij})$ is the difference between positive and negative eigenvalues of L_{ij}. Equation (6) is convenient for theoretical studies, but is not practical for numerical computation. Jaramillo (1998) derives an alternative expression for weight w_{SDM} in terms of second derivatives in the space of seismic data. This weight is given by

$$\begin{aligned} w_{SDM}(\boldsymbol{\xi},\tilde{\boldsymbol{\xi}},\tilde{t}) &= \frac{1}{2\pi}\,\frac{\tilde{a}(\boldsymbol{x},\tilde{\boldsymbol{\xi}})}{a(\boldsymbol{x},\boldsymbol{\xi})}\,\sqrt{|\det(\boldsymbol{H}_D-\boldsymbol{H}_R)|} \\ &\times\ e^{\frac{i\pi}{2}(1-\mathrm{sgn}(\boldsymbol{H}_D-\boldsymbol{H}_R)/2)}. \end{aligned} \tag{7}$$

Here,

$$H_D=\left[\frac{\partial^2\phi}{\partial\xi_i\,\partial\xi_j}\right], \tag{8}$$

represents a matrix of second derivatives of the diffraction time surface. More specifically, this diffraction surface is the support of the data generated by a scatterer located at the stationary phase point (point of contact between the input and output model isochrons).

$$H_R=\left[\frac{\partial^2\phi_{SDM}}{\partial\xi_i\,\partial\xi_j}\right] \tag{9}$$

is the matrix of second derivatives of the time on the stacking somewhat hyperbolic surface used in the data mapping. Figure 3 shows the inline portion of (finite–offset) diffractions corresponding to the points of contact between the input and the output model isochrons displayed in Figure 2. The envelope of those Huygens diffractions builds up the stacking surface, while the stationary phase method guarantees constructive interference along that envelope. The difference in "curvature" between this envelope and each of the diffraction surfaces is quantified by $\det(\boldsymbol{H}_D-\boldsymbol{H}_R)$.

Assumptions and limitations

The assumptions in applying SDM are basically those needed to perform prestack migration and demigration. The main assumptions here are as follows

- We assume only isotropic data. The theory of SDM, however, can be extended directly to anisotropy after the proper use of the corresponding prestack migration and demigration operators for anisotropic media.
- We assume primary scalar wavefields corresponding to a single reflecting layer. The superposition principle applies when a set of layers is to be used.

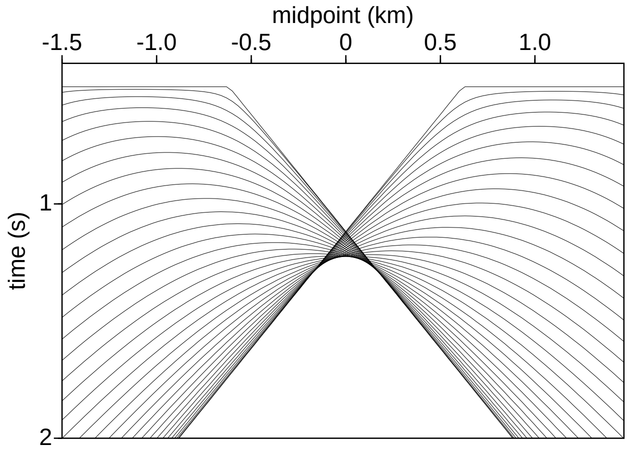

Figure 3. Huygens diffractions (double square–root paths) corresponding to the points of tangency between the circle and the ellipses in Figure 2. The envelope corresponds to the stationary phase path (or surface for a full 3D experiment). The square root of the magnitude of the difference between the second derivatives of these paths and the envelope is proportional to the amplitude of the SDM operator

- The seismic velocity that properly migrates the input data is assumed to be known as is true in conventional migration. The sensitivity of SDM to errors in velocity information for various specific applications needs further study.
- Density is assumed constant, but the theory can be easily extended to media with variable density by using the appropriate Green functions.
- Only the leading asymptotic term is considered (high–frequency approximation).
- Neither the data nor the operators should be aliased.

Applications

Some of the applications for offset–continuation are

- offset–continuation and TZO. Some of the uses of offset–continuation are in binning, reconstruction of missing offsets for multiple suppression and other processes that require filling in of missing data.
- amplitude preserving azimuthal–continuation (Biondi & Chemingui, 1994).
- transformation of P–S to P–P seismic data (Chan & Stewart, 1996). In fact the output data are not truly P–P data since the reflection coefficient of the P–S data remains.
- datuming.
- velocity analysis.
- combinations of the above.

Conclusions

Here we derived a Seismic Data Mapping (SDM) process that unifies a set of known seismic processing techniques. The SDM operator is a surface integral that is computationally more efficient than either the modeling used to produced input data or the migration to image it. Some of the problems that can be solved with this integral are offset–continuation, Transformation to Zero Offset (TZO), datuming, layer replacement and azimuthal–continuation (AMO). To compute the surface of integration for the SDM operator, ray tracing to the output model isochrons must be performed using the input configuration and model parameters. The sensitivity of SDM to errors in velocity information for various specific applications needs further study.

Acknowledgments

This research is partially supported by the Advanced Computational Technology Initiative (ACTI), subcontract number 4731U0015-2F, in conjunction with Los Alamos National Laboratory and industry project partners. It is also partially supported by the Center for Wave Phenomena Consortium Project at the Colorado School of Mines. We thank Ken Larner and Andreas Rüger for proofreading drafts of this paper. Some of the ideas shown here came from discussions of the authors with Paul Fowler.

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