

True Amplitude 3D constant background DMO— an implementation of data mapping

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Summary

Data mapping is a procedure for transforming data from a given input source/receiver configuration and macro earth model to a different output source/receiver configuration and background model. In a companion paper, we described a “platform” for data mapping. The platform is an integral operator derived as a cascade of an inversion operator with a modeling operator. The general formula can be shown to transform geometrical spreading effects, both from point source radiation and from reflector curvature from input to output configurations. To this extent, this is a “true amplitude” mapping procedure. However, the input reflection coefficient is not mapped and retains its dependence on the input specular angle. This angle, in turn, can be estimated by processing two operators and taking a ratio of the outputs.

To apply this platform to a particular data mapping process, it is necessary to eliminate the integration over the modeling variables by analytic methods in order to obtain an operator that directly maps input data and physical model to output data and physical model. That integration is best done as an integral over isochrons—surfaces of constant travel time—and then an integration over travel time. We consider cases of data mapping in which the input data is gathered on parallel lines. In the 3D constant background case, the integration over the isochron in the direction orthogonal to those data acquisition lines is an approximate Dirac delta function, thereby forcing the input and output data to be nearly restricted to the same line. The remaining integrations—one over the remaining variable on the isochron—the other over travel time—are much like those integrals that arise in processing single line data for 2D or 2.5D data mapping. For variable background, the integral over the isochron can be carried out by two dimensional stationary phase. However, the result for constant background suggests that this result obtained by stationary phase has to be invalid for small gradient—when the out-of-plane integration yields an asymptotic delta function.

In this paper, we demonstrate the analysis technique applied to the general data mapping formula to obtain a constant background 3D DMO formula that is true amplitude to the extent described above.

Introduction

Data mapping is the process of transforming observed data from a given source/receiver configuration and model of the propagating medium to another configuration and model, under an assumed model of the propagation mechanism. (This is a generalization of the NMO/DMO process.) Our data mapping is a “true amplitude” process in the following sense. (i) Travel time and geometrical spreading effects of the input configuration are transformed to those effects of the output configuration. (ii) Reflector curvature geometrical spreading effects of the input configuration are transformed to those effects of the output configuration. However, the reflection coefficient of the input configuration is preserved in the output data. The formalism also provides a mechanism for determining both the input and the output geometrical optics incidence angles of the reflection process, thereby providing a basis for amplitude versus angle (AVA) analysis.

In Bleistein and Jaramillo [1998], we derive the general data mapping platform that we specialize here to the case of constant background wave speed 3D DMO. The basic idea of the method is to cascade an inversion formula with a modeling formula. The combined formula maps a given data set to another. The result is an integral over the variables of the input data set to produce an earth model combined with an integral over the coordinates of the earth model to produce the output data set. The operator is a function of the input and output parameters and the earth model variables, as well. The idea, then, is to carry out the integration over the last set of (earth modeling) variables analytically (asymptotically) in order to obtain a weight that is a function of the input and output variables, only. This weight is then applied to the input data set to produce the output data set. This last analysis must be tailored to each specific data mapping implementation.

Motivated by the work of Tygel et al [1998], we propose to carry out that integral over the interior variables by first transforming to a new coordinate system where one of the variables is the input travel time and the other two variables form a coordinate grid on the surfaces of constant travel time—the *isochrons* of the input travel time. See Figure 1. If we attempt to apply the method of stationary phase to this integral over the input isochron, we find that the phase is stationary where the input and output isochrons are tangent. In general, for a variable background medium, we would expect

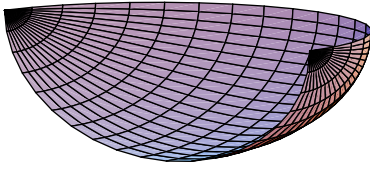


Figure 1. Example of an isochron surface: constant background, offset source and receiver. This isochron would arise in common offset or common shot source/receiver configurations.

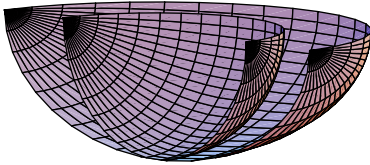


Figure 2. A point of tangency of isochrons from two travel time functions, corresponding to a simple stationary point in the integration over an isochron.

that such tangency would occur at isolated points on the input isochron. See Figure 2. If there were a reflector tangent to the input isochron at such a point, then the input isochron, the output isochron and the reflector would share the same normal. Hence, input specular returns would be transformed into output specular returns, which achieves the kinematics that we would expect of a data mapping formalism.

However, in the simplest of situations, namely, offset continuation and DMO in constant background, the stationary points are not isolated because the isochrons are surfaces of revolution. They can be determined by taking the isochron in the vertical plane below the source and receiver and rotating about the line containing the source and receiver.

For this case, consider data acquisition and data mapping on parallel lines. It is easy to show that there are no stationary points unless the out-of-plane input and output variables are the same. Then, if the isochrons are tangent at a point, *they are tangent along the entire curve of revolution through that point.* See Figure 3. In this case, the order of stationarity in this variable is infinite; all orders of directional derivative of the phase in this direction vanish. This is a manifestation in this approach of a known fact about 3D DMO in constant background media: while its kinematics are straightforward, its dynamics are not, and determination of amplitude dependence of the operator requires great care. However, this also should serve as a warning about 3D processing for the nonconstant background case. Sup-

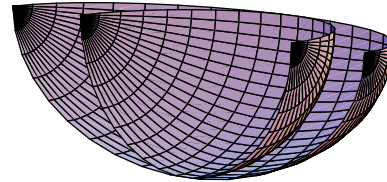


Figure 3. Tangency of isochrons along a curve of revolution.

pose that the wave speed(s) of the model are nearly constant. Then, formally, multi-dimensional stationary phase will seem to work. However, the second derivative in the direction of the curve of near-revolution will be small and the result will not be accurate for realistic frequencies of the exploration experiment. Thus, *uniform asymptotic analysis* is called for, if one is to obtain accurate dynamics in the case of small wave speed gradient.

3D constant background DMO

In this section, we introduce the 3D data mapping formula in constant background that we will analyze. We assume that data is gathered along parallel lines, say, constant value of the coordinate, ξ_{I2} , at constant offset from a midpoint ξ_{I1} , placing the source/receiver at $(\xi_{I1} \mp h, \xi_{I2}, 0)$. The input isochrons are then ellipses,

$$t_I = \tau(\mathbf{x}, \xi_I) = \left[\sqrt{(x - \xi_{O1} + h)^2 + (y - \xi_{O2})^2 + z^2} + \sqrt{(x - \xi_{O1} - h)^2 + (y - \xi_{O2})^2 + z^2} \right] / c \quad (1)$$

$$= [r_s + r_g] / c$$

centered at $(\xi_{I1}, \xi_{I2}, 0)$ with focal length, $2h$; r_s and r_g are the distances from the source and receiver (the foci of the ellipse) to the point at depth. The isochrons are described parametrically, with parameters, ϕ and ψ as follows:

$$x - \xi_{I1} = a \cos \phi, \quad y - \xi_{I2} = b \sin \phi \cos \psi, \quad (2)$$

$$z = b \sin \phi \sin \psi, \quad a = ct_I/2, \quad b = \sqrt{a^2 - h^2}.$$

The lower half of these ellipses are covered by ϕ and ψ in the range, $(0, \pi)$. Similarly, the output isochrons are hemi-spheres, centered at $\xi_O = (\xi_{O1}, \xi_{O2})$

$$t_O = \tau_O(\mathbf{x}, \xi_O) = 2\sqrt{(x - \xi_{O1})^2 + (y - \xi_{O2})^2 + z^2} / c = 2r_O / c. \quad (3)$$

The general 3D data mapping formula can be found in Bleistein and Jaramillo [1998] and in the abstract by the same name in this volume. The specialization of

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that result to constant background leads to the following formula.

$$u_O(\xi_O, \omega_O) = -\frac{i\omega_O}{8\pi^3} \int i\omega_I d\omega_I d^2\xi_I u_I(\xi_I, \omega_I) \cdot \frac{b \sin \phi \sin \psi}{r_O^2} B(\mathbf{x}, \xi_I, \xi_O) e^{i\omega_O \tau_O(\mathbf{x}, \xi_O) - i\omega_I \tau_I(\mathbf{x}, \xi_I)} dV. \quad (4)$$

In this equation,

$$B(\mathbf{x}, \xi_I, \xi_O) = \frac{2 \cos \theta_I}{c^3} \frac{(r_s + r_g)(r_s^2 + r_g^2)}{r_0^2 r_s^2 r_g^2}, \quad (5)$$

with $2\theta_I$ the opening angle between the rays from the input source and receiver at \mathbf{x} , given in terms of the unit vectors, \hat{r}_s and \hat{r}_g , by

$$\cos 2\theta_I = \hat{r}_s \cdot \hat{r}_g. \quad (6)$$

We set

$$dV = b \sin \phi d\gamma d\psi dn_I, \quad (7)$$

with $n_I = d\tau_I/|\nabla_x \tau_I|$ being differential arclength in the direction of the gradient, $d\gamma$ being arclength in the direction of ϕ and $b \sin \phi d\psi$ being arc length in the direction of ψ . These are three orthogonal directions and a simple product yields dV . When we integrate over the isochron, that is, over the variables, ϕ, ψ , in (4), the travel time τ_I is constant, namely, equal to t_I . Furthermore, the other travel time is given by

$$\tau_O = \frac{2r_O}{c} =$$

$$\frac{2}{c} \sqrt{(\delta_1 + a \cos \phi)^2 + b^2 \sin^2 \phi + \delta_2^2 + 2\delta_2 b \sin \phi \cos \psi} \quad (8)$$

with

$$\delta_1 = \xi_{I1} - \xi_{O1}, \text{ and } \delta_2 = \xi_{I2} - \xi_{O2}.$$

The only ψ dependence in the integrand in (4) arises through the explicit sine function and in r_O , with the factor, $b \sin \phi \sin \psi / r_O$ being just $(dr_O/d\psi)/\delta_2$. This is the key to carrying out the ψ integration asymptotically and obtaining a delta function in the argument, δ_2 . One can show that

$$\begin{aligned} I_\psi &= \int_0^\pi \frac{b \sin \phi \sin \psi}{\bar{r}_O^2} e^{2i\omega_O r_O/c} d\psi \\ &= e^{2i\omega_O \bar{r}_O/c} \frac{\sin[2\omega_O b \delta_2 \sin \phi / r_O c]}{\omega_O \delta_2 \bar{r}_O / c} = \frac{c\pi}{\omega \bar{r}_O} \delta(\delta_2), \end{aligned}$$

$$\bar{r}_O = \sqrt{(\delta_1 + a \cos \phi)^2 + (b \sin \phi)^2}. \quad (9)$$

This is an asymptotic result, to leading order in ω_O . The delta function allows us to carry out the integration in ξ_{I2} , forcing the line of the input data to the same second coordinate as that of the output data. Martin Tygel has noted that the middle line of this equation

suggests a sinc interpolation over nearby traces as opposed to the delta function evaluation. This is a subject of future research. Here, we proceed to use the delta function evaluation to obtain

$$u_O(\xi_O, \omega_O) = \frac{-i \text{sgn}(\omega_O)}{8\pi^2} \int d\omega_I d\xi_{I1} u_I(\xi_{I1}, \xi_{O2}, \omega_I) \quad (10)$$

$$\frac{b \sin \phi}{\bar{r}_O} e^{i\omega_O \tau_O(\mathbf{x}, \xi_O) - i\omega_I \tau_I(\mathbf{x}, \xi_I)} d\gamma dn_I.$$

The remaining integrations in γ and n_I are exactly as in the case of 2D or 2.5D DMO. Therefore, we can collect results from Bleistein et al [1998], Bleistein and Jaramillo [1998], and Bleistein [1998a]. The details will be provided in a forthcoming paper. The result is

$$u_O(\xi_O, \omega_O) = \frac{-i \text{sgn}(\omega_O)}{2\pi c} \int_{\omega_I/\omega_O > 1} d\omega_I d\xi_{I1} u_I(\xi_{I1}, \xi_{O2}, \omega_I) \frac{z(2 \cos \theta_I)^{3/2}}{\bar{r}_O \sin \theta_I \sqrt{1 - \cos \theta_I}} \frac{h^2 + \delta_1^2}{h^2 - \delta_1^2} e^{i\omega_O \tau_O(\mathbf{x}, \xi_O) - i\omega_I \tau_I(\mathbf{x}, \xi_I)}. \quad (11)$$

In this equation, we can express \mathbf{x} as a two component vector, (x, z) , with the coordinate geometry just as it is for 2D or 2.5D DMO. This vector (and, hence, the travel times and $\cos \theta_I$) are functions of $\xi_{I1}, \xi_{O1}, \omega_I, \omega_O$ through the stationary phase conditions in γ and n_I that led from (10) to (11). We can define these relationship geometrically in a manner that will be familiar to users of 2D or 2.5D DMO. Given ξ_I and ξ_O , draw the family of isochrons of the input travel time (the DMO ellipses centered at ξ_I and focal length $2h$) and the output travel time (circles centered at ξ_O). The stationary phase condition in γ requires that the normals to the isochrons be colinear. Thus, if there were a reflector at such a point with the same normal, this point would be specular for both the input and the output rays. This also provides a functional dependence between the input and output travel times: given the input travel time, for example, an input isochron is defined; find the output isochron that is tangent to this input isochron; that determines an output travel time.

The stationary phase condition in n_I is

$$\omega_O = \omega_I \cos \theta_I. \quad (12)$$

This equation has no solutions unless ω_I and ω_O have the same sign and, further, $\omega_I/\omega_O > 1$; hence, the constraint on the integration domain in (11). Thus, among the tangency pairs of isochrons and corresponding tangent points determined by the first stationarity condition, we must search for that point at which the opening angle between the rays satisfies (12). In Bleistein et al [1998], there is further discussion of these conditions.

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The relationship, (12) can also be viewed as a scaling in frequency domain imposed by the geometry of the rays. This scaling is just the right factor to assure that there is neither a gain nor a loss in resolution of reflector images after migration or inversion. Again, see Bleistein et al [1998].

We know from the general theory that the proposed data mapping will transform geometrical spreading effects, both from the point source and point receiver and from the curvature effects of a general surface in 3D. Although the result is an integral along a line, just as 2D or 2.5D DMO, this is an added feature; those methods treat reflectors as if they are cylindrical, functions only of depth and the in-line transverse variable. We know also from the theory that the reflection coefficient is not mapped by this formalism; the zero offset data is inaccurate to the extent that the angularly dependent reflection coefficient at the input incidence angle is retained, whereas true zero offset data should have a normal incidence reflection coefficient.

We remark, further, that we have a technique for determining that incidence angle. The trick is to realize that, in the peak amplitude at output back in the time domain, all amplitude factors are evaluate at their stationary values. Thus, if we were to multiply the integrand in (11) by $\cos \theta_I = c|\nabla_x \tau_I|/2$, then the peak value of the new output would be the result above, multiplied the stationary value of θ_I . The stationary value of θ_I is the specular reflection angle. The ratio of amplitudes of the two outputs then yields an estimate of $\cos \theta_I$.

We view this derivation as a prototype for the analysis of other constant background 3D data mappings. We expect in each case to be able to carry out the out-of-plane integrations in a similar fashion to the evaluation of I_ψ in this presentation, yielding a delta function in the out-of-plane direction that will restrict the output data line to the input data line and leaving an in-line processing procedure that mirrors 2D or 2.5D processing. However, the amplitude will always be different from the 2D or 2.5D processing because those formulas assume reflection surfaces with zero surface curvature in one direction.

We have seen here that the transverse integration (in ψ) in the direction orthogonal to the data acquisition line was carried out by integration by parts, *not* by stationary phase. For constant background, we expect this to be generally true, whereas for variable background, this integral will also be done by stationary phase. Thus, the limit from variable background to constant background through progressively smaller gradients must exhibit a pathology to characterize this transition. Consequently, we expect that the second derivative in these

stationary phase evaluations for variable background, will approach zero with decreasing gradient, making the resulting asymptotic result invalid. Thus, alternative—probably uniform asymptotic—analysis will be necessary in this case.

Conclusions

We have outlined the analysis that leads to a 3D constant background true amplitude DMO formula. We have described how the asymptotic analysis restricts the processing to integration along a single line, in this case, dictated by the direction of the input source/receiver configuration and the coordinates of the output configuration; that is, for data acquired on lines of constant second coordinate, that coordinate is restricted by the theory to agree with the second coordinate of the output configuration. The processing is true amplitude to the extent and with the limitations discussed in the introduction.

Deivation of formulas for other such 3D constant background implementations is now in progress.

References

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