

Amplitudes for One-way Extrapolators

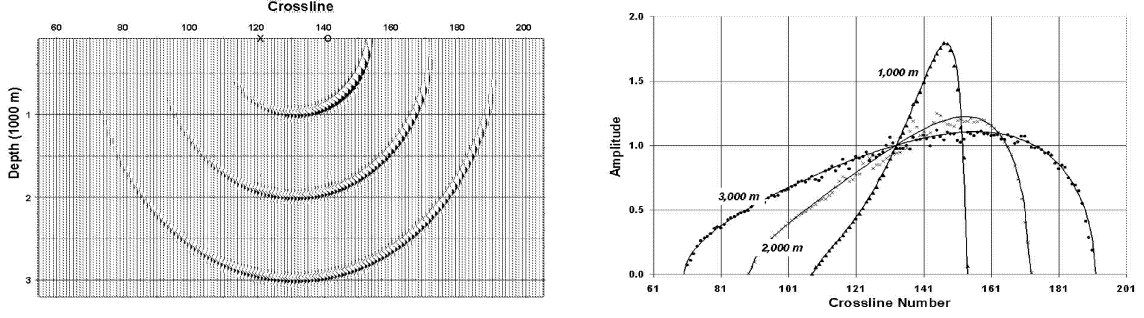


Fig. 2: Left: 3-D phase-shift migrated impulse responses along the center inline. The shot is at crossline 121 and receiver at crossline 141. Right: Amplitudes of the 3-D migrated impulse responses. The solid lines are theoretical predictions and symbols are the peak amplitudes from the left.

shot and receivers, respectively (see Figure 1), and the hat denotes temporal Fourier transform.

For conventional common-shot migration, we downward continue both shot and receiver wavefields:

$$\begin{cases} \left(\frac{\partial}{\partial z} + \Lambda\right) D = 0, \\ D(x, y, z = 0; t) = \delta(\vec{x} - \vec{x}_s)\delta(t), \end{cases} \quad (4)$$

and

$$\begin{cases} \left(\frac{\partial}{\partial z} - \Lambda\right) U = 0, \\ U(x, y, z = 0; t) = Q(x, y; t) \end{cases} \quad (5)$$

where D and U are the downgoing and upgoing waves (Claerbout, 1985), respectively, and

$$\Lambda = \frac{1}{v} \frac{\partial}{\partial t} \sqrt{1 - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2}\right)^{-1}}$$

is the square-root operator. To produce the image, we use the imaging condition

$$R(x, y, z) = \int \frac{\hat{U}(x, y, z; \omega)}{\hat{D}(x, y, z; \omega)} d\omega. \quad (6)$$

For a $v(z)$ medium, Zhang et al. (2001) give an asymptotic expression for the one-way wave fields:

$$\hat{D}(x, y, z; \omega) \sim \frac{i\omega}{2\pi} e^{-i\omega\tau_s} \sqrt{\frac{\cos \alpha_s}{\psi_s \sigma_s}} \quad (7)$$

and

$$\hat{U}(x, y, z; \omega) \sim \iint \frac{i\omega}{2\pi} \sqrt{\frac{\cos \alpha_r}{\psi_r \sigma_r}} e^{i\omega\tau_r} \hat{Q} dx_r dy_r. \quad (8)$$

Substituting (7) and (8) into (6), we obtain

$$R(x, y, z) \sim \iiint \sqrt{\frac{\cos \alpha_r \psi_s \sigma_s}{\cos \alpha_s \psi_r \sigma_r}} e^{i\omega(\tau_r + \tau_s)} \hat{Q} dx_r dy_r d\omega. \quad (9)$$

Comparing (9) with (3), we conclude that the algorithm (4-6) cannot provide the true amplitude image; even the phase term $i\omega$ is missing.

To see why this happens, we observe that for constant velocity, D and U are not components of the full wave fields p , but rather they are related to p by (Zhang, 1993)

$$D = \frac{1}{2} \left(\Lambda - \frac{\partial}{\partial z}\right) p,$$

$$U = \frac{1}{2} \left(\Lambda + \frac{\partial}{\partial z}\right) p,$$

and

$$D + U = \Lambda p.$$

Therefore from (1) and (2) we have

$$\begin{cases} \left(\frac{\partial}{\partial z} + \Lambda\right) D = \frac{1}{2} \delta(t) \delta(\vec{x} - \vec{x}_s), \\ \left(\frac{\partial}{\partial z} - \Lambda\right) U = \frac{1}{2} \delta(t) \delta(\vec{x} - \vec{x}_s), \\ (U + D)|_{z=0} = \Lambda Q(x, y; t), \end{cases} \quad (10)$$

Attaching physical meaning to D and U , we can reformulate (10) as follows:

$$\begin{cases} \left(\frac{\partial}{\partial z} + \Lambda\right) D = 0, \\ \left(\frac{\partial}{\partial z} - \Lambda\right) U = 0, \\ D|_{z=0} = \frac{1}{2} \delta(t) \delta(\vec{x} - \vec{x}_s), \\ U|_{z=0} = \Lambda Q(x, y; t). \end{cases} \quad (11)$$

Noting that the symbol of Λ in the Fourier domain is

$$\lambda = i \frac{\omega}{v} \sqrt{1 - v^2 \frac{k_x^2 + k_y^2}{\omega^2}},$$