Towards Accurate Amplitudes for One-way Wavefield Extrapolation of 3-D Common Shot Records

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Summary

We analyze the amplitudes produced by shot-record migration by one-way wavefield extrapolation. By comparing these amplitudes with those produced by true-amplitude Kirchhoff migration, we show the amplitude and phase errors that come from a standard implementation of migration by one-way wavefield extrapolation. Next, we present a new formulaton of shot-record migration that maintains its high fidelity in imaging complex structures, has correct dynamic behavior at least for constant velocity, and can be easily extended to $v(z)$. This formulation requires that we modify, in a straightforward way, the wavefield that is being downward continued. Our analysis applies equally to all migration methods based on one-way wavefield extrapolators.

Introduction

Until recently, Kirchhoff migration has been used for most 3-D prestack migrations, primarily because of its versatility and efficiency. The demands of imaging increasingly complex geological structures, however, have spurred a demand for increased imaging fidelity. This has led to the growing popularity of imaging methods that handle more than the single arrival (e.g., maximum-energy) that Kirchhoff migration is capable of handling conveniently. Such methods include multi-arrival Kirchhoff migration, which allows for several arrivals at each image location, and finite-difference migration, which allows for an unlimited number of arrivals at each image location. In this paper, we concentrate on one-way wavefield extrapolation, paying particular attention to its amplitude and phase behavior.

The standard formulation of finite-difference migration (Claerbout, 1985) consists of two parts. The first part is the downward continuation of the wavefields from the source and receiver locations using a "wave equation" that splits the wavefields into downgoing and upgoing parts. The second part is the application of an imaging condition, namely the division of the downward continued receiver wavefield by the downward continued source wavefield at each image point. Unfortunately, the one-way "wave equations" used in the downward continuation are not equivalent to the acoustic wave equation whose behavior they are designed to mimic. This lack of equivalence leads to a migrated wavefield that lacks correct amplitude and phase behavior, even though it is kinematically correct. By expressing the downward continued wavefields asymptotically, we are able to compare the imaged wavefield with the reflection coefficient of true amplitude Kirchhoff migration. The latter is our benchmark for amplitude and phase. The former is the downward continued receiver wavefield divided by the downward continued source wavefield. This comparison leads to a corrected equation for the upgoing and downgoing wavefields which, in turn, leads to a corrected expression for the wavefields being downward continued. When these corrections are applied, the migration produces images whose amplitudes and phases agree with true-amplitude Kirchhoff migration. These corrections are essentially without cost, and they do not compromise the migration's structural imaging fidelity, such as finite-difference migration.

Theory

We begin with a layered velocity ($v(z)$) earth and 3D common-shot migration. Given an acoustic wave-field p with source excitation at $\vec{x_s} = (x_s, y_s, 0)$ and $t = 0$,

$$\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} - \triangle\right) p(x,y,z;t) = \delta(\vec{x} - \vec{x_s})\delta(t), \quad (1)$$

(where $\triangle = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$), we record the surface data Q:

$$p(x_r, y_r, z = 0; t) = Q(x_r, y_r; t). \quad (2)$$

According to Bleistein et al.'s (2001) work on inversion, the true-amplitude common shot Kirchhoff inversion formula is (Zhang et al., 2000)

$$R(x,y,z) \sim \iiint i\omega \frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_s\sigma_s}{\psi_r\sigma_r}} \; e^{i\omega(\tau_s+\tau_r)}\hat{Q}(x_r, y_r;\omega)dx_r dy_r d\omega, \quad (3)$$

where ψ and σ are in-plane and out-of-plane geometrical spreading terms and α_{s0} and α_{r0} are surface angles at

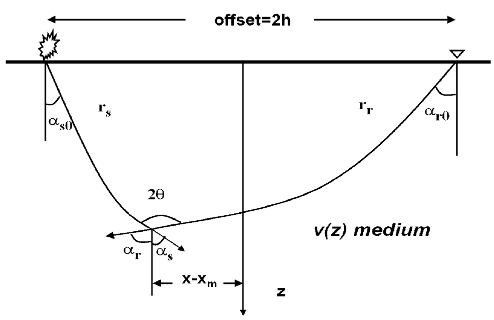

Fig. 1: Ray paths in a $v(z)$ medium.