that we obtain this inversion formula from the intput source/receiver configuration and earth model. That is,

$$\beta(\boldsymbol{x}) = \frac{1}{8\pi^3}\int d^2\xi_I \frac{|h(\boldsymbol{x},\boldsymbol{\xi}_I)|}{a_I(\boldsymbol{x},\boldsymbol{\xi}_I)|\nabla_x\tau_I(\boldsymbol{x},\boldsymbol{\xi}_I)|}\int i\omega_I \, d\omega_I \, e^{-i\omega_I\tau_I(\boldsymbol{x},\boldsymbol{\xi}_I)} u_I(\boldsymbol{\xi}_I,\omega_I). \tag{35}$$

Our objective is to map the input data set, with its source/receiver configuration and background parameters (macro-model) to an output data set, with its source/receiver configuration and background parameters. To achieve this, the representation (35) is substituted into (34) to obtain the following representation for the mapping of data from any input source/receiver configuration and background model to any output source/receiver configuration and background model:

$$\begin{aligned} u_O(\boldsymbol{\xi}_O,\omega_O) \;\sim\; & -\frac{i\omega_O}{8\pi^3}\int i\omega_I \, d\omega_I \, d^2\xi_I \, u_I(\boldsymbol{\xi}_I,\omega_I) \\ & \cdot\int \frac{a_O(\boldsymbol{x},\boldsymbol{\xi}_O)}{a_I(\boldsymbol{x},\boldsymbol{\xi}_I)} \frac{|\nabla_x\tau_O(\boldsymbol{x},\boldsymbol{\xi}_O)|}{|\nabla_x\tau_I(\boldsymbol{x},\boldsymbol{\xi}_I)|} |h(\boldsymbol{x},\boldsymbol{\xi}_I)| e^{[i\omega_O\tau_O(\boldsymbol{x},\boldsymbol{\xi}_O)-i\omega_I\tau_I(\boldsymbol{x},\boldsymbol{\xi}_I)]} d^3x. \end{aligned} \tag{36}$$

Note that the input data in the first line here is independent of the earth modeling variables, $\boldsymbol{x}$. Hence, for each choice of input and output earth model and each choice of input and output source/receiver configuration, the integrations over $\boldsymbol{x}$ in the second and third lines could be carried out to obtain an operator kernel that is a function of $\boldsymbol{\xi}_I$, ω_I, $\boldsymbol{\xi}_O$, and ω_O. Indeed, we anticipate carrying out those integrations by analytical methods including asymptotic methods such as multi-dimensional stationary phase. Numerical integration is out of the question. There are $O(n^3)$ coordinates of integration, with $O(n^3)$ input variables and $O(n^3)$ output variables. Clearly, the n's are different, but this is still an intractably large set of variables. The processing of this formula would require an integration over $O(n^3)$ variables to produce a function of $O(n^6)$ variables. A better choice is the analysis of the volume integral represented by the second and third line to obtain an analytically explicit kernel that depends only on the input and output variables, $\boldsymbol{\xi}_I$, ω_I, $\boldsymbol{\xi}_O$, and ω_O,.

From the derivation and the specific example of 2.5D DMO, it is expected that the result will not transform the reflection coefficient of the input data configuration to the reflection coefficient of the output configuration. However, we do anticipate that the geometrical spreading effects and curvature effects of the input configuration will be transformed to the correct effects for the output configuration. In fact, this has been proven by Tygel, et al, [1998], using a somewhat different (time-domain) approach to the same problem. While their proof does not include all of the cases listed above, it could easily be extended to them. In that sense, we consider their proof as all encompassing for this operator.

There is extensive discussion and interpretation of the structure of this operator in Bleistein [1998a]. Here, we only point out that some implementations have already been carried out and are described in Bleistein [1998b, c], Sheaffer and Bleistein, [1998]. The 2.5D implementation for mapping from common offset data to common shot data has also been worked out, but not yet presented.

2.5D

The specialization of (36) follows from the same type of "thought experiment" as was introduced in the previous section. Except for the subscripts O and I, the variables are as described in that section. The main result is the 2.5D platform for datamapping. That result is

$$\begin{aligned} u_O(\xi_O,\omega_O) \quad \sim \quad & \frac{\sqrt{|\omega_O|}e^{-i\pi\,\mathrm{sgn}(\omega_O)/4}}{4\pi^2}\int\sqrt{|\omega_I|}e^{i\pi\,\mathrm{sgn}(\omega_I)/4}d\omega_I d\xi_I u_I(\xi_I,\omega_I) \\ & \cdot\int\frac{a_O(\boldsymbol{x},\xi_O)}{a_I(\boldsymbol{x},\xi_I)}\frac{|\nabla_x\tau_O(\boldsymbol{x},\xi_O)|}{|\nabla_x\tau_I(\boldsymbol{x},\xi_I)|}\frac{\sqrt{\sigma_{Is}+\sigma_{Ig}}}{\sqrt{\sigma_{Os}+\sigma_{Og}}}\frac{\sqrt{\sigma_{Os}\sigma_{Og}}}{\sqrt{\sigma_{Is}\sigma_{Ig}}} \qquad (37) \\ & \cdot|H(\boldsymbol{x},\xi_I)|\cdot e^{[i\omega_O\tau_O(\boldsymbol{x},\xi_O)-i\omega_I\tau_I(\boldsymbol{x},\xi_I)]}d^2x. \end{aligned}$$

This result has been tested for input data represented by the Kirchhoff approximation, in full generality of an unprescribed source/receiver configuration. We find that, asymptotically, the Kirchhoff data in the input source/receiver variables is mapped to a Kirchhoff approximation in the output source/receiver variables, except that the reflection coefficient is evaluated at the specular angle associated with the input variables.

We know that the Kirchhoff approximation contains the "right" geometrical spreading effects, both due the source and receiver Green's functions and due to the reflector curvature. Therefore, there is no need to check individual implementations separately; the mapping performs as claimed.

REFERENCES.

Artley, C., and Hale, D., 1994, Dip moveout processing for depth-variable velocity: Geophysics, **59**, 4, 610-622.

Berkhout, A. J., and D. J. Verschuur, 1997, Estimation of multiple scattering by iterative inversion, Part I: Theoretical considerations, Geophysics, **62**, 1586-1595.

Berryhill, J. R., 1979, Wave-equation datuming: Geohpysics, **44**, 1329-1344.

Berryhill, J. R., 1984, Wave-equation datuming before stack (short note): Geophysics, **49**, 2064-2066.

Bevc, D., 1995, Imagining under rugged topography and complex velocity structure: Ph. D. Thesis, Stanford University.

Bevc, D., 1999, Flooding the topography: wave equation datuming of land data with rugged acquisition topography: Geophysics, **62**, to appear.

Beylkin, G. [1985], Imaging of discontinuities in the inverse scattering problem by the inversion of a causal Radon transform: J. Math. Phys., **26**, 99-108.

Beylkin, G., Miller, D., and Oristaglio N., Spatial resolution of migration algoritms, *in*, Berkout, A. J., Ridder, J., and van der Waals, L. A., eds., [1985], "Acoustic Imaging 14," Plenum, New York, 155-167.

Beylkin, G., Miller, D., and Oristaglio N., Spatial resolution of migration algorithms, *in*, Berkhout, A. J., Ridder, J., and van der Waals, L. A., eds., [1985], "Acoustic Imaging 14," Plenum, New York, 155-167.

Biondi, B., and Chemingui, N., 1994, Transformation of 3-D prestack data by azimuthal moveout: 64th Ann. Int. Mtg. SEG, Expanded Abstracts, 1541-1544.

Black, J. L., Schleicher, K. L., and Zhang, L., 1993, True amplitude imaging and dip moveout: Geophysics, **58**, 47-66.

Bleistein, N., [1984], "Mathematical Methods for Wave Phenomena," Academic Press Inc. (Harcourt Brace Jovanovich Publishers), New York.

Bleistein, N., [1986a], Kirchhoff inversion for reflector imaging and sound speed and density variations, *in* Proceedings of EAEG/SEG Workshop on Deconvolution and Inversion, Rome, Italy.

Bleistein, N., [1986b], Two-and-one-half dimensional in-plane wave propagation, Geophysical Prospecting, **34**, 686-703.

Bleistein, N., [1988], Analysis of post stacking of common shot inversions for parameter estimation, Res. Rep. CWP-071R Center for Wave Phenomena, Colorado School of Mines.

Bleistein, N., Cohen, J. K., and Hagin F., [1987], Two-and-one-half dimensional Born inversion with an arbitrary reference, Geophysics **52**, 26-36.

Bleistein, N., Cohen, J. K., and Stockwell Jr., J., [1999], Mathematics of Multidimensional Seismic Inversion, Center for Wave Phenomena lecture notes, Colorado School of Mines, Golden, CO.

Bleistein, N., and Gray, S.H., 1985, An extension of the Born inversion method to a depth dependent reference profile: Geophysical Prospecting, **33**, 999-1522.

Bleistein, N., and Jaramillo, H., 1998, A Platform for Data Mapping in Scalar Models of Data Acquisition: Center for Wave Phenomena Research Report number CWP-267.

Bleistein, N., J. K. Cohen, and H. Jaramillo, 1999, True-amplitude transformation to zero offset of data from curved reflectors: Geophysics, Geophysics, **64**, 1,

Chen, W. and Stewart, R., 1996, Transformation of P-S to P-P seismic data: Crews Research Report, 8, 18-1, 18-25.

Červený, V., 1995, Seismic Wavefields in Three-Dimensional Isotropic and Anisotropic Structures: Lecture Notes, University of Trodheim, Norway.

Červený, V., and M. A. de Castro, [1993], Application of dynamic ray tracing in the 3-D inversion of seismic-reflection data.

Claerbout, J. F. [1985], "Imaging the Earth's Interior," Blackwell Scientific Publications, Oxford (Oxfordshire), Boston.

Cohen, J. K. and Hagin, F., [1985], Velocity inversion using a stratified reference, Geophysics, **50**, 1683-1700.

Colton, D. and Kress, R. [1983], "Integral Equation Methods in Scattering Theory," John Wiley & Sons, New York.

de Hoop, M. V., [1998], Asymptotic inversion: Multipathing and caustics: Sixty-eighth International Meeting of the Society of Exploration Geophysicists, New Orleans, September, Expanded Abstracts, 1534-1537.

de Hoop, M., and Bleistein, N., [1997], Generalised Radon Transform inversions for reflectivity in anisotropic elastic media: Inverse Problems, **13**, p. 669-690.

Devaney, A. J. and Oristaglio M. L. [1984], Geophysical diffraction tomography: Presented at the 54th Ann. Internat. Mtg. Soc. Explor. Geophys., 330-333.

Dobrin, M. B. [1976], "Introduction to Geophysical Prospecting," McGraw-Hill, New York.

Docherty, P., [1987], Ray theoretical modeling, migration and inversion in two-and-one-half-dimensional layered acoustic media: CWP Researh Report number CWP-051.

Docherty, P., [1988], CXZ: Fortran program for laterally varying velocity inversion: CWP Program documentation number U09. Ph.D. thesis, Colorado School of Mines.

Docherty, P., [1991], Documentation for the 2.5-D common shot program CSHOT: CWP Program documentation number U08R.

Dong, W., and Bleistein, N., 1990, 2.5D Kirchhoff inversion theory applied to VSP and crosshole data: Center for Wave Phenomena Research Report number CWP-101.

Emanuel, J., [1989], Common shot, prestack inversion and mode conversion analysis of physical model seismic data: CWP Research Report number CWP-083. M.Sc. thesis, Colorado School of Mines. item[] Fomel, S., 1995a, Amplitude preserving offset continuation in theory, Part 1: the offset continuation equation: Stanford Exploration Project preprint, SEP-84, 179-196.

Fomel, S., 1995b, Amplitude preserving offset continuation in theory, Part 2: solving the equation: Stanford Exploration Project preprint, SEP-89.

Fomel, S., 1997, Velocity continuation and the anatomy of prestack residual migration: 67th Annual International Meeting of the Society of Exploration Geophysicists, Expanded Abstracts, Vol II Tulsa, 1762-1765.

Fomel, S., N. Bleistein, H. Jaramillo, J. K. Cohen, 1996, True amplitude DMO, offset continuation and AVA/AVO for curved reflectors: 66th Annual International Meeting of the Society of Exploration Geophysicists, Expanded Abstracts, Vol II, Tulsa, 1731-1734.

French, W. S., [1975], Computer migration of oblique seismic reflection profiles, Geophysics, **40**, 961-980.

Gardner, G. H. F. [1985], “Migration of Seismic Data”, Geophysical reprint series, No. 4, Society of Exploration Geophysics.

Gardner, G. H. F. and Forel, D., 1995, Amplitude preservation equations for dip moveout: Geophysics, **55**, 485-487.

Hale, I. D., 1984, Dip-moveout by Fourier transform: Geophysics, **49**, 6, 741-757.

Gardner, Gerald H. F. and Forel, David, 1988, Amplitude preservation equations for DMO: 58th Annual International Meeting of the Society of Exploration Geophysicists, Expanded Abstracts, 1106-1108, Session: S17.2.

Geoltrain, S., [1989], Asymptotic solutions to direct and inverse scattering in anisotropic elastic media: CWP Research Report number CWP-082. Ph.D. thesis, Colorado School of Mines.

Geoltrain, S., and Chovet, E., [1991], Automatic association of kinematic information to prestack images: Expanded Abstracts of the International Meeting and Exposition of the Society of Exploration Geophysicists, Tulsa, 890-892.

Gray, S. H., [1986], Efficient traveltime calculations for Kirchhoff migration: Geophysics, 51, 8, 1685-1688.

Gray, S. H., [1988], Computational inverse scattering in multi-dimensions: Inverse Problems, 4, 87-101.

Gray, S. H., [1992], Frequency-selective design of the Kirchhoff migration operator: Geophys. Prosp., 40, no. 5, 565-572.

Gray, S. H., [1997], True-amplitude seismic migration: a comparison of three approaches: Geophysics, 62, 3, 929-936.

Gray, S. H., [1998], Speed and accuracy of seismic migration methods: Proceedings of the Mathematical Geophysics Summer School on Seismic Imaging, Stanford, August 3-21, to appear. Available on the web at http:sep//www.stanford.edu/etc/sam_gray/.

Gray, S. H. and May, W. P., [1994], Kirchoff migration using eikonal equation traveltimes: Geophysics, 59, no. 5, 810-817.

Gray, S. H. and Marfurt, K. J., [1995], Migration from topography: improving the near-surface image: Can. J. Expl. Geophys., 31, no. 1/2, 18-24.

Hagedoorn, J. G. [1954], A process of seismic reflection interpretation: Geophysical Prospecting, **2**, 85-127.

Hale, D., and C. Artley, 1993, Squeezing dip moveout for depth-variable velocity: Geophysics, **58**, 2, 257-264.

Hanitzsch, C., 1995, Amplitude preserving prestack Kirchhoff depth migration: PhD Thesis, Universität Karlsruhe, Germany.

Hanitzsch, C., [1995], Amplitude preserving prestack Kirchhoff depth migration/inversion in laterally inhomogeneous media: PhD dissertation, University of Karlsruhe.

Hanitzsch, C., [1997], Comparison of weights in prestack amplitude-preserving Kirchhoff depth migration: Geophysics, 62, 6, 1812-1816.

Hsu, C., [1992], CXZCO: A 2.5D common offset inversion program in a c(x,z) medium: Edited by Z. Liu. CWP Program documentation number U15.

Ikelle, L. T., G. Roberts and A. B. Weglein, [1998], Source signature estimation based on the removal of first-order multiples: Geophysics, **62**, 6, 1904-1920.

Jaramillo, H., 1998, Seismic data mapping: Ph.D. thesis, Colorado School of Mines. Center for Wave Phenomena Research Report number CWP-284.

Jaramillo, H. and Bleistein, N., 1998a , Seismic data mapping: Expanded Abstracts, Society of Exploration Geophysicists International Meeting, September, 1998, to appear.

Jaramillo, H. and Bleistein, N., 1998b, A simplified derivation to migration and demigration in isotropic inhomogeneous media: Center for Wave Phenomena Annual Report, CWP-248, Geophysics, to appear.

Kravtsov, Yu. A., and Orlov, Yu.I., 1990, Geometrical Optics of Inhomogeneous Media: Springer-Verlag, New York.

Lewis, R. M., 1965, Asymptotic theory of wave-propagation: Arch. Rat. Mech. and Anal., **20**, 3, 191-250.

Liner, C. L., 1990, General theory and comparative anatomy of dip moveout: Geophysics, **55**, 595-607.

Liner, C. L., 1991, Born theory of wave-equation dip moveout: Geophysics, **56**, 182-189.

Liu, Z., [1993], SUINVVXZCO: Inversion of common offset seismic data using a Kirchhoff-WKBJ method with the WKBJ parameters computed via finite differencing: CWP Program documentation number U20R.

Ludwig, D. L., 1966, Uniform asymptotic expansions at a caustic: Comm. Pure and Appl. Math., **XIX**, 215-250.

Meng, Z., [1999] Tetrahedral based 3-D depth imaging-modeling and migration velocity analysis: Ph. D. Thesis, CSM, Golden, CO.

Miller, D., Oristaglio, N., and Beylkin G., [1984], A new slant on the seismic imaging: Migration and integral geometry, Geophysics, **52**, 943-964.

Musgrave, A. W. [1961], Wave-front charts and three dimensional migrations: Geophysics, **26**, 738-753.

Najmi, A., [1996], Closed form solutions for the geometrical spreading in inhomogeneous media: Geophysics, 61, 3, 1189-1197.

Ou, Boyi, [1995], 2.5-D common offset inversion in triangulated models: CWP Researh Report number CWP-77, M.Sc. thesis, Colorado School of Mines.

Ratcliff, D. W., Jacewitz, C. A. and Gray, S. H., [1994], Subsalt imaging via target-oriented 3-D prestack depth migration: The Leading Edge, 13, no. 3, 163-170.

Sabatier, P. C. ed. [1987], "Tomography and Inverse Problems," Adam Hilger, IOP Publishing Ltd., Bristol, Philadelphia.

Schneider, W. A. [1978], Integral formulation for migration in two and three dimensions: Geophysics, **43**, 49-76.

Schneider, W. A. [1978], Integral formulation for migration in two and three dimensions: Geophysics, **43**, 49-76.

Sheaffer, S., and Bleistein, N., 1998, 2.5D downward continuation using data mapping theory: Center for Wave Phenomena Research Report number CWP-270.

Slotnick, M. M. [1959], "Lessons in Seismic Computing," Society of Exploration Geophysicists, Tulsa.

Sneddon, I, N., 1972, The Use of Integral Transforms: McGraw-Hill, New York.

Stolt, R. M. [1978], Migration by Fourier Transform: Geophysics, **43**, 23-49.

Sullivan, M. F., Cohen, J. K. [1987], Pre-stack Kirchhoff inversion of common offset data, Geophysics, **52**, 745-754.

Sumner, B., [1988], Asymptotic solutions to forward and inverse problems in isotropic elastic media: CWP Research Report number CWP-075. Ph.D. thesis, Colorado School of Mines.

Symes, W. [1990], Velocity inversion: A case study in infinite- dimensional optimization: Math. Programming, **48**, 71-102.

Sumner, B., [1988a], Asymptotic solutions to forward and inverse problems in isotropic elastic media: CWP Research Report number CWP-075. Ph.D. thesis, Colorado School of Mines.

Sumner, B., [1988b], ELAS: Computer programs for modeling and inversion in isotropic elastic media: CWP Program documentation number U10.

Sumner, B., [1990], CZ: Fortran programs for 2.5-D stratified velocity inversion, descriptions and instructions: CWP Program Documentation number U06R.

Tarantola, A. [1987],"Inverse Problem Theory," Elsevier Science Publishing, Amsterdam, New York.

Telford, W. M. et al. [1976], "Applied Geophysics," Cambridge University Press, Cambridge, London.

Tygel, M., Filpo, E., and Oliveira, A., 1997, True-amplitude MZO in laterally varying media: experiments on synthetic data. Presented at the 5th Intern. Congr. of the Braz. Geoph. Soc., Sao Paulo, Brazil. Expanded Abstracts, 151-154.

Tygel, M., Schleicher, J., and Hubral, P., 1995, True-amplitude migration to zero offset (MZO) by diffraction stack: preprint Insitituto de Matemática Estatística e Ciência da Computação, Universidade Estadadual de Campinas, Brazil.

Tygel, M., Schleicher, J., and Hubral, P., 1995, Dualities between reflectors and reflection time surfaces: Journal of Seismic Exploration, **4**, pp. 123 - 150.

Tygel, M., Schleicher, J., Hubral, P., and Santos, L. T., 1998, 2.5-D true-amplitude Kirchhoff migration to zero offset in laterally inhomogeneous media: Geophysics, **63**, 2, 557-573.

Weglein, A. B., F. A. Gasparotto, P. M. Carvalho and R. H. Stolt, [1998], An inverse scattering series method for attenuating multiples in seismic reflection, Geophysics, **62**, 6, 1975-1989.

Xu, M., [1996], Amplitude calculation for 3-D common offset v(z) inversion: CWP Research Report number CWP-213.

Xu, M., [1996], 3-D common offset inversion in depth dependent media and its parallel implementation: CWP Research Report number CWP-228. M.Sc. thesis, Colorado School of Mines.

Ziolkowski, R. W., and G. A. Deschamps, 1980, The Maslov method and the asmptotic Fourier transform: Caustic analysis. Electromagnetic laboratory Scientific Rep. No. 80-9. University of Illinois, Urbana-Champaign.

Ziolkowski, R. W., and G. A. Deschamps, 1984, Asymptotic evaluation of high frequency fields near a caustic: an introduction to Maslov's method: Radio Science, **19**, 1001-1025.