Me 'n' Ray Theory

An evolutionary tale of mathematics applied to the real world

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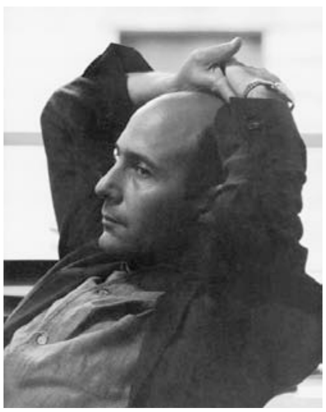
I was going to say a few words before giving a CWP seminar talk on my recent research in Gaussian beams. However, I found that what I wanted to say would take too long. So, I decided instead to write down some thoughts on how I got from being a graduate student to this current project and put my comments on my web page. As with the notes I wrote on Gaussian beams, this has also grown. I apologize in advance for being verbose, but this has been a lifetime evolution with good science and wonderful people along the way. So, read on if you are interested.

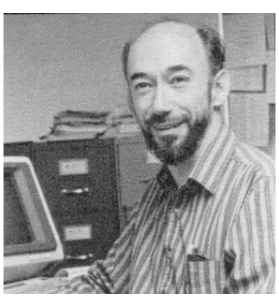
I was a graduate student at the Institute for Mathematical Sciences at New York University—later, Courant Institute for Mathematical Sciences. When, I went there, January 1960, I expressed an interest in partial differential equations and wave propagation, so I was assigned to the Electromagnetics Division with Joe Keller—the intellectual leader of that group—as my faculty advisor.



Joe Keller, 1963, Insitute of Mathematical Sciences, NYU

Joe[1] was (still is) an international giant in applied mathematics, publishing in the literature in electrical engineering, mechanical engineering, fluid dynamics of various sort, biology, and other places. His pièce de'résistance is the geometrical theory of diffraction an extension of ray theory to scattering by edges or conical points, published in the 50's.[2]

Earlier, it was well-known that at smooth interfaces an incident wave gives rise to at least two types of scattered waves, reflected and transmitted (three of each in anisotropic media). Their initiations at an interface are characterized "in the high frequency limit" by *reflection and transmission coefficients,* respectively. Waves incident on edges and points initiate different waves—diffracted waves—characterized by diffraction coefficients. Joe provided recipes for determining the directions of the rays and the diffraction coefficients. (Think of the edges and points as being geometrical elements of a jet plane and you quickly get to describing the scattering of radar signals from an oncoming stealth bomber. Notice, no corners except for those edges at the front and the back and special materials to make those diffraction coefficients smaller than for simple metals.) I was assigned as a research assistant to Robert M. (Bob) Lewis—now deceased (not



Don Ludwig at UBC

Yuri and me, work-shop, Hruba Skala, Czech Rep, June, 2005

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[1] Joe is now in his mid-80's and an active Professor Emeritus in Mechanical Engineering and Mathematics at Stanford. In a recent e-mail he related that he had just returned from skiing in Alta.

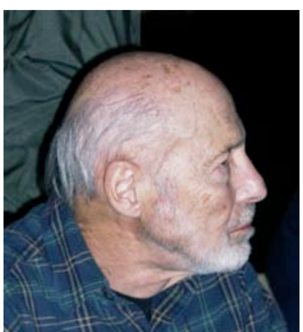
[2] Keller, J. B., 1958, A geometrical theory of diffraction, *in* "Calculus of Variations and its Applications," 27-52, McGraw-Hill, New York.

the Bob Lewis who works with Bill Symes at Rice). Bob was doing research in ray theory for dispersive hyperbolic equations and systems and I became immersed in that work, first checking his calculations, then accepting assignments of my own and finally, a thesis topic (1963 or so).

Near the end of my thesis work (1965), Don Ludwig (in our group) distributed a preprint of a paper[3] on the extension of ray theory to describe waves in the neighborhood of smooth caustics (envelopes of rays). The trick was to describe the waves in terms of Airy functions rather than in the simple $A\exp\{i\omega\tau\}$ that we use more generally. Don did not know that the same work had been published a year earlier by Yuri Kravtsov [4] in the Russian literature. This extension dealt with the convergence of families of rays, using a second parameter to characterize that convergence and creating so-called *uniform asymptotic expansions.* This work led to an entirely new line of research activity to deal with other anomalies of ray theory. Fresnel functions for diffraction by edges, Weber functions for head waves, Hankel functions for point scatterers all came into play.

By that time (1964-1966), I was looking at the same phenomena through their characterization in Fourier integral representations of the wave fields: two stationary points that coalesce as the observation point in physical space approaches a smooth caustic; stationary point approaching an endpoint of integration as the observation point in physical space moves to the shadow or transmission boundary of a wave passing by the edge of a reflector; stationary point approaching a branch point as the observation point moves towards the boundary of a head wave. So, I understood the Kravtsov/Ludwig theory right away and I loved it!

During a post-doc year ('65-'66), I got the opportunity to put this new knowledge into practice. Two years earlier, Courant Institute had a visitor, H. M. Nussenzweig[5] (Brazilian, ultimately at Princeton) who produced a landmark paper in scattering of plane waves and point source waves by a circular cylinder and a sphere. This problem has a great history stretching back to G. N. Watson in the earlier part of the century, responding to the question, "How dark is the dark side of the earth?" Basically, the part of the incoming wave at near-tangential incidence gives rise to a wave that adheres to the sphere in the shadow zone (creeping wave) and then sheds energy out into that shadow zone initiated at progressively decaying amplitude. (The energy lost is proportional to the energy remaining in the creeping wave at any point.)



Joe Keller, 80th birthday workshop, Jan, 2003, Stanford

I remember Nussenzweig's EM seminar talk on a Friday afternoon. He had covered nearly the entire blackboard with a picture in color. A plane wave came from the left past a circular cylinder (a circle on the blackboard). Here was the reflected wave and there was the transmitted wave and on the dark side there was a penumbra, the partially shaded region coming off of each tangential ray to the circle. The boundaries of the

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[3] Ludwig, D., 1966, Uniform asymptotic expansions at a caustic: Comm. Pure and Appl Math, XIX, 215-250.

[4] Kravtsov, Yu. A., 1964a, A modification of the geometrical optics method: Radiofizika, 7 664-673. (In Russian) and Kravtsov, Yu. A., 1964b, Asymptotic solutions of Maxwell's equations near a caustic:Radiofizika, 7, 1049-1056.

[5] Nussenzweig, H.M., 1967, High frequency scattering by an impenetrable sphere: Proceedings URSI Symposium on Electromagnetic Wave Theory, 1967, p 957-960.

penumbra ultimately swallowed up the shadow region far on the right…and there in the "deep" shadow were the creeping waves. Spectacular!

Bob Lewis had the thought that the uniform ray theory for caustics could be applied to this creeping wave phenomenon because the creeping wave has a caustic on the smooth boundary. He called me into his office one day and asked me to explain the uniform theory as I understand it from the ray-theory approach and from the integral approach near smooth caustics. Of course he caught on right away. We then talked through a rough outline of how to extend this theory to the "smooth-body diffraction" arising from near-tangential incidence on smooth surfaces. Here, Bob's knowledge of the known asymptotic expansions of exact solutions was crucial to our understanding of how to develop the ray theory. When we had the idea in place, we went to talk with Don Ludwig, who agreed that our idea should work and was delighted to be part of the project to fill in the details.

We split up the necessary follow-on research among the three of us. My task was to formulate the theory for the propagation of the creeping wave along the surface, essentially describing solutions of the eikonal equation for traveltime and the transport equation for amplitude when the gradient of the traveltime remains tangent to the surface. I remember staying at my apartment for two days rereading the first 90 pages or so of Kreyszig's text on differential geometry, then snapping the book closed and thinking, "OK! I know how to proceed."[6] This work led to a joint paper by the three of us on smooth body diffraction.[7]

Ray theory has been in my blood ever since those early experiences in graduate school. In earlier years here, I would give demonstrations of diffraction in a cookie pan when I taught ray theory. The pan had a lucite bottom. When filled with water and placed on an overhead projector, I could make "tsunamis"that would appear on the screen, making it easy to demonstrate diffraction by edges and creeping waves. (So head-on radar scattering on the stealth now can be seen as all edge diffraction and *creeping waves*.)

During the period at the end of my Courant Institute career ('65-'66), Bob, Joe and Leo Felsen from nearby Brooklyn Polytechnic Institute (now, Polytechnic Institute of New York—PINY)—maybe some others—were trying to get complex-valued traveltimes into ray theory by using complex rays that would only occasionally puncture the real space. But for one American—Luneberg—the idea of just making the traveltime complex and leaving the rays real was a Russian (Babich and Popov) and then Russian/Czech (Popov, Cervèny, Psencik) creation. I believe it was the Czechs who got the idea of ray-centered coordinates. Oh! to have known them and that theory then!

So, this leads to my current project on modeling, imaging and inversion with Gaussian beams. The modelin is essentially ray theory with complex-valued traveltimes. The project started with a request by Sam Gray to explain some results in the open literature. I found that I had to go beyond an "intellectual understanding" of Gaussian beams to an in-depth appreciation of the subtleties, so I started writing a self-tutorial, starting from classical ray theory, then to ray-centered coordinates, then to adding the feature of complex-valued traveltimes to ray theory in ray-centered coordinates. Along the way, I discovered some points that I felt were omissions in the existing theory and one important error in modeling of Green's functions by Gaussian beams.

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[6] I recently returned to studying caustics in Bleistein, N., 2006, Analysis of the neighborhood of a smooth caustic for true-amplitude one-way wave equations: Wave Motion, 43, 4, 323-338.

[7] Uniform Asymptotic Theory of Creeping Waves, 1967, with R. M. Lewis and D. Ludwig, Comm Pure and Appl Math, XX, 2, 295--328.

Further, the current method of derivation in the literature starts with a real traveltime on the central ray and creates the complex correction through a boundary layer analysis of an amplitude transport equation that is more complicated than the one for classical ray theory. I recreated the earlier derivation of Gaussian beams by using complex-valued solutions of the eikonal equation. Then, the transport equation returns to the simpler one that arises in classical ray theory. However, now the solution of that equation is complex valued as well. I consider this to be a much easier and perfectly legitimate extension of the WKBJ method for ordinary differential equations. I also collected and connected the various representations of the amplitude arising from ray theory in Cartesian coordinates and in ray-centered coordinates, including a representation of the amplitude in terms of wave front curvature. I had been meaning to do this last one for a long time because I was not satisfied with the derivations in the literature.

Even trying to be brief, this took a lot of writing! Also, for the new representation of the Green's function by Gaussian beams, I needed a proof that was easy in 2D, much harder in 3D. Here, some discussions with Sam Gray finally got me on the right path for that more difficult proof. Only when I had this done did I feel ready to write down imaging/inversion formulas using Gaussian beams for various source/receiver configurations. So, I did a few: common-shot, plane-wave incidence, common-offset in 2.5D, common-angle written as an integral (a sum) over all source/receivers pairs. (Common-offset in 3D has a computationally expensive integrand that is almost never computed exactly; 'not a practical method for true-amplitude calculations. "If it ain't true amplitude, I ain' in'erested!" Further, common-angle inversion is computationally more feasible and yields a more useful output.[8])

At this point, it has taken 100+ pages of notes, putting my own slant on the forward modeling by ray theory and then describing the inversion. It has been a long and fascinating journey. My original toolbox of ray theory, geometrical theory of diffraction, and asymptotic expansions of integrals[9] beyond the case of simple stationary points still serves me well.

Crude time line

Ray theory, diffraction theory	**Gaussian beams, complex traveltime**
Ray theory: Hamilton, 1828	Luneberg, 1950(?)
Geometrical theory of diffraction, Keller, 1958	Babich, Popov, 1956
Dispersive equations: Lewis et al, 1964	Cerveny, et al, 1977 and much more
Caustics: Kravtsov, 1964; Ludwig, 1966	Hill, 1990, 2001
Creeping waves, Lewis, Bleistein, Ludwig, 1967	Gray, 2005
Edge diffraction, Lewis, Ahluwahlia, Boersma, 1966+	Norm, 2006, 2007

The last word. This was largely stream-of-consciousness writing, without all the details completely filled in. Even this took quite a bit of time with web searches for some of the older paper citations because my copies are cached in storage boxes and not easily accessible. Our library has not been able to get copies of the early Babich references and I have not yet checked on the Luneberg reference. Ivan Psencik put me on to those. I composed this in Word. Unfortunately Word doesn't have the inverted caret that I need for Cervény and Psencik—\v{#} in LaTeX. I took the two pictures of Joe Keller. Judy took the picture of Yuri and me. The

[8] Migration/inversion: think image point coordinates, process in acquisition surface coordinates, 2005, Bleistein, N., Y. Zhang, G. Zhang and S. H. Gray, Inverse Problems, 21, 1715-1744.
[9] Asymptotic Expansions of Integrals, 1987, Bleistein, N., and R. A. Handelsman, Dover, New York

image of Don Ludwig came from his web page; he is now Professor Emeritus at UBC, having moved into mathematical biology after going there.

Some other references

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Note: The papers in SEP-28 are available on the web at http://sepwww.stanford.edu/oldreports/sep28.