

Some empirical relations in earthquake seismology

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INTRODUCTION

The following are some important empirical relations from seismology. See [Kanamori and Anderson, 1975] for a more in-depth analysis of these and other relations. The units in these formulae are expressed largely in the CGS system, reflecting the age of the papers referenced.

ENERGY VERSUS MAGNITUDE

$$\log_{10} E = 1.5M + 11.8 \quad (1)$$

where E is energy in ergs and M is surface wave magnitude or Gutenberg and Richter's unified magnitude. After [Gutenberg and Richter, 1956].

SEISMIC MOMENT, ENERGY, AND MAGNITUDE

$$E = \frac{M_0 \Delta \sigma}{2\mu} \equiv \text{wave energy} \quad (2)$$

Here M_0 is the seismic moment, $\Delta\sigma$ is the static stress drop, μ is the rigidity. After [Orowan, 1960].

Substitute this relation into the equation(1)

$$\log_{10} \left(\frac{M_0 \Delta\sigma}{2\mu} \right) = 1.5M + 11.8. \quad (3)$$

Typical stress drops are $30 \leq \Delta\sigma \leq 60$ bars and typical rigidities are $3.5 \times 10^{11} \leq \mu \leq 7.0 \times 10^{11}$ dynes/cm².

For average values of μ and $\Delta\sigma$, the relation between seismic moment and magnitude is

$$\log_{10} M_0 = 1.5M_w + 16.1. \quad (4)$$

where M_w is the Kanamori wave-energy magnitude. After [Hanks and Kanamori, 1979].

STRESS DROP AND SEISMIC MOMENT

The seismic moment is given by the following relation

$$M_0 = \mu \bar{u} S. \quad (5)$$

Here, S is the area of the rupture in cm², and \bar{u} is the average slip on the rupture.

The static stress drop is given, in general, by the relation

$$\Delta\sigma = c \frac{\mu \bar{u}}{\sqrt{S}} \quad (6)$$

where c is a constant that depends on the geometry of the crack. Combining the previous two equations yields

$$\Delta\sigma = c \frac{M_0}{S^{3/2}}. \quad (7)$$

The above results are after [Aki, 1972].

Some common values for the geometric factor c are

$$\begin{aligned} c &= 7/16\pi^{3/2} && \text{(circular crack)} && \text{[Eshelby, 1952]} \\ &= (4/\pi)(L/W)^{1/2} && \text{(buried strike-slip)} && \text{[Knopoff, 1958]} \\ &= (2/\pi)(L/W)^{1/2} && \text{(surface strike-slip)} && \text{[Knopoff, 1957]} \\ &= (16/3\pi)(L/W)^{1/2} && \text{(buried dip-slip)} && \text{[Starr, 1928].} \end{aligned}$$

Here, $L > W$, where L is the length and W is the width of the rupture. For most earthquake ruptures, $2.4 \leq c \leq 5.0$.

MAGNITUDE, RUPTURE AREA, AND STRESS DROP

Combining several relations of the previous section,

$$M_0 = k_1 \Delta\sigma A^{3/2} \quad (8)$$

where $k_1 = 10^{15}/c$ and A is the rupture area in square kilometers. Taking the \log_{10} of both sides of this equation yields

$$\log_{10} M_0 = \log_{10} \Delta\sigma + \log_{10} k_1 + 1.5 \log_{10} A. \quad (9)$$

Substituting equation (4) into equation (9) and simplifying yields

$$\log_{10} A = M_w + 7.87 - \frac{4}{3} \log_{10} \Delta\sigma - \frac{2}{3} \log_{10} k_1 + \frac{2}{3} \log_{10}(2\mu). \quad (10)$$

Substituting the ranges of μ and k_1 given above, we have

$$\log_{10} A = M_w - \frac{4}{3} \log_{10} \Delta\sigma + 6.3 \pm .1. \quad (11)$$

Here, A is rupture area in square kilometers, M_w is the wave-energy magnitude, and $\Delta\sigma$ is the stress drop in [dyne]/cm².

The general form of the rupture area versus magnitude relation is

$$\log_{10} A = M_w - c_1 \quad (12)$$

Table of Stress drops and c_1 values.

$\Delta\sigma$ (bars)	c_1 value $\pm .25$
0.1	0.5
1.0	1.8
10.0	3.2
30.0	3.8
50.0	4.1
100.0	4.5
200.0	4.9
500.0	5.4
1000.0	5.8

Compare with equation (14) from [Wyss and Brune, 1968] for Parkfield earthquakes, with $\Delta\sigma = 7.3 \pm 1.8$ bar,

$$M = 1.9 \log_{10} A^{1/2} - 6.7$$

which when written in the form above is

$$\log_{10} A = 1.05M - 2.95, \quad (13)$$

which falls within the range expected.

Also compare with [Utsu and Seki, 1954] for (30 bar?) events in Japan

$$\log_{10} A = 1.02M_s - 4.01 \tag{14}$$

where M_s is the surface wave magnitude.

ENERGY VS. MAGNITUDE.

The relationship between energy release and Richter magnitude (M_s) traditionally used by seismologists

$$\log_{10} E = 1.5M + 11.8$$

where E = energy in ergs M == Richter Magnitude. After [Gutenberg and Richter, 1956].

The number 8×10^{20} Ergs is the estimate for the energy released from the Hiroshima bomb as quoted Richter's *Elementary Seismology*. That bomb was a 20 kiloton device, so $1. \times 10^{19}$ Ergs/kiloton is a good number. There are 1000 kilotons per megaton, so a megaton is $1. \times 10^{22}$ Ergs.

The largest earthquake ever recorded was the 20 May 1960 Chilean earthquake, $M_w = 9.5$. This is a 1.1×10^{26} Ergs equivalent, or about 11000 megatons. That's 11 gigtons (billion tons) of TNT!

Note: there are (roughly):

10^{22} ergs/megaton	(million tons of TNT)
10^{19} ergs/kiloton	(thousand tons of TNT)
10^{16} ergs/ton	(ton of TNT)
5×10^{12} ergs/lb	(pound of TNT)

Here is a table of some relative sizes.

Event	Magnitude	energy (Ergs)	Bomb yield (tnt equivalent)
1960 Chile	9.5	$1. \times 10^{26}$	11000 megatons (mt)
2004 Sumatra	9.3	5.6×10^{25}	5600 megatons (mt)
1964 Alaska	9.2	$4. \times 10^{25}$	4000 mt
1906 San. Fr.	8.3	$2. \times 10^{25}$	180 mt
	7.2	$4. \times 10^{22}$	4 mt
	6.5	3.5×10^{21}	360 kilotons (kt)
	6.0	$6. \times 10^{20}$	60 kt
Hiroshima bomb	5.8	$2. \times 10^{20}$	20 kt
	5.5	$1. \times 10^{20}$	10 kt
	5.0	$2. \times 10^{19}$	2 kt
	4.5	3.5×10^{18}	360 tons
	4.0	$6. \times 10^{17}$	60 tons
	3.5	$1. \times 10^{17}$	20 tons
	3.0	$1. \times 10^{16}$	2 tons
	2.5	3.5×10^{15}	700 lb
	2.0	$6. \times 10^{14}$	120 lb
	1.5	$1. \times 10^{14}$	20 lb
	1.0	$1. \times 10^{13}$	4 lb
	0.5	3.5×10^{12}	.7 lb
	0.0	1×10^{11}	.1 lb

EARTHQUAKE-BLAST EQUIVALENCE

If you want to answer the question, "What is the earthquake equivalent of a blast of a certain size?", you can't simply run this table in reverse. The felt size of a particular blast depends on the "seismic efficiency" of the blast source, i.e. how much blast energy is converted to seismic wave energy. Typical air blasts (explosions in air) rarely have a seismic efficiency of greater than 1%, meaning that only 1% of the energy of the blast gets converted into seismic waves. Also, the damage from blasts in air is complicated by the structures and the topography of the land near the blast source.

SMALL EARTHQUAKES AS A SAFETY VALVE?

If you believe that there is a chance that small earthquakes can add up to big ones in such a way that small earthquakes be a "safety valve" earthquake strain, think again. The scaling law for energy says that you get roughly 31.6-fold (the anti-logarithm to the base 10 of 1.5) increase in energy per each magnitude unit. However, statistically, the cumulative number of earthquakes as a function of magnitude is given by

$$\log_{10} N = A - bM,$$

where $b \approx .8$ worldwide. If we use a more liberal value of $b = 1.0$, then, at most, there is only a 10-fold increase in the number of earthquakes as we go down 1 magnitude unit. In short, from equation (1), you need more than 31 earthquakes (the anti-log of 1.5) of a given magnitude to add up to a single earthquake of 1 magnitude greater, however, statistically, you can only expect to get 10 such events. Unless something drastic happens (which is to say $b = 1.5$, in the $\log N$ versus M equation), you will never see small earthquakes acting as a “strain safety valve.” Thus, most of the strain energy is released by the biggest events.

DISCLAIMER

The numbers in the tables above should be taken with a grain of salt. These are rough estimates (within a factor of 2 or 3). A factor of 2 translates into .3 Richter magnitude units.

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