

Common Shot Inversion in 3D

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1 Introduction

This note addresses a simplification of the Kirchhoff common shot inversion formula of Bleistein, et al [2001] (denoted below by **MMSIMI**). In that text, inversion formulas for reflectivities denoted by β and β_1 are expressed in terms of the usual Beylkin determinant. However, a simpler version of the formula that we call β_1 had already appeared in Keho and Beydoun [1988] and was not referenced in our text. Further, Hanitzsch [1997], quoted the earlier formula except for the omission of a factor of $1/2\pi$.

Here, we show how the Keho/Beydoun formula can be derived from information provided in **MMSIMI**. The key is to use the relationship between the common shot Beylkin determinant and the Green's function amplitude for propagation between the image point and the receiver. This relationship, discussed in our text, is due to Najmi [1996].

We will modify the notation of the text to correspond more closely to the notation of Keho and Beydoun and derive their result from ours. Their formula is

$$R(\vec{x}) = 2 \int \frac{\cos \alpha_r}{v_r} \frac{A(\vec{x}_r, \vec{x})}{A(\vec{x}, \vec{x}_s)} D(\vec{x}_s, \vec{x}_r, \vec{x}) dx_r dy_r, \quad (1)$$

with

$$D(\vec{x}_s, \vec{x}_r, \vec{x}) = \frac{1}{2\pi} \int i\omega u(\vec{x}_r, \vec{x}_s, \omega) e^{i\omega\{\tau(\vec{x}_s, \vec{x}) + \tau(\vec{x}, \vec{x}_r)\}} d\omega. \quad (2)$$

Below is a list of the variables and assumptions of these formulas.

1. \vec{x} is the image point. $\vec{x}_s = (x_s, y_s, 0)$ is the source point; $\vec{x}_r = (x_r, y_r, 0)$ is the receiver point.
2. The medium is of constant density with heterogeneous wave speed, $v(\vec{x})$.
3. v_r is the wave speed at the receiver and α_r is the angle that the tangent to the ray from \vec{x}_r to \vec{x} makes with the downward normal to the acquisition surface.
4. $A(\vec{x}, \vec{x}_s)$ and $A(\vec{x}, \vec{x}_r)$ are the ray theoretic or **WKBJ** Green's function amplitudes for the waves from the source or receiver, respectively, to the image point.
5. $u(\vec{x}_r, \vec{x}_s, \omega)$ is the observed data in the frequency domain and $D(\vec{x}_s, \vec{x}_r, \vec{x})$ is the pre-processed (filtered) data of the inversion formula. The same data is used for migration, but with a simplified weight.
6. $\tau(\vec{x}_s, \vec{x})$ and $\tau(\vec{x}_r, \vec{x})$ are the travel times from the source and receiver, respectively, to the image point.

The common shot inversion formula in MMSIMI that is equivalent to this is given by equation (5.2.17) in the text. That result in the current notation is

$$\beta_1(\vec{x}) = \frac{v^2(\vec{x})}{8\pi^2} \int \frac{|h_r(\vec{x}, \vec{\xi})|}{A(\vec{x}_r, \vec{x})A(\vec{x}, \vec{x}_s)} D(\vec{x}_s, \vec{x}_r, \vec{x}) d^2\xi. \quad (3)$$

Here, \vec{x}_r is a function of the two parameters of $\vec{\xi} = (\xi_1, \xi_2)$. In fact, we will consider only the flat upper surface where

$$\vec{x}_r = (x_{r1}, x_{r2}, 0) = (\xi_1, \xi_2, 0).$$

Furthermore, h_r is the Beylkin determinant define in equation (5.2.15) in MMSIMI as

$$h_r(\vec{x}, \vec{\xi}) = \det \begin{bmatrix} \nabla\tau(\vec{x}_r, \vec{x}) \\ \frac{\partial\nabla\tau(\vec{x}_r, \vec{x})}{\partial\xi_1} \\ \frac{\partial\nabla\tau(\vec{x}_r, \vec{x})}{\partial\xi_2} \end{bmatrix}. \quad (4)$$

2 Simplifying h_r

By using a result due to Najmi [1996], the expression for h_r in (4) can be rewritten in terms of the WKB amplitude $A(\vec{x}_r, \vec{x})$. That result was derived in the text for a general upper surface. It is stated in equation (5.3.15) of MMSIMI as

$$h_r(\vec{x}, \vec{\xi}) = \frac{A_r^2(\vec{x}, \vec{\xi})}{K_r(\sigma_1, \sigma_2)} \cdot \frac{\partial\tau}{\partial\sigma_3} \cdot \frac{\partial(q_1, q_2)}{\partial(\sigma_1, \sigma_2)} \cdot \frac{\cos\theta_r\sqrt{g_r}}{q_3 c(\mathbf{x}_r)}. \quad (5)$$

In this equation, the new variables are as follows.

1. $(\sigma_1, \sigma_2, \sigma_3)$ are the parameters that describe the rays from \vec{x}_r to \vec{x} . In particular, σ_3 is the running parameter along the ray, while σ_1 and σ_2 are the initial polar angles of the ray direction at \vec{x}_r .
2. (q_1, q_2, q_3) is a the gradient of the travel time with respect to the variables (x_r, y_r, z_r) evaluated at $z_r = 0$. Thus,

$$\vec{q} = -\nabla\tau(\vec{x}, \vec{x}_r) \Big|_{\vec{x}=\vec{x}_r}.$$

\vec{q} can also be expressed in terms of the initial polar angles as

$$\vec{q} = \frac{1}{v_r} (\cos\sigma_1 \sin\sigma_2, \sin\sigma_1, \sin\sigma_2, \cos\sigma_2). \quad (6)$$

Note that $\sigma_2 = \alpha_r$ in (1).

3. K_r is a constant of the Green's function amplitude A that is specific to the choice of parameters in $\vec{\sigma}$. It is the square of the constant given in equation (E.4.2) in MMSIM1:

$$K_r(\sigma_1, \sigma_2) = \frac{1}{16\pi^2} \frac{\sin \sigma_2}{v_r}. \quad (7)$$

4. g_r is the scaling factor for which

$$\sqrt{g_r} d\xi_1 d\xi_2 = dS,$$

with dS the differential surface area on the acquisition surface at \vec{x}_r . For a flat surface, $g_r = 1$.

We take the derivative of the travel time in (5) from the ray equations; that is,

$$\frac{d\tau}{d\sigma_3} = \frac{1}{v^2(\vec{x})}. \quad (8)$$

The Jacobian of the first two components of \vec{q} with respect to σ_1 and σ_2 appearing in (5) can be directly calculated from equation (6) for \vec{q} :

$$\frac{\partial(q_1, q_2)}{\partial(\sigma_1, \sigma_2)} = \frac{\sin \sigma_2 \cos \sigma_2}{v_r^2}. \quad (9)$$

Finally, we use the third component of (6) for q_3 appearing in (5).

We now use all of the equations below (5) to rewrite that equation as

$$h_r(\vec{x}, \vec{\xi}) = \frac{16\pi^2 A_r^2(\vec{x}, \vec{\xi}) \cos \alpha_r}{v^2(\vec{x}) v_r}. \quad (10)$$

When this result is substituted into (3), the equation for β_1 , the result is the same as the reflectivity R of Kebo and Beydoun [1988] given by (1) and (2).

References

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