Equivalent anisotropy for finely-layered media

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ABSTRACT

Backus averaging over thin, isotropic layers in the earth's subsurface gives anisotropy parameters for an equivalent anisotropic medium. I use Backus averaging to derive equivalent-medium expressions for Thomsen's parameters ϵ and δ and the Alkhalifah and Tsvankin anellipticity parameter η. These three anisotropy parameters are particularly suited for seismic processing applications related to P-wave imaging and AVA analysis. I express these equivalent-medium parameters in terms of density (ρ), P-wave velocity (V_p), and S-wave velocity (V_s) in order to estimate them from well log measurements.

The equivalent-medium value for δ will almost surely be negative for anisotropy of this sort. This constraint is imposed by the (usually positive) sign of the observed correlation between S- wave impedance (ρV_s) and V_s/V_p over a finely-layered sequence. The sign of δ thus distinguishes anisotropy due to fine, isotropic layering from that due to shales, where δ is most often positive. This sign difference points towards shale anisotropy, and away from fine layering, as a reason we often overestimate depth from normal-moveout velocities.

On the other hand, the equivalent-medium value for ϵ can be expected to be positive. As with δ, this sign constraint is not mathematical but physical. More complicated than δ, however, the equivalent ϵ depends on the difference between two correlation coefficients. The first correlation is as stated above for δ, while the statistical relation between V_s and V_p controls the second coefficient.

These discoveries lead one to conclude, qualitatively, that fine layering makes its greatest impact on seismic data in those intervals where the elastic properties are highly correlated. Well log calculations with the Mobil public-domain data produce equivalent-medium η estimates for fine layering that reach at least .05 to .08 over a 600 m interval. I expect that depth migration would be sensitive to the level of anelliptic distortion over that interval.

Key words: Backus averaging, equivalent medium, velocity anisotropy, Thomsen parameters, anellipticity parameter

Introduction

This paper summarizes research that was begun at Conoco, then continued during two visits to CWP, one in July, 1996, the other in March, 1997. Based largely on discussions with a variety of folks during these visits, it is this author's sense that most of the industry associates anisotropy in seismic data with intrinsic anisotropy, due to shale. But the finely-layered nature of well log measurements from around the world begs the question of the importance of fine layering in this respect.

This was the original motivation behind the Backus (1962) investigation. Backus first detailed how to average over fine layering to obtain the stiffness coefficients of an anisotropic medium, now slowly varying over depth, yet equivalent to the fine layering in all respects of the seismic response. These results hold at seismic wavelengths much greater than that of the layering. We refer to this anisotropic medium as the *equivalent medium.*

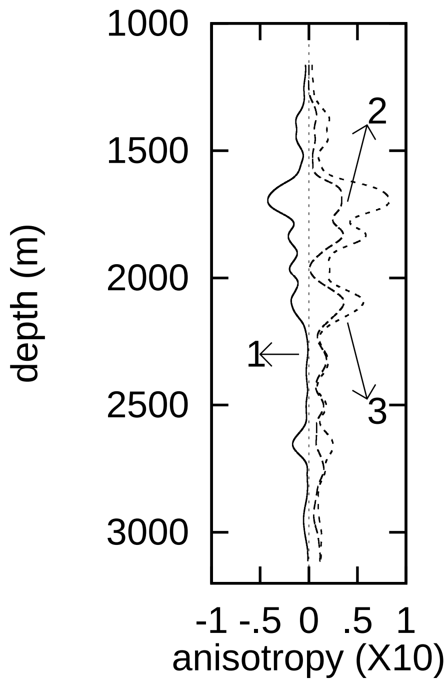

Figure 1. Backus-average estimates of (1) δ, (2) ϵ and (3) η from well logs in the Mobil data set.

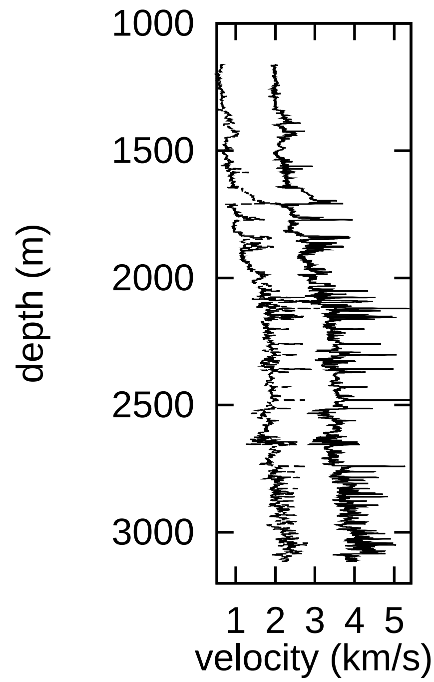

Figure 2. Measured P- and S-wave well log velocities in the Mobil well.

Here I employ the same *Backus averaging* to write down analytical expressions that characterize the equivalent, anisotropic medium in terms of P- and S-wave velocity (V_p and V_s) and density ρ in fine, isotropic layers. These are the quantities available from well log measurements. Concerned mainly with results that bear directly on P-wave data processing, I have chosen to calculate Backus averages for the anisotropy parameters ϵ and δ, introduced in Thomsen (1986), and η (Alkhalifah & Tsvankin, 1995). These three parameters are particularly suited to processing applications. Tsvankin (1996) compiles an extensive applications list. I mention only a few.

The work of Thomsen (1993) and Rüger (1995) shows that the anisotropy parameter δ is important to the AVA response, even at small reflection angles. In an anisotropic subsurface, δ is also required for accurate time-to-depth conversion from normal moveout velocities (Banik,1984; Hake et al.,1984; Thomsen,1986; Tsvankin,1996). Normally defined in terms of stiffness coefficients (Appendix A) a definition in terms of velocities,

$$\delta \equiv \frac{1}{2}\frac{V_n^2 - V_z^2}{V_z^2}, \qquad (1)$$

makes this requirement clear. In the context of horizontal, finely-layered media V_z denotes the equivalent vertical P-wave speed and V_n denotes normal moveout velocity from a horizontal reflecting interface beneath this layering. For non-zero δ the velocity V_z needed for depth conversion differs from V_n, obtained from moveout in traveltimes.

Though many think of the parameter ϵ as "the anisotropy", it is perhaps the least interesting of these three anisotropy parameters. Again in terms of velocities,

$$\epsilon \equiv \frac{1}{2}\frac{V_x^2 - V_z^2}{V_z^2}. \qquad (2)$$

For our purposes V_x denotes the long-wavelength P-wave velocity parallel to fine layering. Thomsen (1993) and Rüger (1995) demonstrate that ϵ contributes to the AVA response at large angles.

More important for seismic processing, however, ϵ enters directly into the definition for the anellipticity parameter, η. That is, Alkhalifah and Tsvankin (1995) introduced η as

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}. \qquad (3)$$

Considering together equations (1) and (2), one can also write this definition in terms of velocities as

$$\eta = \frac{1}{2}\frac{V_x^2 - V_n^2}{V_n^2}. \qquad (4)$$

The P-wave slowness surface is elliptical when $V_x = V_n$ (cf., Daley and Hron,1979; Thomsen,1986).

η is a key parameter for anisotropic imaging applications. Alkhalifah and Tsvankin (1995) demonstrate its

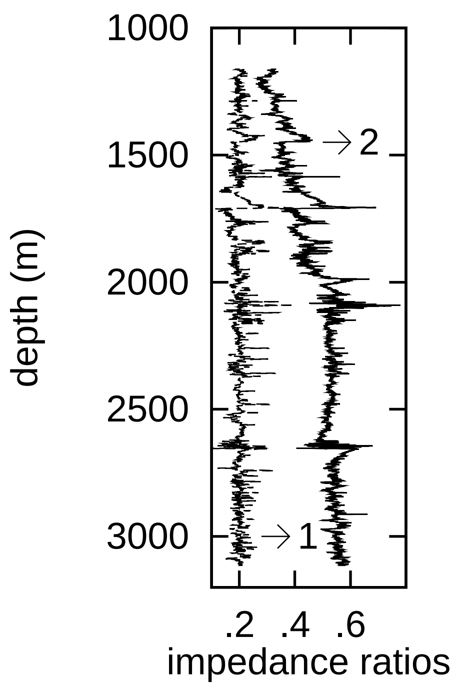

Figure 3. Measured impedance ratios in the Mobil well. Curve (1) is a normalized S-wave impedance $.2\,\rho V_s/\langle\rho V_s\rangle$ and (2) is V_s/V_p.

role in long-spread (non-hyperbolic) moveout and pre- and poststack time migration. Anderson (1996) demonstrates the sensitivity of the DMO operator to small deviations from ellipticity, given by $\eta = 0$.

Equivalent-Medium Parameters

Figure 1 displays these three anisotropy parameters over depth, based on Backus averaging with the ρ, V_s, and V_p logs from the Mobil public-domain data set (Keys, 1994). Figure 2 shows the V_s and V_p logs. Note that while ϵ is positive for all depths, δ is negative. In the sequel I rewrite the righthand sides of equations (1) and (2) to illuminate the physical characteristics of fine layering that determine this outcome. I evaluate through equation (3) the consequences for η. Also evident in Figure 1, η is everywhere positive and takes on the largest magnitude of the three parameters at all depths.

Equivalent δ

Several of the averaging formulas for equivalent-medium stiffness coefficients are complicated and are relegated to Appendix A. They also may be found in their original form in Backus (1962). For a simple example and to establish notation,

$$V_z^2 = \langle\rho\rangle^{-1}\left\langle\frac{1}{\rho V_p^2}\right\rangle^{-1} \tag{5}$$

gives the equivalent vertical speed through isotropic, fine layers. The operator $\langle\rangle$ denotes a vertical average over depth. Substitution of those Backus averages pertinent to δ gives, from Appendix A,

$$\delta = -2\,\frac{\left\langle 1-\frac{V_s^2}{V_p^2}\right\rangle}{\left[1-\overline{\frac{V_s^2}{V_p^2}}\right]}\left[\left\langle\frac{V_s^2}{V_p^2}\right\rangle - \overline{\frac{V_s^2}{V_p^2}}\right]. \tag{6}$$

$$\overline{\frac{V_s^2}{V_p^2}} \equiv \frac{\left\langle\frac{1}{\rho V_p^2}\right\rangle}{\left\langle\frac{1}{\rho V_s^2}\right\rangle} \tag{7}$$

symbolizes the equivalent V_s/V_p (squared) in the vertical direction, normal to fine horizontal layering. I also used the identity

$$\langle 1-x\rangle \equiv 1-\langle x\rangle \tag{8}$$

in deriving equation (6).

Though many forms are possible, equation (6) is convenient for analyzing the sign of δ for only the sign of the bracketed difference on the right is in question. Equation (6) says that the sign of δ varies only with the difference between two V_s/V_p ratios, namely the root-mean-square value over depth and the equivalent-medium value in the vertical direction.

Let us assume that the root-mean-square ratio is greater than the equivalent-medium ratio. Then, using equation (7),

$$\left\langle\frac{1}{\rho V_s^2}\right\rangle\left\langle\frac{V_s^2}{V_p^2}\right\rangle - \left\langle\frac{1}{\rho V_p^2}\right\rangle > 0. \tag{9}$$

Careful inspection identifies the lefthand side of inequality (9) with the statistical correlation (Papoulis, 1991) between $1/\rho V_s^2$ and V_s^2/V_p^2. To indicate the correlation between two parameters x and y I introduce the notation

$$\langle x, y\rangle \equiv \langle xy\rangle - \langle x\rangle\langle y\rangle, \tag{10}$$

and inequality (9) becomes

$$\left\langle\frac{1}{\rho V_s^2}\,,\,\frac{V_s^2}{V_p^2}\right\rangle < 0. \tag{11}$$

That is to say, when $1/\rho V_s^2$ and V_s^2/V_p^2 are anti-correlated over depth, the root-mean-square V_s/V_p ratio exceeds the equivalent-medium ratio and, from equation (6), δ is negative. It is also correct, and simpler, to say that δ will always be negative when ρV_s and V_s/V_p are correlated positively.

Figure 3 demonstrates visually a generally (though not point-for-point) positive correlation between ρV_s and V_s/V_p through the section penetrated by the Mobil well.

But the Backus operators average over depth. In particular, the δ curve in Figure 1 was calculated with the averaging operators in equation (6) applied over 100 m sliding windows. I chose 100 m with the intent that a 30 Hz signal should satisfy (for the velocities measured in this well) Backus' long-wavelength approximation. Over this integration length S-wave impedance and V_s/V_p always correlate positively in this well, making δ negative at all depths. Given a strong correlation, as in the depth interval from 1600 m to 2200 m, δ takes on relatively large magnitudes.

Crossplots of well log data by Eastwood and Castagna (1986) document positive correlations between V_s and V_s/V_p in a wide variety of rocks. Though several of their carbonate examples demonstrate this correlation it appears to be most consistent in their clastics examples. Ignoring the contribution of density in inequality (11), $\delta < 0$ holds for these data as well.

I digress just briefly to underline at this point a fundamental assumption – isotropy – that we make in interpreting these well log data. The reader should be aware that Thomsen (1986) incorporates a comprehensive table with measured anisotropy parameters, including δ, from many sources and for many rock types. I count 20 shale measurements in this table, with only three having $\delta < 0$. This observation in fact nominates δ as a discriminator between anisotropy due to shale or due to fine, isotropic layering.

Finally, recognizing with hindsight the statistical correlation structure in equation (6) one may rewrite it as

$$\delta = -2\left\langle \frac{1}{\rho V_s^2}\right\rangle^{-1} \frac{\left\langle 1-\frac{V_s^2}{V_p^2}\right\rangle}{\left[1-\frac{V_s^2}{V_p^2}\right]} \left\langle \frac{1}{\rho V_s^2}\, ,\, 1-\frac{V_s^2}{V_p^2}\right\rangle \tag{12}$$

to make the correlation explicit. Besides equation (10),

$$\langle x, y\rangle \equiv -\langle x, 1-y\rangle \tag{13}$$

was used to proceed from equation (6) to equation (12). Having posited that inequality (11) characterizes most finely-layered isotropic sequences it follows that

$$\left\langle \frac{1}{\rho V_s^2}\, ,\, 1-\frac{V_s^2}{V_p^2}\right\rangle > 0 \tag{14}$$

in equation (12).

Equivalent ϵ

Referring once again to Appendix A, I form the Backus averages for V_x and V_z in equation (2) to obtain

$$\epsilon = 2\left[\left\langle \frac{1}{\rho V_p^2}\right\rangle \left\langle \rho V_s^2\, ,\, 1-\frac{V_s^2}{V_p^2}\right\rangle - \left\langle 1-\frac{V_s^2}{V_p^2}\right\rangle \left\langle \rho V_s^2\, ,\, \frac{1}{\rho V_p^2}\right\rangle\right]. \tag{15}$$

Also refer to equations (8), (10), and (13) to recover this particular form.

This form for equation (15) reveals that two, independent correlations determine the equivalent ϵ for fine layering. The sign of the first revolves around the relation between ρV_s and V_s/V_p, as before with δ. From inequality (14)

$$\left\langle \rho V_s^2\, ,\, 1-\frac{V_s^2}{V_p^2}\right\rangle < 0. \tag{16}$$

Based on experience with field and laboratory data, folklore and who knows what else, geophysicists take it for granted that wave speed parallel to fine layers always exceeds the speed normal to layering. In view of inequality (16) the second correlation coefficient of equation (15) must therefore be negative.

Indeed, Figure 2 establishes a striking similarity between the V_s and V_p curves from the Mobil well. (Only a scale factor, we can ignore ρ for the moment.) Accordingly, the second correlation coefficient of equation (15) satisfies

$$\left\langle \rho V_s^2\, ,\, \frac{1}{\rho V_p^2}\right\rangle < 0. \tag{17}$$

The Pickett (1963) velocity data record the same correlation between V_s and V_p. The Castagna mudrock equation (Castagna *et al.*, 1985) honors it also.

Driven by their difference, always positive in the Mobil well, the two correlations of equation (15) produce a positive equivalent ϵ in Figure 1 over all depths. As before with δ I chose 100 m operators. Of course, $\epsilon > 0$ also imposes a condition on the relative magnitudes, not just signs, of these two correlations. Not to pursue this here, one normalizes the correlations in equation (15) to be dimensionless, like ϵ, in order to obtain an interpretable result.

I wondered at the outset of this analysis if it is possible, given all the experimental evidence, for the equivalent ϵ to be negative at the dominant frequencies we record. It is now clear from equation (15) that, though mathematically possible, the strong correlation over depth that Nature imposes between V_s and V_p makes $\epsilon < 0$ unlikely.

Equivalent η

Having made the equivalent-medium case for $\epsilon > 0$, $\delta < 0$, equation (3) requires $\eta > 0$. We can also turn to Berryman (1979) for the latter inequality. He derives essentially this result regardless of the sign of any correlation coefficients. As far as the author is aware, however, the physical correlations that determine the sign of ϵ and δ in fine layering have not been identified until now.

These discoveries lead one to conclude, qualitatively, that fine layering makes its greatest impact on seismic data in those intervals where the elastic properties are highly correlated. Figure 1 shows just such an interval between 1600 m and 2200 m. Refer back to inequalities (14), (16) and (17) for quantitative statements about the correlated properties.

In this interval large negative δ's magnify η through both the numerator and denominator of equation (3). ϵ, also attaining its largest values, simultaneously enlarges η through the numerator.

Moreover, the η estimates in Figure 1 should probably be viewed as lower bounds. Sams (1995) shows that well log measurements may underestimate the equivalent ϵ value by more than 50 % due to limited resolution on fine layers. Backus (1962) anticipated this shortcoming of well logs.

Relevance for Seismic Processing

As far as specific processing applications, I expect depth migration to exhibit sensitivity to anelliptic distortion from intervals of the sort identified in Figure 1. After all, depth migration requires parameter estimates over limited depth intervals. Judging from the work of Alkhalifah (1995) and Uzcategui (1995), the η values sustained over that 600 m interval of the Mobil well would locally create significant anellipticity in the wavefield. Anisotropic depth migration should therefore produce a superior focus through such an interval in circumstances where different reflection dips occur at the same position in the image.

The η estimates required for anisotropic time migration and DMO, on the other hand, represent an effective parameter, integrated over all sources of anellipticity from the surface down to the output image time. This integration greatly diminishes the sensitivity of time imaging to anellipticity over a limited interval.

Evidently due to the same integration effect on normal-moveout velocity, equivalent anisotropy from finely-layered, isotropic intervals seems to exert little effect on time-to-depth conversion. Case in point, we generally compute depths from moveout velocities that exceed the true depth measured in wells. Equation (1) identifies this error with $\delta > 0$, pointing instead to shale-induced anisotropy as the source of these misties.

Local reflectivity, however, involves no such integration. One can expect the AVA response at an interface beneath a finely-layered, isotropic interval ($\delta < 0$) to differ from that below most shale intervals ($\delta > 0$). Important for AVA applications, this sign reversal in δ reverses the contribution of anisotropy in the AVA gradient term (Thomsen,1993; Rüger,1995).

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APPENDIX A: Backus averages for isotropic layering

We shall earmark Appendix A as a collection area for the definitions and results required to derive equations (6), (12), and (15). Pertinent to P-wave anisotropy, Thomsen (1986) defines

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}} \tag{A1}$$

and

$$\delta \equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}. \tag{A2}$$

We will also require

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta} \tag{A3}$$

from Alkhalifah and Tsvankin (1995).

Intent on substituting out these stiffness coefficients I gather the Backus (1962) results:

$$c_{33} = \left\langle \frac{1}{\rho V_p^2} \right\rangle^{-1}, \tag{A4}$$

$$c_{13} = c_{33} \left[1 - 2 \left\langle \frac{V_s^2}{V_p^2} \right\rangle \right], \tag{A5}$$

$$c_{66} = \left\langle \rho V_s^2 \right\rangle, \tag{A6}$$

and

$$c_{11} = 4 \left[c_{66} - \left\langle \left(\rho V_s^2 \right) \left(\frac{V_s^2}{V_p^2} \right) \right\rangle \right] + \frac{c_{13}^2}{c_{33}}. \tag{A7}$$

Last of all, one requires an equivalent-medium density in order to form velocities from these stiffnesses. Backus (1962) shows that the equivalent density is simply $\langle \rho \rangle$.