Synthetic aperture Green’s function retrieval

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Equipartitioning

$$\langle u_A \otimes u_B \rangle \propto G(r_A, r_B, t) - G(r_A, r_B, -t)$$
Stationary phase
One source

\[ |t_{SA} - t_{SB} | < t_{AB} \]
Rotating array
Rotating array
Rotating array
Rotating array
Rotating array

$S$
Rotating array
Rotating array
Rotating array

$S$
Stationary phase

\[ S \]
One noise source

\[ u(\mathbf{r}, \omega) = e^{ik\hat{n} \cdot \mathbf{r}} S(\omega) \]
Normalized cross correlation

\[
C_{12} = \frac{\langle u(\mathbf{r}_2)u^*(\mathbf{r}_1) \rangle}{\langle |u(\mathbf{r}_2)| |u(\mathbf{r}_1)| \rangle} = e^{ik\hat{n} \cdot \mathbf{R}_2}
\]
Noise does not propagate between stations!

\[ C_{12} = e^{i\hat{n} \cdot R_2} \]
Normalized cross correlation

\[ C_{12} = e^{i\mathbf{k} \cdot \mathbf{R}_2} \quad C_{13} = e^{i\mathbf{k} \cdot \mathbf{R}_3} \]
Synthetic inter-station direction

\[ R_0 \hat{p}(\varphi) = a(\varphi)R_2 + b(\varphi)R_3 \]
Synthetic inter-station direction

\[ R_0 \hat{p}(\varphi) = a(\varphi)R_2 + b(\varphi)R_3 \]

\[
\begin{pmatrix}
C_{12}^{a(\varphi)} & C_{13}^{b(\varphi)}
\end{pmatrix} = e^{ik\hat{n} \cdot (a(\varphi)R_2 + b(\varphi)R_3)} = e^{ikR_0 \hat{n} \cdot \hat{p}(\varphi)}
\]
Synthetic inter-station direction

\[ R_0 \hat{p}(\varphi) = a(\varphi)R_2 + b(\varphi)R_3 \]

\[
\begin{pmatrix}
C_{12}
\end{pmatrix}^{a(\varphi)} \begin{pmatrix}
C_{13}
\end{pmatrix}^{b(\varphi)} = e^{ik\hat{n} \cdot (a(\varphi)R_2 + b(\varphi)R_3)} = e^{ikR_0 \hat{n} \cdot \hat{p}(\varphi)}
\]

plane wave propagating over distance \( R_0 \hat{p}(\varphi) \)
One incoming wave

(b)
Sum over $\varphi$

![Graph showing the sum over $\varphi$](image)

- **(b)**
  - Graph with labeled axes: $\varphi$ (degree) on the y-axis and Lag-time (s) on the x-axis.
- **(c)**
  - Close-up view of the graph showing peaks at specific lag-times.
Integration in frequency domain

\[ \int_{0}^{2\pi} (C_{12})^{a(\varphi)} (C_{13})^{b(\varphi)} d\varphi = 2\pi J_0(kR_0) \]
Integration in frequency domain

(a)
More incoming waves

(d) Bessel reconstruction

Wavenumber (1/km)
More incoming waves

\[ k < \frac{\pi}{2R_0 (\Delta \theta)^2} \]
You have something you want to say?

Go ahead, I'm all ears!