Estimating \( V_p/V_s \) ratios using smooth dynamic image warping

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SUMMARY

Interval \( V_p/V_s \) ratios can be estimated from derivatives of vertical shifts that align reflections in PP and PS images. This alignment, however, is nontrivial because features in PS images are not simply time-shifted versions of features in PP images. To align PP and PS images we use a smooth dynamic image warping algorithm that can be accurate with respect to problems unrelated to time shifts, such as differences in noise and reflection waveforms. To optimize accuracy of estimated time shifts and \( V_p/V_s \) ratios, we automatically construct a coarse lattice of points located on reflections with high amplitudes, and then estimate time shifts at only those points. By adjusting the coarseness of the lattice, we trade off resolution of changes in \( V_p/V_s \) with increased accuracy in \( V_p/V_s \) estimates. By processing 3D PP and PS images we learned that our estimates of \( V_p/V_s \) cannot be obtained by simply smoothing estimates obtained using other methods.

INTRODUCTION

The vertical shifts that align PP and PS images can be used to quantify the ratio of P- and S-wave velocities (\( V_p/V_s \)), an important attribute that is the target of much P-S analysis (Stewart et al., 2003). However, PP and PS image registration is complicated by differences in noise and reflection waveforms between the two images, and thus this registration is often done manually. Many techniques have nevertheless been developed to automate the registration process and to estimate \( V_p/V_s \) ratios (Gaiser, 1996; Fomel and Backus, 2003; Nickel and Sonneland, 2004; Yuan et al., 2008; Liang and Hale, 2012).

Liang and Hale (2012) used dynamic image warping (Hale, 2013) for this purpose. Dynamic image warping computes vertical time shifts that align events in PP and PS images; these shifts are a globally optimal solution of a minimization problem with many possible local minima. The dynamic warping algorithm also honors bounds on time shifts and on the rate at which time shifts vary. The method discussed here is a modified dynamic warping algorithm that estimates changes in time shifts more accurately than the previous method. For PP and PS image registration, time shifts are related to integrals of \( V_p/V_s \) ratios, and thus, are often smoothly varying. Our modified algorithm exploits this fact. We show in application to 3D PP and PS images that our new method is more robust and accurate in automatic image registration and estimation of \( V_p/V_s \) ratios.

THEORY

Let \( t_{pp} \) denote the PP traveltime for a reflector apparent in a PP image, and \( t_{ps} \) denote the PS traveltime for the same reflector in the corresponding PS image. For each \( t_{pp} \), we have a corresponding \( t_{ps} \) defined by the function

\[
t_{ps}(t_{pp}) = t_{pp} + u(t_{pp}).
\]

where \( u(t_{pp}) \) denotes the time shift, which varies with \( t_{pp} \). To compute the time shift function \( u(t_{pp}) \) we use dynamic image warping (Hale, 2013), but modified to estimate time shifts on a coarse sampling grid. These time shifts are related to interval \( V_p/V_s \) ratios

\[
\gamma(t_{pp}) = 1 + 2 \frac{du(t_{pp})}{dt_{pp}}.
\]

A key feature of dynamic warping is that it honors specified bounds on both values and derivatives of the time shift function \( u(t_{pp}) \). Let \( u_l \) and \( u_u \) denote lower and upper bounds on time shifts, and \( r_l \) and \( r_u \) denote lower and upper bounds on the derivatives of time shifts, so that

\[
u_l \leq u(t_{pp}) \leq u_u, \quad r_l \leq \frac{du(t_{pp})}{dt_{pp}} \leq r_u.
\]

To specify a lower bound \( u_l \), we use the fact that \( V_p \geq V_s \), which implies that \( t_{ps} \geq t_{pp} \) and that all shifts \( u(t_{pp}) \) must be non-negative, so that \( u_l = 0 \). A precise upper bound \( u_u \) is unnecessary. We need only ensure that we do not set this bound too low, so that the correct shifts \( u(t_{pp}) \) will not exceed \( u_u \).

SMOOTH DYNAMIC IMAGE WARping

In the dynamic image warping (DIW) algorithm described by Hale (2013), the computed time shifts \( u(x,y,t_{pp}) \) are integer multiples of the time sampling interval \( \Delta t \). DIW is used by Liang and Hale (2012) to register PP and PS images and to estimate \( V_p/V_s \) ratios. Because the shifts are integer multiples of \( \Delta t \), the number of possible values for \( du(t_{pp})/dt_{pp} \) is small; thus from equation 2, the number of possible \( \gamma(x,y,t_{pp}) \) values is small. To increase the number of possible changes in time shifts, and hence the number of possible values for \( V_p/V_s \), Liang and Hale (2012) smooth the computed shifts.

Here, we instead use our modified smooth dynamic image warping (SDIW) algorithm to directly compute a smooth and more accurate time-shift function \( u(x,y,t_{pp}) \). With the modified algorithm, we decrease the temporal and spatial resolution with which we estimate \( V_p/V_s \). However, by forcing time shifts to vary smoothly between samples in our coarse grid, we increase the accuracy of \( V_p/V_s \) estimates. Such tradeoffs between resolution and accuracy are common in signal processing.

Figure 1 shows 1D time-shift curves \( u(t_{pp}) \) extracted from 3D time shifts \( u(x,y,t_{pp}) \) computed from DIW (black curve) and SDIW (blue curve). Note the roughness of the black curve...
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Figure 1: The black curve shows rough time shifts computed from DIW (Hale, 2013) and the dashed red curve shows these shifts after smoothing. The blue curve shows time shifts computed using our smooth warping (SDIW) algorithm.

which results from the fact that DIW computes time shifts at every time sample. In fact, the black time-shift curve contains only two slopes, 0 and 1, and thus from equation 2, represents only two $V_p/V_s$ values, 1 and 3. The dashed red curve in Figure 1 shows the black time-shift curve after smoothing. The blue curve in Figure 1 shows the time shifts computed using SDIW. Time shifts from SDIW are computed only at specific subsampled locations and then are interpolated at all other locations. The subsampling greatly increases the number of possible changes in shift within subsampled intervals, at the cost of decreasing resolution of changes in shifts within those intervals.

Note the large discrepancy in the blue and red time-shift curves in Figure 1 between 0 and 0.6 s. To see that SDIW is more accurate, compare the time slices at 0.38 s shown in Figure 2. This figure compares time slices from the PP image (Figure 2a) with the PS image warped using DIW time shifts that have been smoothed (Figure 2b), and the PS image warped using SDIW time shifts (Figure 2c). The warped PS image in Figure 2c is a better match to the PP image in Figure 2a.

Figure 3 shows the interval $V_p/V_s$ ratios estimated from DIW time shifts that have been smoothed. Interval $V_p/V_s$ ratios estimated from SDIW time shifts are shown in Figure 5d. The two images exhibit similar trends of high or low $V_p/V_s$; Figure 3, however, shows a high level of detail that is unwarranted given the resolution of seismic reflections and differences in noise and reflection waveforms between the PP and PS images. Moreover, Figure 2 shows that attempting to resolve changes in time shifts at every image sample can result in misalignment of reflectors.

COARSE SAMPLING LOCATIONS

Coarse sampling improves the accuracy of computed shifts, but the placement of our coarse lattice of sample points is important. The SDIW algorithm requires the same number of coarse-grid time samples for all inline and crossline sample locations, and therefore, one simple approach is to use a regular grid. However, we can estimate shifts best at locations of strong reflectors.

Because reflectors in seismic images are generally not horizon-

tal, aligning coarse-grid samples with strong reflectors throughout an image is not trivial. However, when images are not structurally complex, we make reflectors horizontal using seismic image flattening (Parks et al., 2008; Parks, 2010). In the flattened image space we then stack the envelopes of all image traces and select times corresponding to the strongest reflectors in the flattened 3D image. We select a coarse lattice of points whose coordinates are mapped back to the original image space to obtain a reflector-aligned 3D lattice on which to compute time shifts using SDIW.

We compare a simple regular-interval grid shown in Figure 5a, with a reflector-aligned coarse grid in Figure 5b. Both grids have nearly uniform sampling in the inline and crossline directions. Interval $V_p/V_s$ ratios estimated from the regular-interval grid and the reflector-aligned grid are shown in Figures 5c
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Figure 3: Interval $V_p/V_s$ ratios estimated from rough DIW time shifts that have been smoothed.

and 5d, respectively. Note the step in $V_p/V_s$ at 1.7 and 1.8 s for the layers in the crossline slice of Figure 5c compared to the more continuous layer with low $V_p/V_s$ in Figure 5d. This step compensates somewhat for the less appropriate regular-interval coarse grid sampling.

INTERPOLATION

The SDIW algorithm computes time shifts only at specified subsampled locations, but to align PP and PS images we need time shifts at every image location. We first interpolate laterally using a simple bilinear interpolation. For vertical interpolation of time shifts, we should consider the effect of the interpolation on the $V_p/V_s$ ratios we seek to estimate.

Time shifts computed by SDIW are piecewise-linear; therefore the most consistent way to interpolate time shifts vertically is with linear interpolation. Figure 4 shows interval $V_p/V_s$ ratios estimated from linearly interpolated time shifts. These $V_p/V_s$ are piecewise-constant between subsampled time locations. When smoother estimates are required, we can interpolate time shifts using a piecewise-cubic interpolation method as in Figure 5c and 5d. However, in SDIW we compute time shifts at only coarse sample locations, and thus truly know changes in shift only between those locations.

CONCLUSION

We showed that a smooth dynamic image warping algorithm can be used to directly compute time shifts that register 3D PP and PS images with increased accuracy compared to an alternative but similar method. The SDIW algorithm computes time shifts on a coarse lattice of subsampled locations in a PP image, from which we interpolate time shifts at all image sample locations. We aligned the coarse lattice with strong reflectors in the PP image, further increasing accuracy of computed changes in time shifts, and thus estimated $V_p/V_s$ ratios.

Our estimated $V_p/V_s$ ratios do not exhibit fine vertical resolution, because of a fundamental tradeoff between accuracy and resolution. We showed that trying to resolve changes in time shifts at every image location yields less accuracy. Furthermore, smoothing rough time shifts did not improve accuracy and did not yield the time shifts computed by our smooth dynamic image warping algorithm. Because of inevitable differences in noise and reflection waveforms between PP and PS images, fine temporal resolution of changes in time shifts is infeasible.

Lastly, an additional benefit of smooth dynamic image warping is that it improves computational efficiency and dramatically reduces memory requirements, especially for 3D images.

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Figure 5: The regular-interval coarse grid (a) indicated by the red dots has minimum intervals of 500 m in the inline and crossline directions and 160 ms in time. The reflector-aligned coarse grid (b) has the same minimum intervals of 500 m in the inline and crossline directions, but is aligned with strong reflectors in the PP image, and are also a minimum of 100 ms apart. $V_p/V_s$ estimates for the reflector-aligned grid (d) are more continuous laterally than those for the regular grid (c).
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REFERENCES


