Role of the inhomogeneity angle in anisotropic attenuation analysis

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SUMMARY

The inhomogeneity angle (the angle between the real and imaginary parts of the wave vector) is seldom taken into account in estimating attenuation coefficients from seismic data. Wave propagation through the subsurface, however, can result in relatively large inhomogeneity angles ξ, especially for models with significant attenuation contrasts across layer boundaries. Here, we study the influence of the angle ξ on phase and group attenuation in arbitrarily anisotropic media using the first-order perturbation theory verified by exact numerical modeling.

Application of the spectral-ratio method to transmitted or reflected waves yields the normalized group attenuation coefficient $\mathcal{A}_g$, which is responsible for the amplitude decay along seismic rays. Our analytic solutions show that unless the inhomogeneity angle is uncommonly large, the coefficient $\mathcal{A}_g$ is close to the normalized *phase* attenuation coefficient $\mathcal{A}$ computed for $\xi = 0^\circ$ ($\mathcal{A}|_{\xi=0^\circ}$). The coefficient $\mathcal{A}|_{\xi=0^\circ}$ can be inverted directly for the attenuation-anisotropy parameters, so no knowledge of the inhomogeneity angle is required for attenuation analysis of seismic data. This conclusion remains valid even for high attenuation with the quality factor Q less than 10 and strong velocity and attenuation anisotropy.

INTRODUCTION

For a plane wave propagating in attenuative media, the direction of maximum attenuation (determined by the attenuation vector $\mathbf{k}^I$) can deviate from the propagation vector $\mathbf{k}^R$ (Figure 1). The angle ξ between the real and imaginary parts of the wave vector is called the "inhomogeneity angle." When $\xi = 0^\circ$, the plane wave is often characterized as "homogeneous;" when $\xi \neq 0^\circ$, the wave is called "inhomogeneous." For plane-wave propagation, ξ represents a free parameter except for certain "forbidden directions" where solutions of the wave equation do not exist (Krebes & Le, 1994; Carcione & Cavallini, 1995; Červený & Pšenčík, 2005a,b).

The influence of the inhomogeneity angle on point-source radiation in homogeneous media is relatively weak, unless the model is anomalously attenuative and anisotropic (Zhu, 2006; Vavryčuk, 2007). During wave propagation in layered media, however, the angle ξ can attain significant values. For the model in Figure 2, the wave vector in the elastic cap rock is real, while that in the attenuative reservoir is complex. Because the projections of the incident (real) and transmitted (complex) wave vectors onto the interface have to be the same according to Snell's law, the imaginary part $\mathbf{k}^I$ of the wave vector in the reservoir is orthogonal to the interface. This implies that the inhomogeneity angle of the transmitted wave is equal to the transmission angle θ_T, which can reach 90°. Although the angle ξ can be significant, its estimation from seismic data is extremely difficult.

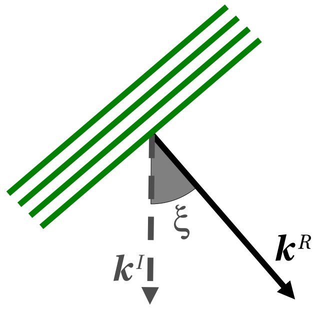

Fig. 1: Plane wave with a nonzero inhomogeneity angle ξ. $\mathbf{k}^R$ and $\mathbf{k}^I$ are the real and imaginary parts of the wave vector, respectively. The wave propagates in the direction $\mathbf{k}^R$ (perpendicular to the planes of constant phase) and attenuates most rapidly in the direction $\mathbf{k}^I$.

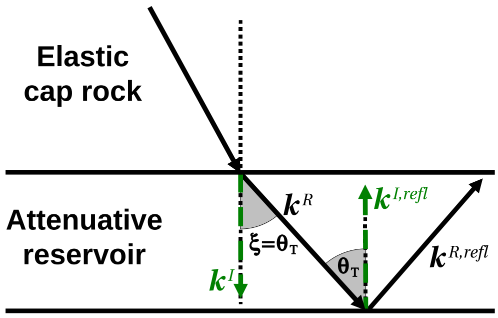

Fig. 2: Illustration of the reflection/transmission problem at the interface between a purely elastic cap rock and an attenuative reservoir. $\mathbf{k}^R$ and $\mathbf{k}^I$ are the real and imaginary parts of the wave vector of the transmitted wave, while $\mathbf{k}^{R,refl}$ and $\mathbf{k}^{I,refl}$ correspond to the reflected wave.

Here, we present closed-form linearized expressions for group and phase attenuation in arbitrarily anisotropic media, which provide useful physical insight into the influence of the angle ξ.

PHASE VS. GROUP ATTENUATION

The ratio $|\mathbf{k}^I|/|\mathbf{k}^R| = k^I/k^R)$ yields the phase attenuation per wavelength, which is called the normalized phase attenuation coefficient $\mathcal{A}$ (Zhu & Tsvankin, 2006). For a nonzero inhomogeneity angle ξ, the coefficient $\mathcal{A}$ for a plane wave determines attenuation along the vector $\mathbf{k}^I$ rather than $\mathbf{k}^R$. In seismic data processing, attenuation is estimated along the raypath, which deviates from the phase direction $\mathbf{k}^R$. The measure of attenuation obtained from seismic data is $\mathcal{A}_g = k_g^I/k_g^R$ (Behura & Tsvankin, 2008) where k_g^I is the group attenuation coefficient and $k_g^R = \omega/V_g$ (V_g is the group velocity).

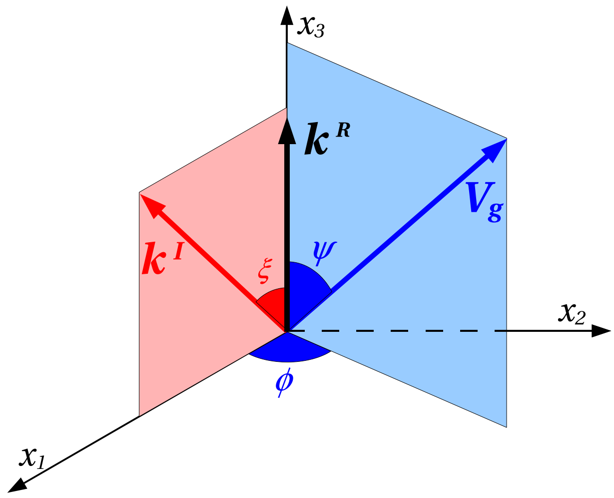

Fig. 3: Plane wave propagating along the coordinate axis x_3 in an anisotropic attenuative medium. The group angle ψ is the deviation of the group-velocity vector $\mathbf{V}_g$ from the real part $\mathbf{k}^R$ of the wave vector.

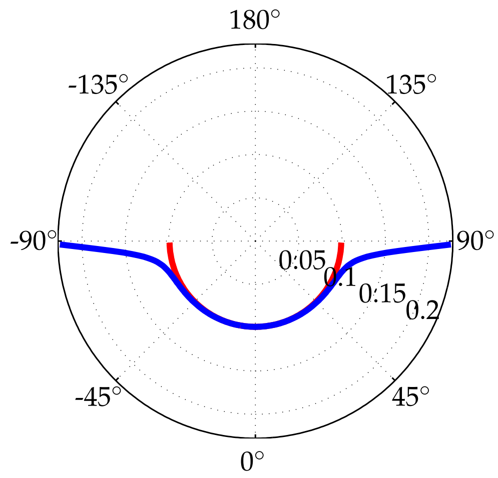

Fig. 4: Exact P-wave coefficients $\mathcal{A}|_{\xi=0^\circ}$ (red curve) and $\mathcal{A}_g$ (blue) in strongly attenuative isotropic media as a function of the inhomogeneity angle ξ (numbers on the perimeter). The P-wave quality factor is $Q_P = 5$.

The coefficient k_g^I can be found by projecting the phase attenuation vector $\mathbf{k}^I$ onto the group direction:

$$k_g^I = \frac{1}{V_g}\left(\mathbf{k}^I \cdot \mathbf{V}_g\right) = k^I(\cos\xi\cos\psi + \sin\xi\sin\psi\cos\phi)\,, \quad (1)$$

where ψ is the angle between $\mathbf{k}^R$ and $\mathbf{V}_g$ (group angle) and ϕ is the azimuth of $\mathbf{V}_g$ with respect to the plane formed by $\mathbf{k}^R$ and $\mathbf{k}^I$ (Figure 3). If the vectors $\mathbf{V}_g$, $\mathbf{k}^R$, and $\mathbf{k}^I$ lie in the same plane (i.e., $\phi = 0$), k_g^I is given by

$$k_g^I = k^I\cos(\xi - \psi)\,. \quad (2)$$

Using equation 1, the normalized group attenuation coefficient $\mathcal{A}_g$ can be represented as

$$\mathcal{A}_g = \frac{k_g^I}{k_g^R} = \frac{k^I\cos\xi\cos\psi\,(1 + \tan\xi\tan\psi\cos\phi)}{\omega/V_g}\,. \quad (3)$$

For a zero inhomogeneity angle, the coefficient $\mathcal{A}_g$ reduces to

$$\mathcal{A}_g(\xi = 0^\circ) = \frac{k^I}{\omega/(V_g\,\cos\psi)} = \left.\frac{k^I}{k^R}\right|_{\xi=0^\circ} = \mathcal{A}|_{\xi=0^\circ}\,. \quad (4)$$

Equation 4 demonstrates that if $\xi = 0^\circ$, $\mathcal{A}_g$ coincides with $\mathcal{A}|_{\xi=0^\circ}$ even for arbitrary anisotropy. It is unclear, however, how the coefficient $\mathcal{A}_g$ is related to phase attenuation for a nonzero ξ and what role is played by the inhomogeneity angle in the estimation of the attenuation coefficients.

ISOTROPIC MEDIA

To evaluate the influence of the inhomogeneity angle on velocity and attenuation in isotropic media, we obtain the real and imaginary parts of the vector $\mathbf{k}$ from the wave equation. The solution for the wave vector exists only if $\mathbf{k}^I$ deviates from $\mathbf{k}^R$ by no more than 90°, so the inhomogeneity angles $|\xi| > 90^\circ$ correspond to forbidden directions.

If the angle ξ is not close to 90° (we assume $\xi > 0$) and the medium does not have uncommonly strong attenuation [i.e., $(Q\cos\xi) \gg 1$], the real and imaginary parts of the wave vector are given by

$$k^R = \frac{\omega}{V}\,, \quad k^I = \frac{\omega}{2VQ\cos\xi}\,. \quad (5)$$

According to equation 5, for $(Q\cos\xi) \gg 1$ the velocity of wave propagation is equal to V and is independent of the inhomogeneity angle and of attenuation as a whole. Using equation 5, we find the normalized phase attenuation coefficient $\mathcal{A}$ as

$$\mathcal{A} = \frac{k^I}{k^R} = \frac{1}{2Q\cos\xi}\,. \quad (6)$$

In general, the inhomogeneity angle also influences the group velocity V_g and the group angle ψ. For $(Q\cos\xi) \gg 1$, however, the influence of ξ on the group quantities is negligible:

$$\tan\psi = \frac{\tan\xi}{1 + 2Q^2} \ll 1\,, \quad (7)$$

and $V_g \approx V$. The normalized group attenuation coefficient $\mathcal{A}_g$ in equation 3 then becomes

$$\mathcal{A}_g = \frac{k^I\cos\xi}{k^R} = \frac{1}{2Q} = \mathcal{A}|_{\xi=0^\circ}\,. \quad (8)$$

Therefore, for a wide range of common inhomogeneity angles, the coefficient $\mathcal{A}_g$ for both P- and S-waves does not

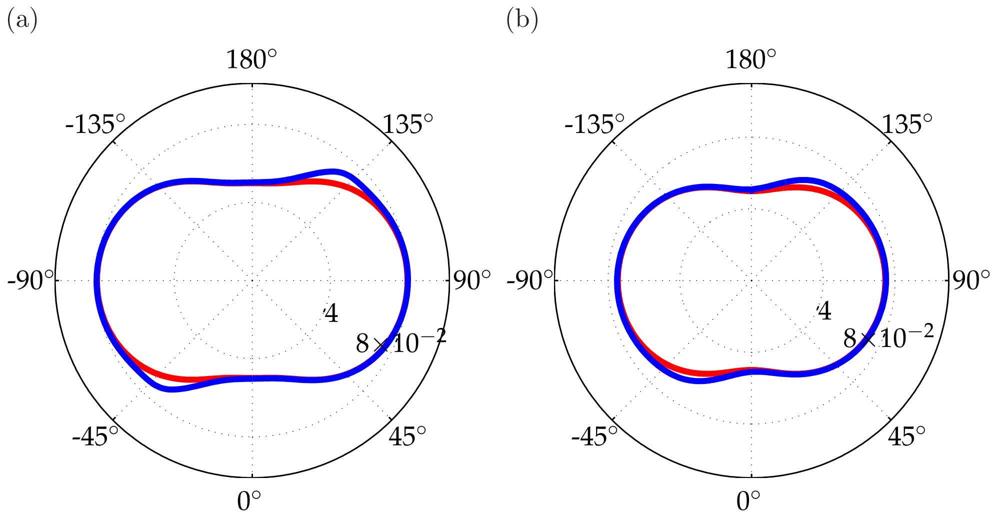

Fig. 5: Exact P-wave (a) and SH-wave (b) coefficients $\mathcal{A}|_{\xi=0^\circ}$ (red curve) and $\mathcal{A}_g$ (blue) in TI media as a function of the phase angle θ for $\xi = 60^\circ$. The model parameters are: (a) $V_{P0} = 2800$ m/s, $\epsilon = 0.6$, $\delta = 0.4$, $Q_{P0} = 10$, $\epsilon_Q = 0.6$, and $\delta_Q = 0.4$; and (b) $V_{S0} = 1700$ m/s, $\gamma = 0.5$, $Q_{S0} = 10$, $\gamma_Q = 0.5$. V_{P0} and V_{S0} are the P- and S-wave symmetry-direction velocities, Q_{P0} and Q_{S0} are the symmetry-direction quality factors, ϵ, δ, and γ are velocity-anisotropy parameters (Thomsen, 1986), and ϵ_Q, δ_Q, and γ_Q are attenuation-anisotropy parameters (Zhu & Tsvankin, 2006).

depend on the angle ξ and is close to the phase attenuation coefficient $\mathcal{A}$ computed for $\xi = 0^\circ$ (Figure 4). Later we demonstrate that this result remains valid for much more complicated models with anisotropic velocity and attenuation functions. Equation 8 also shows that seismic attenuation measurements (i.e., the coefficient $\mathcal{A}_g$) for isotropic media provide a direct estimate of the quality factor Q.

For large inhomogeneity angles approaching 90°, the assumption $(Q\cos\xi) \gg 1$ is no longer satisfied. Dropping quadratic and higher-order terms in $(Q\cos\xi)$, we find

$$\mathcal{A} = \frac{k^I}{k^R} = 1 - Q\cos\xi\,. \tag{9}$$

When $\xi \rightarrow 90^\circ$, the influence of the inhomogeneity angle on the group quantities ψ, V_g, and $\mathcal{A}_g$ is no longer negligible. The group angle for large inhomogeneity angles becomes

$$\tan\psi = \frac{1}{Q} - \cos\xi\,. \tag{10}$$

Equation 10 demonstrates that for strong attenuation (small Q) and large angles ξ the group-velocity vector deviates from the phase direction.

The coefficient $\mathcal{A}_g$ for $(Q\cos\xi) \ll 1$ is given by

$$\mathcal{A}_g = \frac{1}{Q} - \cos\xi\,. \tag{11}$$

Therefore, as $\xi \rightarrow 90^\circ$, the group attenuation coefficient approaches $1/Q$ and is about twice as large as $\mathcal{A}|_{\xi=0^\circ}$ (Figure 4). Seismic attenuation measurements no longer yield the quality factor directly because $\mathcal{A}_g$ rapidly increases with ξ from $1/(2Q)$ to $1/Q$. Several key conclusions drawn above prove to be valid for models with anisotropic velocity and attenuation functions.

ANISOTROPIC MEDIA

The dependence of attenuation on the inhomogeneity angle ξ in anisotropic media is influenced by the angular variation of the phase quantities and by the difference between the group and phase directions. Using the Christoffel equation, the phase attenuation coefficient $\mathcal{A}$ can be computed for arbitrary values of the angle ξ. Then general group-velocity equations (e.g., Tsvankin, 2005) can be employed to obtain the group attenuation coefficient. To derive closed-form expressions for $\mathbf{k}^R$, $\mathbf{k}^I$, and $\mathcal{A}_g$ in arbitrarily anisotropic media, we use the first-order perturbation theory and verify the analytic solutions by numerical modeling.

Normalized group attenuation coefficient

According to equation 4, for a zero inhomogeneity angle the normalized group attenuation coefficient $\mathcal{A}_g$ coincides with $\mathcal{A}|_{\xi=0^\circ}$ for all wave modes. To examine the influence of the angle ξ on the coefficient $\mathcal{A}_g$, we linearize the wave vector and then $\mathcal{A}_g$ in the density-normalized stiffness perturbations Δa_{ij} to obtain

$$\mathcal{A}_{g,P} = \frac{1}{2Q_{P0}} - \frac{1}{2V_{P0}^2}\left(\frac{\Delta a_{33}^R}{Q_{P0}} - \Delta a_{33}^I\right), \tag{12}$$

where Q_{P0} and V_{P0} are the P-wave quality factor and velocity, respectively, in the isotropic attenuative background medium. Equation 12 is derived in a Cartesian coordinate system with the x_3-axis parallel to the vector $\mathbf{k}^R$ and the vector $\mathbf{k}^I$ confined to the $[x_1, x_3]$-plane (Figure 3).

Using perturbation analysis, we also obtained closed-form expressions for the coefficient $\mathcal{A}|_{\xi=0^\circ}$ in arbitrarily anisotropic media. For a wide range of angles ξ (except for values close to 90°), the approximate P-wave phase attenuation coefficient $\mathcal{A}|_{\xi=0^\circ}$ is given by equation 12 and, therefore, coincides with $\mathcal{A}_{g,P}$. This conclusion is also valid for the split shear waves S_1 and S_2.

For the special case of P-wave propagation in TI media with arbitrary symmetry-axis orientation, we find $\mathcal{A}_g$ as

$$\mathcal{A}_{g,P} = \frac{1}{2Q_{P0}} \left(1 + \delta_Q \sin^2\theta \cos^2\theta + \epsilon_Q \sin^4\theta\right), \qquad (13)$$

where θ is the phase angle with the symmetry axis, and δ_Q and ϵ_Q are the attenuation-anisotropy parameters (Zhu & Tsvankin, 2006). As is the case for arbitrary anisotropy, equation 13 coincides with the P-wave phase attenuation coefficient $\mathcal{A}|_{\xi=0^\circ}$ derived by Zhu & Tsvankin (2006).

Therefore, the inhomogeneity angle has no influence on the approximate coefficient $\mathcal{A}_{g,P}$. Equations 12 and 13 deviate from the exact solutions only when the angle ξ approaches forbidden directions; the behavior of $\mathcal{A}_g$ for large inhomogeneity angles is analyzed below. Note that the angle dependence of the linearized $\mathcal{A}_g$ in equation 13 is controlled by attenuation anisotropy and does not depend on the velocity-anisotropy parameters.

Figure 5 demonstrates that the maximum difference between the exact coefficients $\mathcal{A}_g$ and $\mathcal{A}|_{\xi=0^\circ}$ does not exceed 10% even for strong attenuation ($Q = 10$) and uncommonly large anisotropy parameters.

$\mathcal{A}_g$ for large inhomogeneity angles

The above conclusions about the influence of the inhomogeneity angle on phase velocity and attenuation no longer hold for large inhomogeneity angles approaching forbidden directions. To study the attenuation coefficients for large ξ, we follow the same perturbation-based approach but with different background values of the wave vector, group velocity, and group angle. As is the case for isotropic media, when $(Q\cos\xi) \ll 1$, the group coefficient $\mathcal{A}_g$ varies with the angle ξ and differs from $\mathcal{A}|_{\xi=0^\circ}$. Even though for large inhomogeneity angles the real and imaginary parts of the wave vector become infinite, $\mathcal{A}_g$ in anisotropic media remains finite and does not go to zero. While for isotropic media the angle ξ can vary between -90° and 90°, velocity anisotropy makes the bounds on the inhomogeneity angle asymmetric with respect to $\xi = 0^\circ$ (Figure 6).

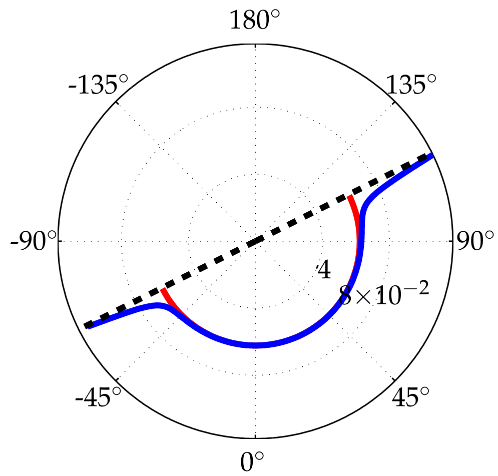

Fig. 6: Exact SH-wave coefficients $\mathcal{A}|_{\xi=0^\circ}$ (red curve) and $\mathcal{A}_g$ (blue) as a function of ξ (numbers on the perimeter) for $\theta = 45^\circ$, $Q_{S0} = 5$, $\gamma = 1.0$, and $\gamma_Q = -0.5$. The dashed line marks the bounds of ξ computed from perturbation theory.

DISCUSSION AND CONCLUSIONS

We applied the first-order perturbation theory to study the influence of the inhomogeneity angle on velocity and attenuation in arbitrarily anisotropic media. For a wide range of small and moderate angles ξ, the phase-velocity function is practically independent of attenuation, while the normalized group attenuation coefficient $\mathcal{A}_g$, which is measured from seismic data, is insensitive to the inhomogeneity angle. Furthermore, $\mathcal{A}_g$ practically coincides with the phase attenuation coefficient $\mathcal{A}|_{\xi=0^\circ}$, which is inversely proportional to the angle-dependent quality factor in anisotropic media. The influence of the inhomogeneity angle on both attenuation and phase velocity becomes pronounced only for uncommonly large ξ approaching forbidden directions. Behura & Tsvankin (2008) corroborate this conclusion by applying attenuation layer stripping and the spectral-ratio method to full-waveform P-wave synthetic data generated by a point source in layered anisotropic models. The interval coefficients $\mathcal{A}_g$ and $\mathcal{A}|_{\xi=0^\circ}$ estimated by Behura & Tsvankin (2008) are practically identical even at large offsets where the inhomogeneity angle reaches 60°.

The negligible difference between $\mathcal{A}_g$ and $\mathcal{A}|_{\xi=0^\circ}$ suggests that seismic data can be used to evaluate attenuation anisotropy without knowledge of the inhomogeneity angle. In particular, the coefficient $\mathcal{A}|_{\xi=0^\circ}$ in TI and orthorhombic media can be inverted for the Thomsen-style attenuation-anisotropy parameters using the formalism developed by Zhu & Tsvankin (2006, 2007). Note that estimation of the attenuation-anisotropy parameters from $\mathcal{A}|_{\xi=0^\circ}$ requires computation of the corresponding phase angle, which is determined by the anisotropic velocity field. However, the dependence of velocity on attenuation typically is weak, which implies that velocity analysis can be performed using existing methods.

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