AVO-sensitive semblance analysis for wide-azimuth data
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Summary
Conventional semblance-based moveout analysis models prestack reflection data with events that have hyperbolic moveout and no amplitude variation with offset (AVO). It has been shown that polarity reversals associated with a change in the sign of the reflection coefficient may cause conventional semblance to fail. Here, we present an AVO-sensitive algorithm designed to correct for polarity reversals in semblance analysis of long-spread, wide-azimuth data from anisotropic media. The amplitude variation is described by Shuey’s approximation, in which the ratio $K$ of the AVO gradient and intercept varies with azimuth but is kept constant within each semblance window. In our implementation, the amplitude-sensitive semblance operator is combined with a nonhyperbolic moveout inversion algorithm devised for orthorhombic media.

Synthetic tests show that conventional semblance produces substantial errors in both the NMO ellipse and azimuthally varying parameter $\eta$ not just for type 2 AVO response, but also for some orthorhombic models with type 1 AVO. In contrast, the AK semblance algorithm gives accurate estimates of the moveout parameters even when the position of the polarity reversal varies with azimuth. The AK method not only helps to flatten wide-azimuth reflection events prior to stacking and azimuthal AVO analysis, but also provides the input parameters for time processing and the anisotropic geometrical-spreading correction.

Introduction
Conventional semblance-based moveout analysis is applied under the assumption that reflection amplitudes in a gather are constant. In the presence of polarity reversals, the conventional operator often breaks down (Sarkar et al., 2001) and gives strongly distorted NMO-velocity values. Sarkar et al. (2002) developed the so-called “AK semblance” method for 2D data with polarity reversals by introducing offset-dependent amplitudes into the semblance operator $S_G$:

$$S_G(V, t_0) = 1 - \frac{\| M - D_V \|^2}{\| D_V \|^2}, \quad (1)$$

where $t_0$ is the zero-offset time at the center of the semblance window, $D_V = D_V(t_1, x)$ is the data with the zero-offset time $t_1$ after a hyperbolic moveout correction with the velocity $V = V_{\text{nmo}}$ and $M = M(t_1, x)$ is the modeled variation of the trace amplitudes. The velocity $V_{\text{nmo}}$ and amplitude parameters that govern $M(t_1, x)$ are estimated by matching the model $M$ and data $D$, which can be achieved by maximizing the semblance. The function $M$ can be approximately described by Shuey’s (1985) linearized equation for the reflection coefficient in terms of the AVO intercept $A$ and gradient $B$:

$$M(t_1, x) = A(t_1) + B(t_1) \sin^2 \theta_x$$

$$\approx A(t_1) \left( 1 + K \frac{x^2}{x^2 + t_0^2 V_{\text{nmo}}^2} \right), \quad (2)$$

where $\theta_x$ is the incidence angle at the reflector, which is expressed through the offset $x$ under the assumption that the medium is homogeneous and isotropic. Since the only role of equation 2 is to introduce a smooth amplitude variation into the semblance operator, there is no need for an accurate estimate of $\theta_x$. To increase velocity resolution, the ratio $B/A = K$ is kept constant inside the semblance window, which implies that the wavelet shape does not change with offset (that explains the name “AK semblance”). The algorithm of Sarkar et al. (2002), however, is restricted to 2D data with hyperbolic moveout.

Here, we present an extension of the AK semblance method to nonhyperbolic (long-spread) moveout and wide-azimuth data. Our generalized AK algorithm is particularly important for processing of reflection data from anisotropic media because anisotropy usually enhances nonhyperbolic moveout of P-waves (Tsvankin, 2005) and moves the polarity reversal toward smaller offsets.

Methodology
A key issue in the implementation of our semblance algorithm is the choice of the moveout equation. Azimuthally anisotropic formations (e.g., fractured reservoirs) are commonly described by an effective orthorhombic model (Bakulin et al., 2000; Grechka and Kachanov, 2006). P-wave reflection traveltimes in a horizontal orthorhombic layer can be well-approximated by the generalized Alkhalifah-Tsvankin (1995) nonhyperbolic moveout equation (Xu and Tsvankin, 2006):

$$t^2(x, \alpha) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2(\alpha)} - \frac{2\eta(\alpha)}{V_{\text{nmo}}^2(\alpha)} \left[ t_0^2 V_{\text{nmo}}^2(\alpha) + (1 + 2\eta(\alpha)) x^2 \right], \quad (3)$$

where $\alpha$ is the azimuth, and

$$V_{\text{nmo}}^2(\alpha) = \frac{\cos^2(\alpha - \varphi)}{V_{(2)}^2(\alpha)} + \frac{\sin^2(\alpha - \varphi)}{V_{(1)}^2(\alpha)}, \quad (4)$$

$$\eta(\alpha) = \eta^{(2)} \cos^2(\alpha - \varphi) + \eta^{(1)} \sin^2(\alpha - \varphi) - \eta^{(3)} \cos^2(\alpha - \varphi) \sin^2(\alpha - \varphi). \quad (5)$$

Here, $t_0$ is the zero-offset time at the center of the semblance window, $D_V = D_V(t_1, x)$ is the data with the zero-offset time $t_1$ after a hyperbolic moveout correction with the velocity $V = V_{\text{nmo}}$, and $M = M(t_1, x)$ is the modeled variation of the trace amplitudes. The velocity $V_{\text{nmo}}$ and amplitude parameters that govern $M(t_1, x)$ are estimated by matching the model $M$ and data $D$, which can be achieved by maximizing the semblance. The function $M$ can be approximately described by Shuey’s (1985) linearized equation for the reflection coefficient in terms of the AVO intercept $A$ and gradient $B$:

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Fig. 1: Shot record of a P-wave reflection from an interface between VTI (top) and isotropic (bottom) media. The vertical P- and S-wave velocities in the VTI medium are \( V_{P0} = 2.96 \) km/s and \( V_{S0} = 1.38 \) km/s, the density \( \rho = 2.43 \) g/cm\(^3\) and the Thomsen parameters \( \epsilon = 0.065 \) and \( \delta = -0.096 \); for the isotropic medium, \( V_{P} = 3.49 \) km/s, \( V_{S} = 2.29 \) km/s, and \( \rho = 2.14 \) g/cm\(^3\). The event has a type 2 AVO response with the polarity reversal at an offset-to-depth ratio close to 0.5.

Here, \( \varphi \) is the azimuth of the \([x_1, x_3]\) symmetry plane, \( V_{\text{nmo}}^{(1)} \) and \( V_{\text{nmo}}^{(2)} \) are the NMO velocities in the symmetry planes \([x_2, x_3]\) and \([x_1, x_3]\), respectively (i.e., the semi-axes of the NMO ellipse), and \( \eta^{(1)}, \eta^{(2)}, \) and \( \eta^{(3)} \) are the anellipticity parameters defined (respectively) in the symmetry planes \([x_2, x_3], [x_1, x_3], \) and \([x_1, x_2]\). If \( V_{\text{nmo}}^{(1,2)} \) and \( \eta^{(1,2,3)} \) are treated as effective parameters, equations 3–5 can be applied to P-wave moveout from layered orthorhombic media with uniform orientation of the symmetry planes.

The normalized AVO gradient \( K = B/A \) for a boundary between orthorhombic media with the same azimuth of the vertical symmetry planes is given by (Rüger, 2001):

\[
K(\alpha) = K^{(2)} \cos^2(\alpha - \varphi) + K^{(1)} \sin^2(\alpha - \varphi). \tag{6}
\]

Substitution of \( K(\alpha) \) from equation 6 into equation 2 yields an azimuthally dependent amplitude function that we combine with the moveout equations 3–5 to devise an AK semblance operator (equation 1) for long-spread, wide-azimuth data. The parameter \( A \) can be found analytically by setting the derivative of \( S_G \) with respect to \( A \) to zero (Sarkar et al., 2002), while \( K^{(2)} \) and \( K^{(3)} \) are estimated along with the moveout parameters by maximizing semblance.

Our AK semblance algorithm is based on the nonhyperbolic moveout inversion method of Vasconcelos and Tsvankin (2006). Estimation of \( K^{(1)}, K^{(2)}, \) and the moveout parameters \( \varphi, V_{\text{nmo}}^{(1,2)} \) and \( \eta^{(1,2,3)} \) is carried out in three steps. First, we invert for the NMO ellipse (described by \( \varphi, V_{\text{nmo}}^{(1)}, \) and \( V_{\text{nmo}}^{(2)} \)) and for an azimuthally-invariant value of \( K \) using conventional-spread, wide-azimuth data. Second, nonhyperbolic AK semblance is applied in narrow sectors around the identified symmetry planes to find approximate values of \( \eta^{(1)}, \eta^{(2)}, K^{(1)}, K^{(2)}, \) and updated estimates of \( V_{\text{nmo}}^{(1)} \) and \( V_{\text{nmo}}^{(2)} \). Third, we use the initial model obtained in the first two steps to perform nonhyperbolic moveout inversion with the 3D AK semblance operator for all offsets and azimuths. The best-fit parameters \( \varphi, V_{\text{nmo}}^{(1,2)}, \eta^{(1,2,3)}, \) and \( K^{(1)} \) are obtained via Powell minimization.

### Tests on synthetic data

Here, the AK semblance algorithm is applied to long-offset P-wave data from VTI (transversely isotropic with a vertical symmetry axis) and orthorhombic media. In addition to type 2 AVO responses (Rutherford and Williams, 1989), for which the polarity reversal is observed at relatively small offsets, we compare the performance of conventional and AK semblance for type 1 AVO. All synthetic data used below are generated by anisotropic ray tracing code ANRAY of Gajewski and Pšencík (1987).

#### 2D semblance for type 2 AVO

We consider a model that includes an isotropic halfspace beneath a VTI layer with relatively small absolute values of the Thomsen parameters \( \epsilon \) and \( \delta \) (Figure 1). The anellipticity parameter \( \eta \), however, is substantial (0.2), which leads to pronounced nonhyperbolic moveout at offsets approaching two reflector depths. The event has a type 2 AVO response, with the polarity reversal at an offset close to half the reflector depth.

The semblance computation is performed using the 2D version of the nonhyperbolic moveout equation 3 (Figure 2). Although this equation provides a close approximation to the exact traveltimes, the best-fit parameters \( V_{\text{nmo}}^{(1)} \) and \( \eta^{(1,2)} \) estimated by the conventional semblance algorithm are severely distorted. Clearly, this error is caused by the polarity reversal because the semblance that corresponds to the correct moveout parameters is relatively low. In contrast, the AK method yields accurate estimates of both \( V_{\text{nmo}}^{(1)} \) and \( \eta \), as well as a much higher semblance (0.94).

#### 3D semblance for type 2 AVO

Next, we apply the 3D AK semblance operator to wide-azimuth P-wave reflections from an interface between orthorhombic (incidence) and isotropic (reflecting) media. The event has a type 2 AVO response with the polarity reversal at offset-to-depth ratios close to 0.5; the offset of the polarity reversal is weakly dependent on azimuth. The NMO ellipse reconstructed by the conventional semblance operator for offset-to-depth ratios limited by unity has the correct orientation but highly distorted semi-axes equal to 3.20 km/s and 3.44 km/s (the actual values are 2.66 km/s and 2.87 km/s). Clearly, for this model conventional processing cannot be used even for stacking of moderate-offset data.

The final inversion results for two long spreads are displayed in Figure 3. Because of the influence of the polarity reversal, conventional semblance produces highly erroneous NMO velocities and \( \eta \) values for both
Fig. 2: Semblance scans over $V_{nmo}$ and $\eta$ computed for the event from Figure 1 with the conventional (left) and AK semblance (right) operators. The black dots mark the true model parameters ($V_{nmo} = 2.66$ km/s and $\eta = 0.2$). The best-fit values for the conventional method are $V_{nmo} = 1.98$ km/s and $\eta = 0.72$ (the maximum semblance is 0.72); for the AK method, $V_{nmo}=2.68$ km/s and $\eta=0.18$ (the maximum semblance is 0.94).

Fig. 3: Moveout-inversion results for a wide-azimuth, long-offset P-wave reflection from an interface between orthorhombic (top) and isotropic (bottom) media. The event has a type 2 AVO response. (a) and (c) are the NMO ellipses; (b) and (d) are the azimuthally-dependent $\eta$ values. The maximum offset-to-depth ratio is two for the top row, and three for the bottom row. The solid lines are the actual NMO ellipses and $\eta$-curves, the dashed lines are estimated by the conventional semblance algorithm, and the dotted lines by AK semblance (they are hardly visible because of the high accuracy of the AK algorithm). The azimuth with respect to the symmetry plane $[x_1, x_3]$ is shown on the perimeter. The parameters of the orthorhombic medium (the notation is explained in Tsvankin, 2005) are $V_{P0} = 2.96$ km/s, $V_{S0} = 1.38$ km/s, $\rho = 2.43$ g/cm$^3$, $\epsilon^{(1)} = 0.065$, $\delta^{(1)} = -0.029$, $\gamma^{(1)} = 0.18$, $\epsilon^{(2)} = 0.065$, $\delta^{(2)} = -0.096$, $\gamma^{(2)} = 0.05$, and $\delta^{(3)} = -0.08$. For the isotropic medium, $V_P = 3.49$ km/s, $V_S = 2.29$ km/s, and $\rho = 2.14$ g/cm$^3$. 
spreadlengths. As discussed above, muting out long offsets does not help the conventional algorithm to reconstruct the NMO ellipse. The AK semblance method gives far superior results for both \( V_{nmo} \) and \( \eta \), with almost negligible errors for the whole range of azimuths.

3D semblance for type 1 AVO

Type 1 AVO response is typically characterized by a larger offset of the polarity reversal. Figure 4 shows the results for a model in which the position of the polarity reversal varies with azimuth and corresponds to offset-to-depth ratios between unity and 1.5. Although the conventional algorithm performs better than it did for type 2 AVO, the errors in both the NMO ellipse and parameter \( \eta \) are noticeable. A more accurate reconstruction of the NMO ellipse using conventional semblance can be achieved by reducing the maximum offset-to-depth ratio to unity.

While AK semblance yields a close approximation for the NMO ellipse, the inverted \( \eta \) for the spreadlength-to-depth ratio equal to two somewhat deviates from the actual curve. This distortion in \( \eta \), which is caused by the small bias in the nonhyperbolic moveout equation (Tsvankin, 2005), decreases for the larger spread (Figure 4d). In contrast, the \( \eta \) estimate produced by the conventional method deteriorates with increasing spreadlength, as the semblance becomes more influenced by traces at post-reversal offsets.

Conclusions

We presented an efficient AVO-sensitive (“AK semblance”) methodology for moveout analysis of long-offset 2D and 3D (wide-azimuth) P-wave reflection data. The amplitude function in the 3D AK semblance operator is based on the azimuthal variation of the normalized AVO gradient \( K \) for orthorhombic media. Our algorithm is designed to invert for the two symmetry-plane values of \( K \) along with the moveout parameters of orthorhombic media, which include the azimuth of one of the symmetry planes, the symmetry-plane NMO velocities \( V_{nmo}^{(1,2)} \), and the anellipticity coefficients \( \eta^{(1,2,3)} \). Note that the parameters \( V_{nmo} \) and \( \eta^{(1,2,3)} \) control time processing and geometrical spreading of P-wave data in orthorhombic media.

The improvement achieved by the 3D AK semblance is especially significant for models with type 2 AVO response. Because of the relatively small offset of the polarity reversal, conventional semblance analysis completely breaks down. Despite the approximate character of its amplitude function, the AK algorithm properly accounts for the azimuthally varying polarity reversal in the estimation of both the NMO ellipse and the \( \eta \)-curve. Although for type 1 AVO it may be possible to reconstruct the NMO ellipse by computing conventional semblance for a truncated gather, accurate estimation of the \( \eta \)-parameters requires application of AK semblance.

Our results demonstrate that a smooth amplitude function based on Shuey’s approximation for the AVO gradient is sufficient for removing the influence of polarity reversals on semblance analysis. Still, AK semblance should not be regarded as a substitute for 2D or 3D (azimuthal) AVO analysis.