Kirchhoff inversion for Incident Waves Synthesized from Common-Shot Data Gathers

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Abstract

Synthesis is a process for combining common-shot data gathers tp produce reflection responses from more general sources or from prescribed incident waves. Synthesis can provide survey-wide data sets, similar in that regard to common-offset data gathers, with the advantage that each synthesized data set is a solution to a single wave equation. A common-offset data set does not have this last feature. Thus, synthesized data sets can be processed by true amplitude wave equation migration. The output is then known to be true amplitude, as well, in the same sense as the output of Kirchhoff inversion. We present here a theory of data synthesis based on application of Green's theorem to the ensemble of common-shot gathers and using prescribed more general sources or prescribed incident waves. Specific examples include delayed-shot line sources and incident dipping plane waves at the upper surface. We also discuss two cases in which waves are prescribed at depth, back-projected to the upper surface and then used to generate a synthesized data set.

Introduction

Zhang et al [2005] propose a method of synthesizing data for a delayed-shot line source from the ensemble of common-shot data sets. They also suggest synthesis of reflection data for incident plane waves. Synthesized sources and their responses can cover the entire range of a survey, in contrast to the individual shot gathers whose surface range is limited by the receiver coverage of each experiment. This process produces a data set that is again the solution of a single wave equation. Thus we invert by applying the true-amplitude (WEM) imaging condition to achieve our inversion even when using Kirchhoff representations for modeling.

Y. Zhang (personal communication) suggested exploring for an underlying principle behind the two examples of Zhang et al [2005]. S. H. Gray observed that this notion is closely related to the results of Wapenaar et al [1989], Rietveld et al [1992] and Rietveld [1995]. Here we provide a classic continuum approach to synthesis by using Green's theorem to synthesize the wave fields. In this regard, this paper has much in common with Wapenaar's, although our point of view and objectives are somewhat different.

We also discuss starting with an incident wave at depth to derive the controlled illumination application of Rietveld's [1995] thesis and Rietveld et al [1992] in a heterogeneous medium. Here, the incident wave is a vertically incident plane wave at depth. For a homogeneous medium or depth-dependent medium, It is possible to use an obliquely incident wave at depth. We discuss this case, as well.

Finally, we observe without discussion that the operations of synthesis and modeling the synthesized source and response can be commuted. There is a trade-off in efficiency here: in the commuted order, we need to compute all of the common-shot inversions first, but examination of the common-shot inversion output in a target zone could lead to a smaller, more efficiently chosen suite of synthesized common-source inversions.

A numerical implementation of plane wave synthesis is provided in W. Zhang [2004]. Synthesis with directional plane wave patches is discussed in Xie and Wu [2004], among other papers from Wu and colleagues.

Defining the common-shot experiment

We define $G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}_s,\omega)$ to be the free-space Green's function that models the common-shot experiment, with $\boldsymbol{x}_s = (x_{s1}, x_{s2}, 0)$, [and later $\boldsymbol{x}_r = (x_{r1}, x_{r2}, 0)$], being the source (and receiver) coordinates. This function satisfies the following Helmholtz equation and Sommerfeld radiation conditions:

$$\left[\nabla^2 + \frac{\omega^2}{\mathrm{v}^2(\boldsymbol{x})}\right] G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}_s,\omega) = -\delta(\boldsymbol{x}-\boldsymbol{x}_s), \tag{1}$$

$$rG_{\mathrm{FS}} \text{ bounded}, \; r\left[\frac{\partial G_{\mathrm{FS}}}{\partial r} - \frac{i\omega G_{\mathrm{FS}}}{\mathrm{v}(\boldsymbol{x})}\right] \to 0, \; r = |\boldsymbol{x}| \to \infty.$$

We will also use the *incoming* Green's functions to represent incoming waves. This is just the complex conjugate $\overline{G_{\mathrm{FS}}}$ of G_{FS} in equation (1).

We set

$$G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}_s,\omega) = G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_s,\omega) + G_{\mathrm{R}}(\boldsymbol{x},\boldsymbol{x}_s,\omega), \tag{2}$$

with G_{D} being the downgoing part of the Green's function and G_{R} being the upgoing part.

Green's Theorem and its Consequences

Let $u(\boldsymbol{x},\omega)$ be any solution of the wave equation and radiation conditions of equation (1), with downgoing and upgoing parts, u_{D} and u_{R}, respectively:

$$u(\boldsymbol{x},\omega) = u_{\mathrm{D}}(\boldsymbol{x},\omega) + u_{\mathrm{R}}(\boldsymbol{x},\omega). \tag{3}$$

We will apply Green's theorem in one of two domains depending on the mechanism by which we generate u—a prescribed incident wave on the upper surface (or at depth) or a prescribed source. We state Green's theorem as follows.

$$u(\boldsymbol{x},\omega) = \int_{\mathcal{D}} f(\boldsymbol{x}',\omega)G(\boldsymbol{x},\boldsymbol{x}',\omega)dV' \tag{4}$$
$$-\int_{\partial\mathcal{D}} \left[u(\boldsymbol{x}',\omega)\frac{\partial G(\boldsymbol{x},\boldsymbol{x}',\omega)}{\partial n'} - G(\boldsymbol{x},\boldsymbol{x}',\omega)\frac{\partial u(\boldsymbol{x}',\omega)}{\partial n'}\right] dS'.$$

Here, $f(\boldsymbol{x}',\omega)$ is a source; $\partial\mathcal{D}$ is the boundary of the domain $\mathcal{D}$; $\partial n'$ denotes the outward normal derivative on $\partial\mathcal{D}$; dV' is the differential volume element in $\boldsymbol{x}'$; dS' is the differential surface element in $\boldsymbol{x}'$ for the integration on $\partial\mathcal{D}$.

Synthesis through a defined source function

We consider the Green's identity of equation (4) with $\mathcal{D}$ being the sphere depicted in Figure 1. We can show that the surface integral in equation (4) approaches zero as the radius of the sphere approaches infinity. Thus,

$$u(\boldsymbol{x},\omega) = \int_{\mathcal{D}} f(\boldsymbol{x}',\omega)G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}',\omega)dV'. \tag{5}$$

We now write this last result in terms of the downward and upward propagating wave forms comprising G_{FS} in equation (2) thereby defining the downward and upward propagating wave forms of u in equation (3):

$$\begin{aligned} u_{\mathrm{D}}(\boldsymbol{x},\omega) &= \int_{\mathcal{D}} f(\boldsymbol{x}',\omega)G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}',\omega)dV', \\ u_{\mathrm{R}}(\boldsymbol{x},\omega) &= \int_{\mathcal{D}} f(\boldsymbol{x}',\omega)G_{\mathrm{R}}(\boldsymbol{x},\boldsymbol{x}',\omega)dV'. \end{aligned} \tag{6}$$

Zhang et al [2005] described the wave equation migration of a delayed-shot line and its reflection response. We show how this example fits into the theory proposed here. We prescribe the line source of equation (5) as follows:

$$f(\boldsymbol{x}',\omega) = \delta(x_2'-x_{s2})\delta(x_3')\exp\{i\omega p_{01}x_1'\}. \tag{7}$$

We apply the wave field representation formula of equation (5) to obtain

$$\begin{aligned} u_{\mathrm{D}}(\boldsymbol{x},p_{01},x_{s2},\omega) &= \int_{\mathcal{D}} G_{\mathrm{D}}(\boldsymbol{x},x_1',x_{s2},0,\omega) \\ &\qquad \cdot\exp\{i\omega p_{01}x_1'\}dx_1', \\ u_{\mathrm{R}}(\boldsymbol{x}_r,p_{01},x_{s2},\omega) &= \int_{\mathcal{D}} G_{\mathrm{R}}(\boldsymbol{x}_r,x_1',x_{s2},0,\omega) \\ &\qquad \cdot\exp\{i\omega p_{01}x_1'\}dx_1'. \end{aligned} \tag{8}$$

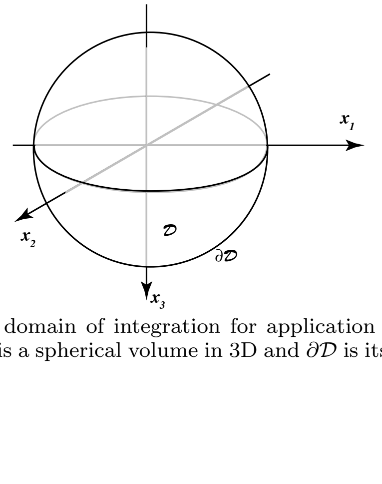

Fig. 1: The domain of integration for application of Green's theorem. $\mathcal{D}$ is a spherical volume in 3D and $\partial\mathcal{D}$ is its bounding surface.

The added arguments on the left here reflect the special variables of this example. We find the point source solution G_{D} everywhere in the subsurface, so the first line provides u_{D} everywhere. However, we only know the observed data G_{R} at the receivers; hence synthesis provides the data u_{R} at the receivers. Below, we downward continue this synthesized data as part of the inversion process. These outputs are just the Fourier transform of the common-shot data sets at wavenumber ωp_{01}

For homogeneous media, we can insert the exact Green's function for G_{D} and apply the method of stationary phase to obtain the asymptotic expression for u_{D} in equation (8):

$$\begin{aligned} &u_{\mathrm{D}}(\boldsymbol{x},p_{01},x_{s2},\omega) \\ &\qquad \sim \frac{\exp\{i\omega[x_1\sin\gamma_0+\rho\cos\gamma_0]+i\pi/4\mathrm{sgn}(\omega)\}}{2\sqrt{2|\omega|qr/\mathrm{v}}}, \\ &q=\sqrt{1/\mathrm{v}^2-p_{01}^2}, \qquad \sin\gamma_0=\mathrm{v}p_{01}, \qquad \cos\gamma_0=\mathrm{v}q. \end{aligned} \tag{9}$$

Here, it is easy to see that the wave front forms a "line" in x_1,ρ, making a dip angle γ_0 with the x_1 direction. Since ρ is a radial variable in (x_2-x_{s2},x_3) we see that the wave front lies on a cone.

Synthesis through prescribed boundary data

We return to Green's theorem as stated in equation (4). Now, however, we set $f=0$ and integrate over the hemispherical domain of Figure 2. Thus, we restate equation (4) as

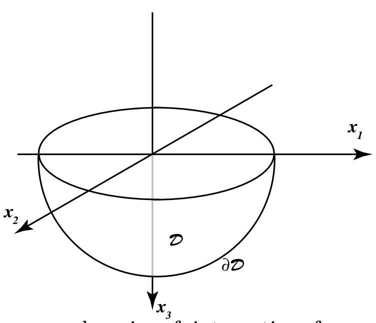

Fig. 2: The new domain of integration for application of Green's theorem. $\mathcal{D}$ is a spherical volume in 3D and $\partial\mathcal{D}$ is its bounding surface.

$$\begin{aligned} u(\boldsymbol{x},\omega) = -\int_{\partial\mathcal{D}} \Bigg[&u(\boldsymbol{x}',\omega)\frac{\partial G(\boldsymbol{x},\boldsymbol{x}',\omega)}{\partial n'} \\ &- G(\boldsymbol{x},\boldsymbol{x}',\omega)\frac{\partial u(\boldsymbol{x}',\omega)}{\partial n'}\Bigg] dS' \end{aligned} \tag{10}$$

As above, the integral on the hemisphere can be shown to approach zero as $r\to\infty$ by applying the Sommerfeld radiation conditions given in equation (1). The domain in this last equation then reduces to the acquisition surface where $x_3'=0$. We further use the method of images to define a Green's function in terms G_{FS}. We set

$$G(\boldsymbol{x},\boldsymbol{x}',\omega) = G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}',\omega) + G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}'^*,\omega), \tag{11}$$

with $\mathrm{v}(\boldsymbol{x}^*)=\mathrm{v}(\boldsymbol{x})$, $\boldsymbol{x}^*=(x_1,x_2,-x_3)$. For this function, when $x_3'=0$, the arguments of the two functions on the right side of the equation are equal and $G=2G_{\mathrm{FS}}$, $x_3=0$. On the other hand, one can check that the x_3' derivative of the Green's function defined in equation (11) is equal to zero when $x_3' = 0$.[1]

We use the Green's function of equation (1), and rewrite the representation of u in equation (10) as

$$u(\boldsymbol{x},\omega) = -2\int_{x_3'=0} G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}',\omega)\frac{\partial u(\boldsymbol{x}',\omega)}{\partial x_3'}dx_1'dx_2'. \quad (12)$$

Again we can decompose this representation into the incident downward propagating synthesized wave and the upward propagating synthesized response by using equation (2) for G_{FS}. That is,

$$\begin{aligned} u_{\mathrm{D}}(\boldsymbol{x},\omega) &= -2\int_{x_3'=0} G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}',\omega)\frac{\partial u(\boldsymbol{x}',\omega)}{\partial x_3'}dx_1'dx_2', \\ u_{\mathrm{R}}(\boldsymbol{x}_r,\omega) &= -2\int_{x_3'=0} G_{\mathrm{R}}(\boldsymbol{x}_r,\boldsymbol{x}',\omega)\frac{\partial u(\boldsymbol{x}',\omega)}{\partial x_3'}dx_1'dx_2'. \end{aligned}$$

with $u(\boldsymbol{x},\omega) = u_{\mathrm{D}}(\boldsymbol{x},\omega) + u_{\mathrm{R}}(\boldsymbol{x},\omega)$. (13)

Downward propagation of $u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)$

Given the data $u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)$ in equation (13), we propagate it downward by using $\overline{G(\boldsymbol{x},\boldsymbol{x}_r,\omega)}$. Since we do not have data for the x_{r3}-derivative of u_{R} at $x_{r3} = 0$, we choose $\overline{G}$ with boundary value is zero; that is,

$$\overline{G(\boldsymbol{x},\boldsymbol{x}_r,\omega)} = \overline{G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_r,\omega)} - \overline{G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_r^*,\omega)} \quad (14)$$

By using this Green's function in the Green's identity (10), with $\boldsymbol{x}' = \boldsymbol{x}_r$ and $u = u_R$, we find that for $x_3 > 0$,

$$u_{\mathrm{R}}(\boldsymbol{x},\omega) = 2\int_{x_{r3}=0} u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)\frac{\partial\overline{G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_r,\omega)}}{\partial x_{r3}}dx_{r1}dx_{r2}. \quad (15)$$

Note that the downward propagation of the data $u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)$ by equation (15) also applies to data synthesized via a source mechanism as discussed earlier in this section. That is, we use equation (8) to determine the data $u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)$ and then use equation (15) to project these data into the Earth. We postponed the discussion of downward continuation of data generated by a synthesized source until after we had the machinery of the present discussion in place.

As an example, we consider a plane wave incident at $x_3 = 0$ from above in a medium that is homogeneous above through $x_3 = 0+$. We set

$$\begin{aligned} u_{\mathrm{D}}(\boldsymbol{x},\omega) &= -2i\omega p_{03}\int_{x_3'=0} G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}',\omega)\exp\{i\omega\boldsymbol{p}_0\cdot\boldsymbol{x}'\}dx_1'dx_2' \\ u_{\mathrm{R}}(\boldsymbol{x}_r,\omega) &= -2i\omega p_{03}\int_{x_3'=0} G_{\mathrm{R}}(\boldsymbol{x}_r,\boldsymbol{x}',\omega)\exp\{i\omega\boldsymbol{p}_0\cdot\boldsymbol{x}'\}dx_1'dx_2' \end{aligned} \quad (16)$$

The synthesized wave forms are 2D Fourier transforms of the elements of the common shot source and response, multiplied by $-2i\omega p_{03}$. When inverting, the constant factor of $-2i\omega p_{03}$ would appear in both the numerator and the denominator of the imaging condition and, thus, can be disregarded.

[1]Note that the method of images will not work when the acquisition surface is not flat. Wapenaar et al [1989] discuss the errors that occur when we use the approximation in equation (12) anyway.

Inversion

We now have a method for synthesizing an up- and downgoing wave at the upper surface and downward continuing the data into the Earth. To invert for the reflectivity function $\mathcal{R}(\boldsymbol{x},\theta)$ we use the true amplitude imaging condition of wave equation migration, namely,

$$\mathcal{R}(\boldsymbol{x},\theta) = \frac{1}{2\pi}\int\frac{u_{\mathrm{R}}(\boldsymbol{x},\omega)}{u_{\mathrm{D}}(\boldsymbol{x},\omega)}d\omega. \quad (17)$$

If we now use the downward propagator of equation (15) to express $u_{\mathrm{R}}(\boldsymbol{x},\omega)$ in terms of $u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)$, then this last result becomes

$$\mathcal{R}(\boldsymbol{x},\theta) = \frac{1}{\pi}\int d\omega\int_{x_{r3}=0}\frac{u_{\mathrm{R}}(\boldsymbol{x}_r,\omega)}{u_{\mathrm{D}}(\boldsymbol{x},\omega)}\frac{\partial\overline{G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_r,\omega)}}{\partial x_{r3}}dx_{r1}dx_{r2}. \quad (18)$$

As an inversion example, we set $f(\boldsymbol{x},\omega) = \delta(\boldsymbol{x}-\boldsymbol{x}_s)$ in equation (6) and use ray-theoretic Green's functions in the inversion formula, (18), we obtain

$$\begin{aligned} \mathcal{R}(\boldsymbol{x},\theta) &= 2\int\frac{\cos\alpha_r}{\mathrm{v}(\boldsymbol{x}_r)}\frac{A(\boldsymbol{x},\boldsymbol{x}_r)}{A(\boldsymbol{x},\boldsymbol{x}_s)}D(\boldsymbol{x}_r,\boldsymbol{x}_s,\boldsymbol{x})dx_{r1}dx_{r2}, \\ D(\boldsymbol{x}_r,\boldsymbol{x}_s,\boldsymbol{x}) &= \frac{1}{2\pi}\int i\omega\,d\omega u_{\mathrm{R}}(\boldsymbol{x}_r,\omega) \\ &\quad\cdot\exp\{-i\omega\left[\tau(\boldsymbol{x},\boldsymbol{x}_r)+\tau(\boldsymbol{x},\boldsymbol{x}_s)\right]\}. \end{aligned} \quad (19)$$

This is the standard form of common-shot Kirchhoff inversion as found in Keho and Beydoun [1988] and, with minor constant correction, in Hanitzsch [1997]. This result provides the simplest of checks on the general formula of inversion of synthesized waves as expressed by equation (18).

Prescribing a Wave at Depth

At depth in a heterogeneous medium, we can not easily prescribe a convenient wave, say with a prescribed gradient. The reason is that the line integral connecting values of the travel time in the plane is not single valued except under certain restrictive conditions. One alternative to overcome the constraint on traveltime is to prescribe a wave with constant traveltime and constant amplitude at a given depth. This is the the controlled illumination problem that Rietveld [1995] and Rietveld et al [1992] considered. For such a wave, we set

$$u_{\mathrm{H}}(\boldsymbol{x}_H,\omega) \equiv 1, \quad \boldsymbol{x}_H = (x_1,x_2,H). \quad (20)$$

The back-projected wave at the upper surface can be derived using the anti-causal Green's function, leading to a representation similar to the result in equation (13):

$$u_{\mathrm{H}}(\boldsymbol{x},\omega) = 2i\omega\int_{x_3'=H}\frac{\overline{G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}',\omega)}}{\mathrm{v}(\boldsymbol{x}_H')}dx_1'dx_2', \; x_3 < H. \quad (21)$$

We use the synthesis method for prescribed boundary data, with $u_{\mathrm{H}}(\boldsymbol{x},\omega)$ being the function $u(\boldsymbol{x},\omega)$ in equations (13). We only use this solution at $x_3 = H$; there the down going wave is just the unidirectional wave of unit amplitude given by equation (20). We can use $u_{\mathrm{H}}(\boldsymbol{x},\omega)$ to compute $u_{\mathrm{R}}(\boldsymbol{x},\omega)$ from its representation in equation in (13). That is,

$$u_{\mathrm{R}}(\boldsymbol{x}_r,\omega) = -2\int_{x_3'=0} G_{\mathrm{R}}(\boldsymbol{x}_r,\boldsymbol{x}',\omega)\frac{\partial u_{\mathrm{H}}(\boldsymbol{x}',\omega)}{\partial x_3'}dx_1''dx_2''. \quad (22)$$

Next, we downward continue the data $u_{\rm R}(\boldsymbol{x}_r,\omega)$ into the Earth by using equation (15). We can now use the imaging condition of equation (17) for imaging and inversion at depth H. That is, by using equations (15) and (22) in equation (17) we find that

$$\mathcal{R}(\boldsymbol{x}_H,\theta) = -\frac{2}{\pi}\int d\omega \int_{x_{r3}=0} dx_{r1}dx_{r2}\frac{\partial \overline{G_{\rm D}(\boldsymbol{x},\boldsymbol{x}_r,\omega)}}{\partial x_{r3}} \cdot \int_{x_3''=0} dx_1' dx_2' G_{\rm R}(\boldsymbol{x}_r,\boldsymbol{x}',\omega)\frac{\partial u_{\rm H}(\boldsymbol{x}',\omega)}{\partial x_3'}. \tag{23}$$

This is inversion using Rietveld [1995] and Rietveld et al [1992] for controlled illumination.

Let us now suppose that the wave speed depends at most on the depth x_3: $\mathrm{v} \equiv \mathrm{v}(x_3)$. Now, the gradient of the travel time has a constant horizontal component, even for oblique incidence. In this case the traveltime is a single-valued function in the plane. Thus, in place of the unit wave of equation (20), we introduce the following.

$$u_{\rm H}(\boldsymbol{x}_H,\omega) = \exp\{i\omega \boldsymbol{p}_H\cdot\boldsymbol{x}_H\}, \quad \boldsymbol{p}_H = \hat{\boldsymbol{p}}_H/\mathrm{v}(H), \tag{24}$$

with $\hat{\boldsymbol{p}}_H$ a constant unit vector. This wave is unidirectional at the depth H. The back-projected wave in the upward direction is represented as only a minor modification of the previous upward projected wave of equation (21); that is, for $x_3 < H$,

$$u_{\rm H}(\boldsymbol{x},\omega) = \frac{2i\omega\hat{p}_{H3}}{\mathrm{v}(H)}\int_{x_3'=H}\exp\{i\omega\boldsymbol{p}_H\cdot\boldsymbol{x}_H\}\overline{G_{\rm D}(\boldsymbol{x},\boldsymbol{x}',\omega)}dx_1'dx_2', \tag{25}$$

Given this representation of the back-projected wave field at $x_3 = 0$, the representation of equation (22) for the synthesized reflection response remains the same, as does the downward continuation of these synthesized data given by equation (15). We do need to modify the inversion formula of equation (23), since the incident wave at depth H was identically one in the previous case, while here it is given by equation (24). We find that the reflectivity is now given by

$$\mathcal{R}(\boldsymbol{x}_H,\theta) = -\frac{2}{\pi}\int d\omega\int_{x_{r3}=0}dx_{r1}dx_{r2}\exp\{-i\omega\boldsymbol{p}_H\cdot\boldsymbol{x}_H\} \cdot\frac{\partial\overline{G_{\rm D}(\boldsymbol{x},\boldsymbol{x}_r,\omega)}}{\partial x_{r3}}\int_{x_3''=0}dx_1'dx_2'G_{\rm R}(\boldsymbol{x}_r,\boldsymbol{x}',\omega)\frac{\partial u_{\rm H}(\boldsymbol{x}',\omega)}{\partial x_3'}. \tag{26}$$

Note that the formulas presented in this last discussion remain unchanged when the wave speed is constant, that is, in a homogeneous medium. Only the specific Green's functions in these formulas change.

Summary and Conclusions

We have described a general theory of synthesis of sources or incident waves and their reflection responses from an ensemble of common-shot data records. The theory is close to the method expounded in Wapenaar et al [1989] with one application, controlled illumination, being the method of Rietveld et al [1992] and Rietveld [1995]. The method here is based on careful application of Green's theorem for waves generated by boundary data or waves generated by sources. Synthesis starting from a unidirectional wave at depth, with appropriate constraint on the wavespeed, has not been previously presented.

Our results are expressed in terms of general Green's functions, leaving to the user the option of how accurate a Green's function will be used in the implementation. We expect that the hierarchy of progressively more accurate Green's functions will provide higher quality images. However, in all cases, the output of the inversion will be an angularly dependent reflection coefficient scaled by the area under the source signature whenever the multiple speculars at the image point can be separated.

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