Plane-wave attenuation anisotropy in orthorhombic media

Yaping Zhu and Ilya Tsvankin, Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines, Golden, CO 80401-1887, USA

Summary

We develop an analytic framework for describing the attenuation coefficients in orthorhombic media with orthorhombic attenuation. To take full advantage of the analogy between the Christoffel equations in the symmetry planes of orthorhombic and TI (transversely isotropic) media, we introduce a set of attenuation-anisotropy parameters similar to the TI attenuation parameters ϵ_Q, δ_Q, and γ_Q. This notation, based on the same principle as Tsvankin's velocity-anisotropy parameters for orthorhombic media, leads to simple linearized equations for the symmetry-plane attenuation coefficients of all three modes (P, S_1, and S_2). The attenuation-anisotropy parameters also allow us to simplify the P-wave attenuation coefficient $\mathcal{A}_P$ outside the symmetry planes under the assumption of weak attenuation and weak velocity and attenuation anisotropy. The approximate $\mathcal{A}_P$ has the same form as the linearized phase-velocity function, with Tsvankin's velocity parameters $\epsilon^{(1,2)}$ and $\delta^{(1,2,3)}$ replaced by the attenuation parameters $\epsilon_Q^{(1,2)}$ and $\delta_Q^{(1,2,3)}$.

The simple expression for the coefficient $\mathcal{A}_P$ and the reduction in the number of parameters responsible for P-wave attenuation provide a basis for inverting attenuation measurements from orthorhombic media. The attenuation processing has to be preceded by anisotropic velocity analysis that can be performed (in the absence of pronounced velocity dispersion) using existing algorithms for nonattenuative media.

Introduction

Effective velocity models of fractured reservoirs often have orthorhombic or an even lower symmetry (Schoenberg and Helbig, 1997; Bakulin et al., 2000). Existing experimental data indicate that attenuation in fractured rocks also varies with angle, and the magnitude of the attenuation anisotropy may exceed that of the velocity anisotropy.

This paper is devoted to plane-wave attenuation in a homogeneous medium that has orthorhombic symmetry for both the velocity function and attenuation coefficient. The two main assumptions used here to simplify the analytic description of attenuation are as follows:
1) Wave propagation is "homogeneous," which means that the real ($\mathbf{k}$) and imaginary ($\mathbf{k}^I$) parts of the wave vector are parallel to each other ($\mathbf{k} \parallel \mathbf{k}^I$).
2) The symmetry of the imaginary part of the stiffness matrix coincides with that of the real part, which ensures that the quality-factor matrix $\mathbf{Q}$ (Carcione, 2001) has the same structure as the real part of the stiffness matrix that governs the velocity anisotropy. As in our previous work on transversely isotropic (TI) media (Zhu and Tsvankin, 2004; hereafter, referred to as Paper I), we study the *normalized* attenuation coefficient defined as $\mathcal{A} \equiv k^I/k$.

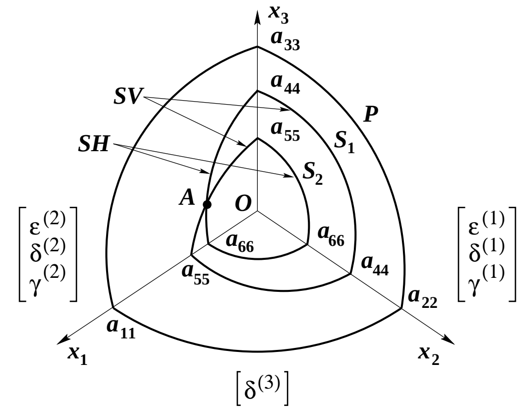

Fig. 1: Sketch of body-wave phase-velocity surfaces and Tsvankin's parameters for orthorhombic media ($a_{ij} \equiv \sqrt{c_{ij}/\rho}$, where c_{ij} are the stiffness coefficients and ρ is the density). The coordinate planes represent planes of symmetry for both velocity and attenuation. Point A marks a shear-wave singularity.

Because of the coupling between the velocity and attenuation anisotropy, the coefficient $\mathcal{A}$ depends (for a fixed orientation of the symmetry planes) on nine real stiffness coefficients and nine elements Q_{ij}. Here, we show that substantial simplifications in the description of orthorhombic attenuation can be achieved by extending the principle of Tsvankin's (1997, 2001) notation for velocity anisotropy to attenuative media.

Attenuation-anisotropy parameters

The Thomsen-style parameters for orthorhombic media introduced by Tsvankin (1997, 2001; see Figure 3) provided a basis for developing efficient inversion and processing methods operating with traveltimes and reflection coefficients. Here, we extend Tsvankin's approach to attenuative orthorhombic media with the main goal of defining the parameter combinations that govern the directionally dependent attenuation coefficients.

Since our notation is designed primarily for reflection data, we choose the P- and S-wave attenuation coefficients in the vertical (x_3) direction ($\mathcal{A}_{P0}$ and $\mathcal{A}_{S0}$) as the reference isotropic quantities:

$$\mathcal{A}_{P0} \equiv \frac{1}{2Q_{33}}, \qquad (1)$$

$$\mathcal{A}_{S0} \equiv \frac{1}{2Q_{55}}. \qquad (2)$$

The coefficient $\mathcal{A}_{S0}$ corresponds to the S-wave polarized in the x_1-direction, which may be either the fast or slow shear mode depending on the relationship between the stiffnesses c_{44} and c_{55}.

To characterize the attenuation of waves propagating in the $[x_1, x_3]$-plane (Figure 3), we define three attenuation-anisotropy parameters analogous to the Thomsen-style parameters ϵ_Q, δ_Q, and γ_Q introduced for VTI (TI with a vertical symmetry axis) media with VTI attenuation in Paper I. The parameters $\epsilon_Q^{(2)}$ and $\gamma_Q^{(2)}$ (the superscript "(2)" stands for the x_2-axis perpendicular to the $[x_1, x_3]$-plane) determine the fractional difference between the normalized attenuation coefficients in the x_1- and x_3-directions for the P- and SH-waves, respectively. Another parameter, $\delta_Q^{(2)}$, is expressed through the second derivative of the P-wave attenuation coefficient in the vertical direction and, therefore, governs the P-wave attenuation for near-vertical propagation in the $[x_1, x_3]$-plane.

$$\epsilon_Q^{(2)} \equiv \frac{Q_{33} - Q_{11}}{Q_{11}}, \tag{3}$$

$$\delta_Q^{(2)} \equiv \frac{\dfrac{Q_{33} - Q_{55}}{Q_{55}} c_{55} \dfrac{(c_{13} + c_{33})^2}{(c_{33} - c_{55})} + 2 \dfrac{Q_{33} - Q_{13}}{Q_{13}} c_{13}(c_{13} + c_{55})}{c_{33}(c_{33} - c_{55})}$$

$$\approx 4 \, \frac{Q_{33} - Q_{55}}{Q_{55}} \, g^{(2)} + 2 \, \frac{Q_{33} - Q_{13}}{Q_{13}} \left(1 + 2\delta^{(2)} - 2g^{(2)}\right), \tag{4}$$

$$\gamma_Q^{(2)} \equiv \frac{Q_{44} - Q_{66}}{Q_{66}}, \tag{5}$$

where equation (4) for $\delta_Q^{(2)}$ is simplified by assuming that the ratio $g^{(2)} \equiv c_{55}/c_{33}$ and the absolute value of Tsvankin's velocity-anisotropy parameter $\delta^{(2)}$ are small. Since the Christoffel equation in the $[x_1, x_3]$-plane has the same form as in VTI media, equations (3)–(5) are identical to the definitions of the corresponding VTI parameters. In contrast to VTI models, however, the parameters of orthorhombic media with the subscripts "55" and "44" are generally different.

Similarly, we introduce three attenuation-anisotropy parameters in the $[x_2, x_3]$-plane:

$$\epsilon_Q^{(1)} \equiv \frac{Q_{33} - Q_{22}}{Q_{22}}, \tag{6}$$

$$\delta_Q^{(1)} \equiv \frac{\dfrac{Q_{33} - Q_{44}}{Q_{44}} c_{44} \dfrac{(c_{23} + c_{33})^2}{(c_{33} - c_{44})} + 2 \dfrac{Q_{33} - Q_{23}}{Q_{23}} c_{23}(c_{23} + c_{44})}{c_{33}(c_{33} - c_{44})}$$

$$\approx 4 \, \frac{Q_{33} - Q_{44}}{Q_{44}} \, g^{(1)} + 2 \, \frac{Q_{33} - Q_{23}}{Q_{23}} \left(1 + 2\delta^{(1)} - 2g^{(1)}\right), \tag{7}$$

$$\gamma_Q^{(1)} \equiv \frac{Q_{55} - Q_{66}}{Q_{66}}. \tag{8}$$

In equation (7), $\delta^{(1)}$ is Tsvankin's velocity-anisotropy parameter defined in the $[x_2, x_3]$-plane, and $g^{(1)} \equiv c_{44}/c_{33}$.

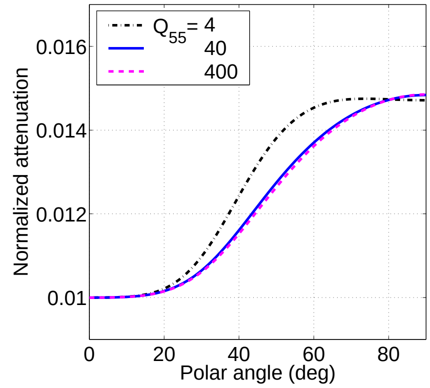

Fig. 2: Influence of the parameter $\mathcal{A}_{S0} = 1/(2Q_{55})$ (marked on the plot) on the normalized P-wave attenuation coefficient at the azimuth $\phi = 50°$. The velocity parameters correspond to an orthorhombic model formed by vertical cracks embedded in a VTI background (Schoenberg and Helbig, 1997): $V_{P0} = 2.437$ km/s, $V_{S0} = 1.265$ km/s, $\epsilon^{(1)} = 0.329$, $\epsilon^{(2)} = 0.258$, $\delta^{(1)} = 0.083$, $\delta^{(2)} = -0.078$, $\delta^{(3)} = -0.106$, $\gamma^{(1)} = 0.182$, and $\gamma^{(2)} = 0.0455$. The P-wave vertical attenuation coefficient $\mathcal{A}_{P0} = 0.01$ ($Q_{33} = 50$); each attenuation-anisotropy parameter is twice the corresponding velocity-anisotropy parameter: $\epsilon_Q^{(1)} = 0.658$, $\epsilon_Q^{(2)} = 0.516$, $\delta_Q^{(1)} = 0.166$, $\delta_Q^{(2)} = -0.156$, $\delta_Q^{(3)} = -0.212$, $\gamma_Q^{(1)} = 0.364$, and $\gamma_Q^{(2)} = 0.091$.

The remaining anisotropy parameter, $\delta_Q^{(3)}$, plays the role of the VTI parameter δ_Q in the $[x_1, x_2]$-plane (x_1 is taken as the symmetry axis):

$$\delta_Q^{(3)} \equiv \frac{\dfrac{Q_{11} - Q_{66}}{Q_{66}} c_{66} \dfrac{(c_{11} + c_{12})^2}{(c_{11} - c_{66})} + 2 \dfrac{Q_{11} - Q_{12}}{Q_{12}} c_{12}(c_{12} + c_{66})}{c_{11}(c_{11} - c_{66})}$$

$$\approx 4 \, \frac{Q_{11} - Q_{66}}{Q_{66}} \, g^{(3)} + 2 \, \frac{Q_{11} - Q_{12}}{Q_{12}} \left(1 + 2\delta^{(3)} - 2g^{(3)}\right), \tag{9}$$

where $\delta^{(3)}$ is another Tsvankin's velocity-anisotropy parameter defined in the $[x_1, x_2]$-plane, and $g^{(3)} \equiv c_{66}/c_{11}$. The nine attenuation-anisotropy parameters defined in equations (1)–(9), combined with the corresponding velocity-anisotropy parameters, are sufficient to fully characterize plane-wave attenuation in orthorhombic media.

Symmetry-plane attenuation coefficients

The equivalence between plane-wave propagation in the symmetry planes of orthorhombic and VTI media means that the symmetry-plane attenuation coefficients of all three modes can be found by adapting the VTI equations of Paper I. While the exact attenuation coefficients are rather complicated, much simpler, linearized solutions can be obtained by assuming simultaneously weak attenuation and weak velocity and attenuation anisotropy. For example, the approximate attenuation coefficients of the

Orthorhombic attenuation

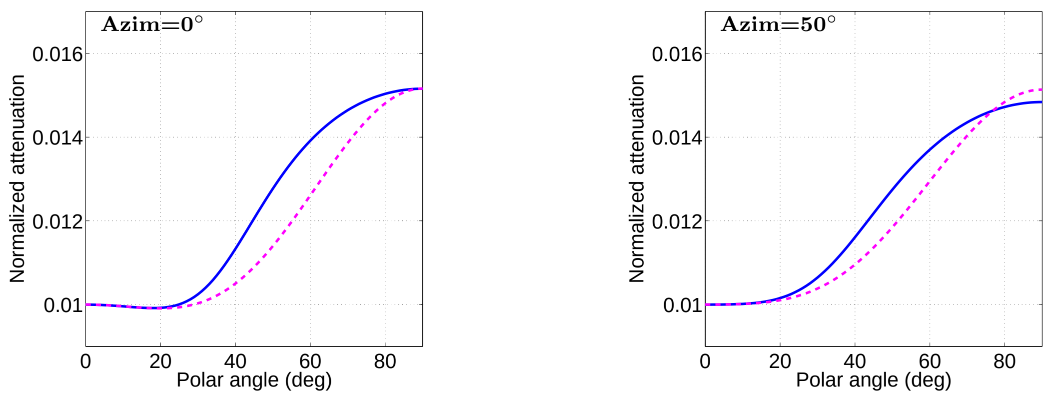

Fig. 3: Comparison of the exact coefficient $\mathcal{A}_P$ (solid curves) with the linearized approximation (14) (dashed) for an orthorhombic medium with orthorhombic attenuation. The model parameters are the same as in Figure 2 ($Q_{55} = 40$).

P- and SV-waves in the $[x_1, x_3]$-plane are given by

$$\mathcal{A}_P^{(2)} = \mathcal{A}_{P0}\left(1 + \delta_Q^{(2)} \sin^2\theta \cos^2\theta + \epsilon_Q^{(2)} \sin^4\theta\right), \quad (10)$$

$$\mathcal{A}_{SV}^{(2)} = \mathcal{A}_{S0}\left(1 + \sigma_Q^{(2)} \sin^2\theta \cos^2\theta\right), \quad (11)$$

where θ is the phase angle with the vertical,

$$\sigma_Q^{(2)} \equiv \frac{1}{g_Q^{(2)}}\left[2(1 - g_Q^{(2)})\,\sigma^{(2)} + \frac{\epsilon_Q^{(2)} - \delta_Q^{(2)}}{g^{(2)}}\right], \quad (12)$$

$g_Q^{(2)} \equiv \dfrac{Q_{33}}{Q_{55}} = \dfrac{\mathcal{A}_{S0}}{\mathcal{A}_{P0}}$, and $\sigma^{(2)} \equiv \dfrac{\epsilon^{(2)} - \delta^{(2)}}{g^{(2)}}$. Although equations (10) and (11) have exactly the same form as the corresponding linearized phase-velocity functions, the dependence of the attenuation-anisotropy parameter $\delta_Q^{(2)}$ on the real parts of the stiffness coefficients reflects the coupling between the attenuation and velocity anisotropy.

An important role in the attenuation analysis of reflection shear-wave data should be played by the *attenuation splitting parameter* $\gamma_Q^{(S)}$ defined as the fractional difference between the attenuation coefficients of the split S-waves at vertical incidence:

$$\gamma_Q^{(S)} \equiv \frac{|\gamma_Q^{(1)} - \gamma_Q^{(2)}|}{1 + \gamma_Q^{(2)}}. \quad (13)$$

P-wave attenuation outside symmetry planes

The linearized approximation for the P-wave attenuation coefficient can be extended to arbitrary phase directions outside the symmetry planes:

$$\mathcal{A}_P(\theta, \phi) = \mathcal{A}_{P0}\left[1 + \delta_Q(\phi) \sin^2\theta \cos^2\theta + \epsilon_Q(\phi) \sin^4\theta\right], \quad (14)$$

where ϕ is the azimuthal phase angle, and

$$\epsilon_Q(\phi) = \epsilon_Q^{(1)} \sin^4\phi + \epsilon_Q^{(2)} \cos^4\phi + (2\epsilon_Q^{(2)} + \delta_Q^{(3)}) \sin^2\phi \cos^2\phi, \quad (15)$$

$$\delta_Q(\phi) = \delta_Q^{(1)} \sin^2\phi + \delta_Q^{(2)} \cos^2\phi. \quad (16)$$

Therefore, the approximate coefficient $\mathcal{A}_P$ in any vertical plane of orthorhombic media is described by the VTI equation with the azimuthally varying parameters $\epsilon_Q(\phi)$ and $\delta_Q(\phi)$. Remarkably, equations (14)–(16) have exactly the same form as the linearized P-wave phase-velocity equations (1.107)–(1.109) in Tsvankin (2001). This similarity is explained by the identical (orthorhombic) symmetry imposed on both the real and imaginary parts of the stiffness matrix and the assumption of homogeneous wave propagation. However, an important difference between the coefficient $\mathcal{A}_P$ and phase velocity is that the parameters $\delta_Q^{(1)}$, $\delta_Q^{(2)}$, and $\delta_Q^{(3)}$ include a contribution of the velocity anisotropy.

Transversely isotropic models with both vertical (VTI) and horizontal (HTI) symmetry axis can be treated as special cases of orthorhombic media. For VTI media with VTI attenuation, all vertical planes are identical ($\epsilon_Q(\phi) = \epsilon_Q$, $\delta_Q(\phi) = \delta_Q$), and there is no velocity or attenuation variation with angle in the horizontal (isotropy) plane. Equations (14)–(16) then yield the VTI result (Paper I):

$$\mathcal{A}_P^{(\mathrm{VTI})} = \mathcal{A}_{P0}\left(1 + \delta_Q \sin^2\theta \cos^2\theta + \epsilon_Q \sin^4\theta\right). \quad (17)$$

The linearized P-wave attenuation coefficient (14) does not contain the parameters $\mathcal{A}_{S0}$, $\gamma_Q^{(1)}$, and $\gamma_Q^{(2)}$, which are primarily responsible for shear-wave attenuation. An important practical issue is whether or not this conclusion remains valid for models with strong attenuation and pronounced velocity and attenuation anisotropy. As illustrated by Figure 2, the dependence of $\mathcal{A}_P$ on the shear-wave vertical attenuation coefficient $\mathcal{A}_{S0}$ becomes noticeable only for extremely high attenuation (i.e., uncommonly small values of Q_{55}). The influence of the parameters $\gamma_Q^{(1)}$ and $\gamma_Q^{(2)}$ on the coefficient $\mathcal{A}_P$ (not shown here) for typical moderately attenuative models is also negligible. Therefore, for a fixed orientation of the symmetry planes and fixed velocity parameters, P-wave attenuation is controlled by $\mathcal{A}_{P0}$ and five attenuation-anisotropy parameters — $\epsilon_Q^{(1,2)}$ and $\delta_Q^{(1,2,3)}$.

To evaluate the accuracy of the weak-anisotropy approx-

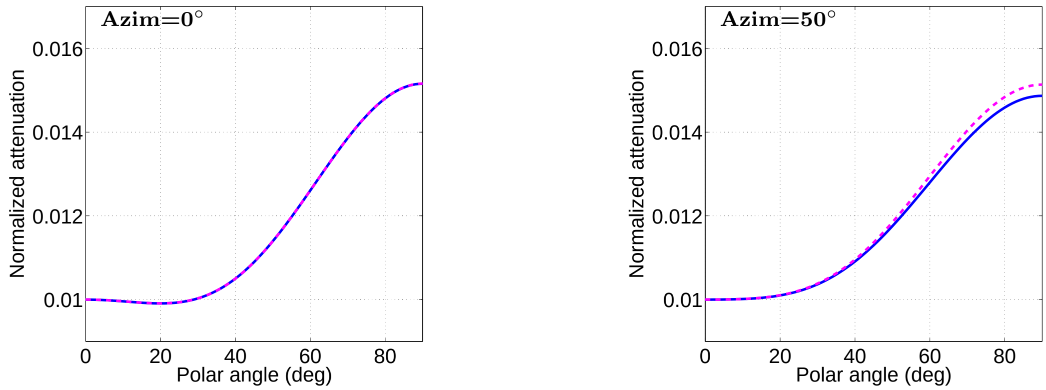

Fig. 4: Comparison of the exact coefficient $\mathcal{A}_P$ (solid curves) with the linearized approximation (14) (dashed) for a medium with orthorhombic attenuation but a purely isotropic velocity function. The attenuation parameters are the same as in Figures 2 and 3, but the velocity $V_{P0} = 2.437$ km/s is constant in all directions.

imation (14) outside the symmetry planes, we compare it with the exact coefficient $\mathcal{A}_P$ (obtained by solving the Christoffel equation) for a model with pronounced orthorhombic attenuation (Figure 3). As expected, the weak-anisotropy approximation gives the best results for near-vertical propagation directions with polar angles up to about 30°. When the vertical incidence plane is close to either vertical symmetry plane (i.e., the azimuth ϕ approaches 0° or 90°), the approximate solution also yields an accurate estimate of $\mathcal{A}_P$ near the horizontal direction. Overall, the error of the weak-anisotropy approximation for the full range of polar and azimuthal angles is less than 10%. Note that while the velocity anisotropy for this model is moderate (both $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are about 0.3), the attenuation anisotropy is much more pronounced.

To identify the source of errors in the weak-anisotropy approximation, we repeat the test in Figure 3 using a purely isotropic velocity model (Figure 4). The approximate solution (dashed curves) in Figure 4 coincides with that in Figure 3b because both models have identical attenuation-anisotropy parameters. The exact coefficient $\mathcal{A}_P$ (solid curves), however, is influenced by the velocity-anisotropy parameters in such a way that the error of the weak-anisotropy approximation becomes much smaller when the velocity field is isotropic (Figure 4). Hence, the accuracy of the approximation (14) is controlled primarily by the strength of the velocity anisotropy, even if the magnitude of the attenuation anisotropy is much higher.

Conclusions

Analysis of plane-wave attenuation coefficients in orthorhombic media can be significantly facilitated by introducing attenuation-anisotropy parameters similar to Tsvankin's parameters for the orthorhombic velocity function. Our notation includes two reference (isotropic) P- and S-wave attenuation coefficients in the vertical direction ($\mathcal{A}_{P0}$ and $\mathcal{A}_{S0}$) and seven dimensionless anisotropy parameters ($\epsilon_Q^{(1,2)}$, $\delta_Q^{(1,2,3)}$, and $\gamma_Q^{(1,2)}$). The P-wave attenuation coefficient, both within and outside the symmetry planes, depends only on $\mathcal{A}_{P0}$ and the parameters $\epsilon_Q^{(1,2)}$ and $\delta_Q^{(1,2,3)}$.

In the absence of pronounced velocity dispersion, the influence of attenuation (i.e., of the imaginary part of the stiffness tensor) on velocity is practically negligible. In contrast, the definitions of the attenuation-anisotropy parameters $\delta_Q^{(1,2,3)}$ include the velocity-anisotropy parameters $\delta^{(1,2,3)}$. Also, although the velocity anisotropy does not explicitly contribute to the linearized expressions for attenuation, the exact attenuation coefficient $\mathcal{A}_P$ does vary with the velocity-anisotropy parameters even for fixed values of $\delta_Q^{(1,2,3)}$. Moreover, the accuracy of the linearized approximation for $\mathcal{A}_P$ is controlled to a large degree by the strength of the velocity anisotropy.

References

Bakulin, A., Grechka, V., and Tsvankin, I., 2000, Estimation of fracture parameters from reflection seismic data – Part II: Fractured models with orthorhombic symmetry: Geophysics, **65**, 1803–1817.

Carcione, J. M., 2001, Wave fields in real media: wave propagation in anisotropic, anelastic, and porous media: Pergamon Press.

Schoenberg, M., and Helbig, K., 1997, Orthorhombic media: Modeling elastic wave behavior in a vertically fractured earth: Geophysics, **62**, 1954–1974.

Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: Geophysics, **62**, 1292–1309.

Tsvankin, I., 2001, Seismic signatures and analysis of reflection data in anisotropic media: Elsevier.

Zhu, Y., and Tsvankin, I., 2004, Plane-wave propagation and radiation patterns in attenuative TI media: 74th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 139–142.