Nonhyperbolic moveout analysis in VTI media using rational interpolation

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Summary

We present a rational interpolation approach to nonhyperbolic moveout (NHMO) correction in the $t-x$ domain, for qP-waves in a single homogeneous transversely isotropic layer with a vertical symmetry axis. The accuracy of the method is compared to the NHMO equation of Alkhalifah and Tsvankin (1995) using both synthetic and field data. Both data types confirm that for $\eta \gtrsim 0.1$ this method significantly outperforms the Alkhalifah-Tsvankin (A-T) approximation in terms of combining unbiased parameter estimation with accurate moveout correction, for arbitrary offset-to-depth ratios (ODR) and arbitrary levels of anellipticity. The proposed method has no additional computational overhead compared to using expressions explicit in the relevant parameters η and V_{NMO}. The lack of such additional overhead stems from the observation that, for a fixed value of η and a fixed zero-offset two-way traveltime t_0, the moveout curve for different values of V_{NMO} can be calculated by simple stretching (or squeezing) of the offset axis, where the amount of stretch depends on the change in V_{NMO}. This observation is based on the generally accepted assumptions that the traveltimes of qP-waves in transversely isotropic media, depend mainly on η and V_{NMO} (i.e., we use $\delta = 0$), and that the shear-wave velocity along the symmetry axis has a negligible influence on these traveltimes (i.e., $V_{S0} = 0$ km/s). The accuracy of the rational interpolation method is as good as that of these approximations. The method can be tuned to be accurate to any offset range of interest by increasing the order of the interpolation, making it accurate for arbitrary ODR.

Introduction

For velocity analysis in VTI media using qP-waves, the NHMO equation for a single horizontal VTI layer, derived by Tsvankin and Thomsen (1994) and rewritten in terms of V_{NMO} and η by Alkhalifah and Tsvankin (1995), is the current standard in seismic data processing. Even though this approximation is exact at zero offset and infinite offset, Grechka and Tsvankin (1998) mention that at intermediate offsets "this approximation can be somewhat improved ... ." This was also noted by other authors [e.g., Zhang and Uren (2001), van der Baan and Kendall (2002), Stovas and Ursin (2004), and Fomel (2004)]. Here, we propose a rational interpolation (RI) approach to increase the accuracy of NHMO analysis in VTI media at larger ODR.

Dependence of NHMO on η and V_{NMO} for a single VTI layer

From simple geometric considerations, it follows that for a homogeneous horizontal VTI layer, the traveltime t and associated offset x are given by

$$t = \frac{V_{P0}\, t_0}{v \cos\psi}\,, \qquad x = V_{P0}\, t_0 \tan\psi\,, \tag{1}$$

where V_{P0} is the P-wave traveltime along the vertical symmetry axis, ψ is the group angle, and v is the group velocity for propagation in direction ψ. In a TI medium, the expressions for the group angle and group velocity are given by Tsvankin (2001, p.29 and p.22)

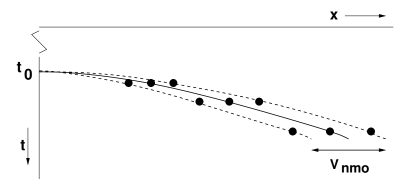

Fig. 1: Under the assumptions that $V_{S0} = 0$ and $\delta = 0$, the influence of V_{NMO} on NHMO of qP-waves in a single horizontal VTI layer is limited to a stretch along the offset axis.

$$\tan\psi = \frac{\tan\theta + \frac{1}{V}\frac{dV}{d\theta}}{1 - \frac{\tan\theta}{V}\frac{dV}{d\theta}}\,, \quad v = V\sqrt{1 + \left(\frac{1}{V}\frac{dV}{d\theta}\right)^2}\,, \tag{2}$$

where $V = V(\theta)$ is the phase velocity at phase angle θ [given by equation (1.59) in (Tsvankin, 2001, p.22)].

The qP-wave phase-velocity in TI media depends only weakly on V_{S0} (Tsvankin & Thomsen, 1994), and is usually ignored in kinematic problems regarding qP-waves (i.e, $V_{S0} = 0$). Alkhalifah and Tsvankin (1995) showed that the traveltimes of qP-waves in homogeneous VTI media depend mainly on the (zero-dip) normal-moveout velocity V_{NMO} and the anellipticity parameter η, with an almost negligible influence of V_{P0}. Hence we can choose $\delta = 0$ and thus $V_{P0} = V_{NMO}$ and $\epsilon = \eta$. The equation for the phase velocity then becomes

$$V(\theta) = V_{NMO}\left\{\eta\sin^2\theta + \frac{1}{2}\left(1 + \sqrt{\left(1 + 2\eta\sin^2\theta\right)^2 - 2\eta\sin^2 2\theta}\right)\right\}^{1/2} \tag{3}$$

while equation (1) for the traveltime t and the associated offset x become

$$t = \frac{V_{NMO}\, t_0}{v\cos\psi}\,, \qquad x = V_{NMO}\, t_0 \tan\psi\,. \tag{4}$$

Note that the phase velocity $V(\theta)$ now depends linearly on V_{NMO}. This linearity causes the term $\frac{1}{V(\theta)}\frac{dV}{d\theta}$ in equation (2) to be independent of V_{NMO}. Since the dependence of the group angle on the anisotropic parameters is governed by the term $\frac{1}{V(\theta)}\frac{dV}{d\theta}$ [cf. equation (2)], the group angle ψ is independent of V_{NMO} and depends only on η. In addition, it follows from equation (2) that the group velocity v depends linearly on V_{NMO} since the phase velocity depends linearly on V_{NMO}. From equation (4) it then follows that *the traveltime t becomes independent of V_{NMO} and that the associated offset x depends linearly on V_{NMO}*. Also, t_0 is a simple scaling factor for both the traveltime t and the associated offset x. In a single horizontal VTI layer, this means that *for fixed η and t_0, the moveout curve for different values of V_{NMO} can be calculated by simple horizontal stretching along the offset axis* (see Figure 1). This observation is a straightforward consequence of the negligible influence of V_{S0} on qP-wave traveltimes in TI media and the fact that the kinematics of qP-waves in homogeneous VTI media depend mainly on V_{NMO} and η; i.e., we can use $V_{S0} = 0$ and $\delta = 0$. To make explicit the independence of the traveltimes of V_{NMO}, we rewrite the traveltimes in equation (4) as

$$t = \frac{t_0}{v|_{V_{NMO}=1} \cos\psi} \;, \tag{5}$$

where $v|_{V_{NMO}=1}$ denotes the group velocity calculated for $V_{NMO} = 1$ km/s.

NHMO using rational interpolation

A [2/2] rational approximation to the squared traveltimes is given by

$$T(X) \approx \frac{T_0 + n_1 X + n_2 X^2}{1 + d_1 X + d_2 X^2} \quad , \tag{6}$$

where $T(X) = t^2(x)$ and $X = x^2$. The notation [2/2] refers to both a quadratic numerator and denominator in the rational approximation. Given four function values T_i at squared offsets X_i, with $i = 1, \ldots, 4$, we have a linear system of four equations with four unknowns, i.e., the coefficients of the polynomials (i.e., n_1, n_2, d_1, and d_2). We solve for the coefficients in terms of T_i and X_i using Mathematica. The resulting equations can be used to find the coefficients for particular values of T_i and X_i. These coefficients can in turn be used to find the function values $T(X)$ at values of X different from the interpolation points X_i.

To calculate the four coefficients n_1, n_2, d_1, and d_2, we need four traveltimes and four associated offsets. Let t_i ($i = 1, \ldots, 4$) be the traveltime for a fixed ODR k_i (or group angle ψ_i), and let the associated offset be x_i. From the previous section, we know that the traveltimes t_i are independent of V_{NMO} and depend on t_0 through a simple scaling only. Therefore the traveltimes t_i can be calculated from a small subset of traveltimes calculated for a fixed reference value of t_0, denoted as t_0^{ref}, and a range of η values, say from -0.2 to 1.0 in steps of 0.01. This subset can be precomputed and stored in a table. The traveltimes t_i for a particular combination of t_0 and η are found from a simple lookup in this table for the particular η, combined with scaling with t_0/t_0^{ref} [cf. equation (5)]. Since the offsets are linear in V_{NMO}, the desired traveltime t_i and offset x_i for given values of t_0, η, and V_{NMO}, are thus obtained from

$$t_i = t_i^{table}\left(\frac{t_0}{t_0^{ref}}\right) \;, \quad x_i = \frac{V_{NMO}\, t_0\, k_i}{2} \;, \tag{7}$$

where t_i^{table} is the value of t_i obtained from the table (evaluated for $t_0 = t_0^{ref}$), for ODR k_i and the desired value of η. Evaluating the table for $t_0^{ref} = 1$ s allows the calculation of t_i to be done by scaling of t_i^{table} with t_0 only. The values of t_i and x_i thus obtained can be used to determine the coefficients n_1, n_2, d_1, and d_2 in equation (6), which allows calculation of the NHMO curve.

The traveltimes and offsets obtained using the method outlined above allow velocity analysis in VTI media using [2/2] RI. The efficiency is comparable to that of current velocity analysis using the A-T equation, since the small subset of traveltimes for a range of η values is precomputed and stored in a table. Hence, no computational overhead is required compared to that of current methods; the NHMO equation is replaced simply with the RI formula, and traveltimes needed to calculate the coefficients n_1, n_2, d_1, and d_2 are read from the precomputed table. To precompute the table of traveltimes, a standard anisotropic ray-tracing algorithm can be used. Here, we solve

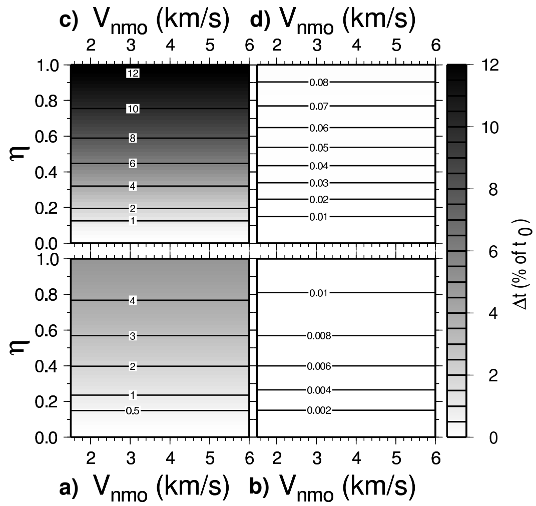

Fig. 2: Accuracy of A-T equation (a and c) and RI (b and d) for maximum ODR of two (a and b) and four (c and d).

$$\frac{k_i}{2} = \frac{\tan\theta_i + \dfrac{1}{V_i}\left.\dfrac{dV}{d\theta}\right|_{\theta_i}}{1 - \dfrac{\tan\theta_i}{V_i}\left.\dfrac{dV}{d\theta}\right|_{\theta_i}} \;, \tag{8}$$

(where we used $\tan\psi_i = k_i/2$) for θ_i numerically, using the Matlab function 'fsolve'. In equation (8), V_i is the phase velocity associated with ODR k_i. The traveltime is then found through calculation of the group velocity $v_i|_{V_{NMO}=1}$ using equations (3) and (2) with $V_{NMO} = 1$ km/s, and subsequent use of this velocity in

$$t_i^{table} = \frac{t_0^{ref}}{v_i|_{V_{NMO}=1} \cos\left(\tan^{-1}\dfrac{k_i}{2}\right)} \;, \tag{9}$$

[cf. equation (5)]. We found that using the group angle as an initial guess for the phase angle generally worked well. Calculating four traveltimes for 100 values of η in this way, takes about one minute on a modern PC.

Figures 2a-d show contours of the maximum absolute traveltime difference (in percentage of t_0) between ray-traced traveltimes for a single horizontal VTI layer and (a and c) the A-T equation and (b and d) the [2/2] RI method, for a maximum ODR of two (a and b) and four (c and d), for virtually all combinations of η and V_{NMO} of practical interest. For the RI method we used $k_1 = 1/2$, $k_2 = 1$, $k_3 = 3/2$, and $k_4 = 2$ for a maximum ODR of two, and $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, and $k_4 = 4$ for a maximum ODR of four. The differences in the traveltimes for the A-T equation (a and c) are of the order of a percent, which in this case amounts to 10 ms. Such errors are not negligible and cause errors in the estimation of η and V_{NMO} in velocity analysis. The [2/2] RI method has traveltimes errors of the order 10^{-3}% of t_0 (or 0.01 ms in this case) for a maximum ODR of two, and of the order of 10^{-2}% of t_0 (or 0.1 ms) for a maximum ODR of four, hence showing the significant

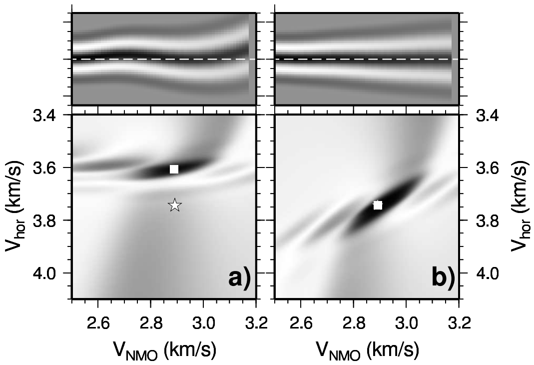

Fig. 3: Semblance scans and moveout-corrected gathers for a maximum ODR of four using the A-T equation (a) and RI (b). Moveout correction was done with parameters derived from the maximum semblance (indicated by the star) as opposed to the true model parameters (indicated by the square).

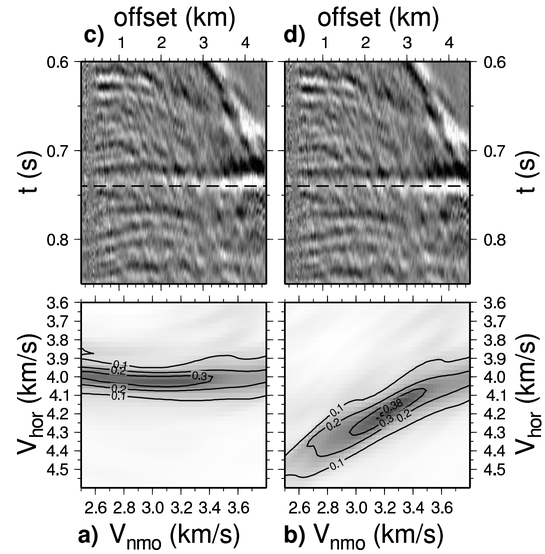

Fig. 4: Semblance scans (a and b) and moveout corrected gathers (c and d) for one CMP gather, calculated using the A-T equation (a) and the RI method (b).

improvement in accuracy when compared to the A-T equation, for arbitrary levels of anellipticity. With [2/2] RI we achieved accuracy $O(10^{-1})$% of t_0 up to maximum ODR of 8. If accuracy is needed beyond such ODR, or higher accuracy is needed for ODR between 4 and 8, higher order interpolation can be used. Note how the traveltime errors are independent of V_{NMO} because the raytracing is done with $V_{S0} = 0$ km/s and $\delta = 0$; i.e., since under these assumptions the influence of V_{NMO} on the moveout is limited to a stretch along the offset axis, the errors in the traveltimes are independent of V_{NMO}. A comparison of the RI method with Fomel's NHMO equation (Fomel, 2004) for Greenhorn-shale anisotropy established almost identical accuracy of both methods (with a maximum difference in absolute traveltime error of 1 ms over ODR up to 20).

Synthetic data example

Figures 3a and b show semblance scans (at fixed t_0) as a function of V_{NMO} and V_{hor} calculated using (a) the A-T moveout equation and (b) the [2/2] RI, for a maximum ODR of four. The true model parameters are indicated by the star ($\eta = 0.34$) while the semblance maxima are indicated by the squares. The residual moveout for the parameters V_{NMO} and V_{hor} associated with the semblance maxima are shown on top of the semblance scans. The RI method combines unbiased parameter estimation with accurate moveout correction, while the A-T equations yields biased estimates of V_{hor} (or η) combined with substantial residual moveout. In general, the combination of biased estimates of V_{hor} (or η) with substantial residual moveout for the A-T equation method was found to hold for $\eta \gtrsim 0.1$. We found that the A-T equation generally underestimates η, as is evident from this example also. For a maximum ODR of two (not shown) we found that the A-T equation allowed accurate moveout correction, but with biased values of η (for $\eta \lesssim 0.1$ the bias in the estimated η was small). The high accuracy of the RI method yields both unbiased parameter estimation and accurate moveout correction for arbitrary levels of anellipticity.

Field data example

We calculated semblance as a function of V_{NMO} and V_{hor} for a land dataset over a 100-ms window centered on an event of interest, illuminated with ODR of approximately four. The geology consists of relatively flat (dip less than two degrees), predominantly shale layers, such that a layered VTI model seems appropriate for these data. Figures 4a and b show the semblance scans calculated using (a) the A-T equation and (b) the [2/2] RI method, for a randomly selected CMP gather. The semblance calculated with the A-T equation exhibits no clear evidence of the inherent trade-off relation between η and V_{NMO}, whereas semblance calculated with the RI method does display this known trade-off. Figures 4c and d show the moveout-corrected gather for both methods, with the semblance-derived values of η and V_{NMO} used for the moveout correction. Note that the residual moveout from the A-T equation is substantial, whereas the RI method provides well-corrected moveout. Notice the resemblance between the semblance scans and the residual moveouts obtained from both the synthetic and field data (cf. Figure 3a and b). For this CMP gather, the estimated values of η are indeed close to the η value in the synthetic example. Note that the estimated values of η and V_{NMO} are 'effective' quantities resulting from applying the method for a single VTI layer to layered media.

Figures 5a-d show map views of the values of η obtained for the event of interest over the entire dataset, using (a and c) the A-T equation and (b and d) the [2/2] RI method, for maximum ODR of two (a and b) and four (c and d). For both methods, the estimated η values are spatially less variable for maximum ODR of four compared to maximum ODR of two. This can be understood in light of the improved resolution in η for larger ODR. The η values resulting from the RI method are on average slightly higher than those from the A-T equation method; for a maximum ODR of four, $\eta_{av} = 0.28$ for the RI method compared to $\eta_{av} = 0.25$ for the A-T equation. The lower values of η from the A-T equation method, are consistent with the results from the synthetic example (cf. Figure 3). The [2/2] RI method resulted in more spatially smooth and continuous values of both η (and V_{NMO}, not shown) that in some parts seem to correlate with the geologic trend. This spatial continuity was not imposed but followed from a straightforward application of the RI method. The relatively smooth spatial variation of η is consistent with the

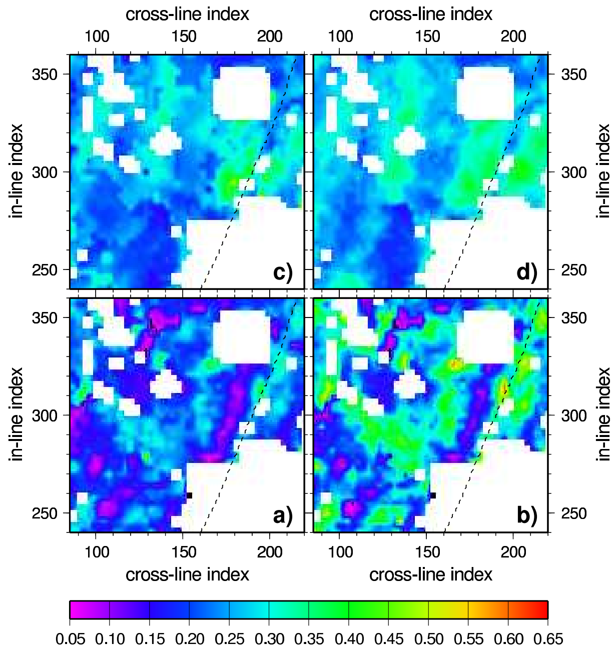

Fig. 5: Map views of η derived from the field data with maximum ODR ≈ 2 (a and b), and maximum ODR ≈ 4 (c and d) using the A-T equation (a and c) and the RI method (b and d).

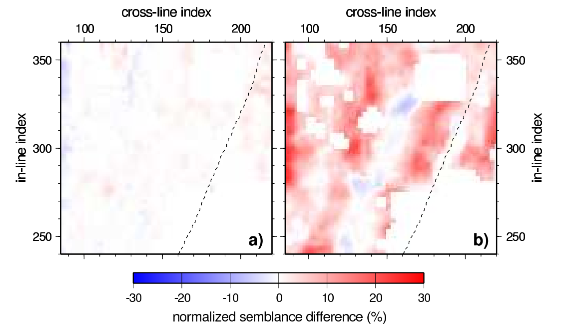

Fig. 6: Normalized semblance difference between A-T method and the RI method for (a) maximum ODR ≈ 2 and (b) maximum ODR ≈ 4.

seismic data, which showed no evidence of substantial lateral heterogeneity. This increases our confidence in the η values obtained using the RI method.

Figure 6a shows the normalized semblance difference between the A-T equation method and the [2/2] RI method, for a maximum ODR of two. The normalized semblance difference is calculated as the difference between the semblances from both methods divided by the semblance obtained using the A-T equation. Figure 6b is as Figure 6a, but here the maximum ODR is four. For a maximum ODR of two, both methods result in similar semblance values, indicating similar ability to flatten the gathers. For a maximum ODR of four, the [2/2] RI method has on average 10% higher semblance values. This supports (for the whole dataset) our findings from the synthetic example that the RI method results in more accurate moveout correction, and confirms the applicability of the method to layered media.

Conclusions

We have presented a RI approach to NHMO analysis in VTI media that has no additional computational overhead relative to that of methods that use equations explicit in η and V_{NMO}. This stems from the observation that, in a single VTI layer, for fixed η and t_0, the influence of V_{NMO} on the NHMO curve is limited to a stretch along the offset axis. This observation is based on the generally accepted assumptions that V_{S0} influences the qP-wave velocity only weakly and that the traveltimes of qP-waves in VTI media depend mainly on η and V_{NMO}. Using this, the calculation of the traveltimes necessary for the RI can be computed from a small precomputed table of traveltimes for a range of η values and a reference value of t_0. Due to the high accuracy of the RI combined with the known accuracy of the above-mentioned assumptions, the method combines accurate moveout correction with unbiased parameter estimation, for arbitrary levels of anellipticity and arbitrary ODR. Using both synthetic and field data, we showed that for $\eta \gtrsim 0.1$ the method significantly outperforms the A-T equation based method. The application to the field data confirms the applicability of the method to layered media.

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