Kirchhoff Inversion in Image Point Coordinates Recast as Source/Receiver Point Processing

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Abstract

Kirchhoff inversion theory tells us that reflection data provides information about the Fourier transform of the reflectivity function at each point in the illuminated subsurface. Thus, inversion formulas expressed as integrals in image point coordinates that closely characterize that Fourier domain are attractive for their relative simplicity. On the other hand, integrals over source/receiver coordinates are more natural to implement on seismic data. We propose a general principle for seismic migration/inversion (MI) processes: think image point coordinates; compute in surface coordinates. This principle allows the natural separation of multiple travel paths of energy from a source to a reflector to a receiver. Furthermore, the Beylkin determinant is particularly simple in this formalism, and transforming to surface coordinates transforms deconvolution-type imaging and inversion operators into convolution-type operators with the promise of better numerical stability.

Introduction

Kirchhoff inversion theory as described in Bleistein et al [2001] reveals that reflection data provides information in a Fourier domain about the reflectivity function at each illuminated point in the subsurface. Important coordinates and vectors of this Fourier domain are shown in Figure 1 and defined in the caption. At each image point, $\boldsymbol{k} = \omega\nabla\tau$ defines a wave vector. For fixed $\boldsymbol{x}$, the direction and magnitude of this vector varies with the choice of source point $\boldsymbol{x}_s$, receiver point $\boldsymbol{x}_r$ and frequency ω, sweeping out a volume in the $\boldsymbol{k}$-domain. It is from this *limited aperture* in the $\boldsymbol{k}$-domain that we reconstruct one or more band limited reflectivity functions for each reflector in the subsurface. For a given survey, the $\boldsymbol{k}$-domain aperture changes with each image point. In that sense, it is a *micro-local* property of the image point.

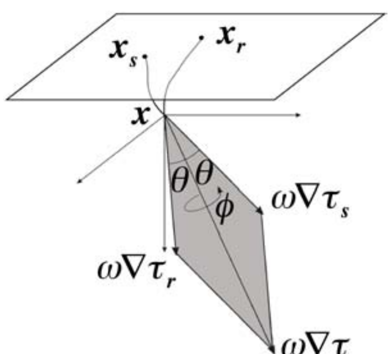

Fig. 1:

Coordinates of the 2D inversion process. $\boldsymbol{x}$; the image point. $\boldsymbol{x}_s$ and $\boldsymbol{x}_r$; source and specular receiver, respectively. θ; incident specular angle of the source ray, also the reflection angle with respect to the normal. $\nabla\tau_s$ and $\nabla\tau_r$; gradients of traveltime from source and receiver, respectively, with sum $\nabla\tau$. θ and ϕ are the half-opening angle between the rays and azimuth angle of the plane of the rays, as indicated.

We define the normal to a reflector to be the *reflector dip*. At a given $\boldsymbol{x}$ can think of the vector $\omega\nabla\tau$ as defining a migration dip for each choice of source and receiver point. It is a fundamental result of the theory that a reflector at an image point $\boldsymbol{x}$ will only be imaged if the reflector dip lies in the aperture of migration dips. With this insight, it should not be too surprising to discover that from a theoretical point of view, inversion formulas are likely to take on their simplest form when expressed as closely as possible in terms of the Fourier variables of the described $\boldsymbol{k}$-domain; hence, our interest in integration in image point coordinates. On the other hand, from the point of view of computation, we prefer processing formulas as integrals over source and receiver coordinates. We will show here that these two points of view are not in conflict and, in fact, have merged through a series of recent results developed by the authors. Further, this merging of objectives has led to relatively simple and likely more numerically stable inversion formulas: the transformation of image point integrals to surface coordinates transforms deconvolution-type MI formulas into correlation-type formulas, eliminating the need to divide by wavefield amplitudes to achieve an image and amplitude estimate. Further, we anticipate better numerical stability when the divisions of deconvolution processing are replaced by the multiplications of correlation processing.

Xu et al [2001] presented a 2D Kirchhoff inversion formula as an integral over dip angles at an image point for each fixed value of the opening (or scattering) angle between the rays from source and receiver at the image point. They then recast the result as an integral over source and receiver points on the acquisition surface. Bleistein and Gray [2002] presented a 3D version of that formula. The transformation between image point coordinates and surface coordinates involves ray Jacobians for the rays from the image point to the source and receiver point.

More recently, in a true-amplitude wave equation migration (WEM), Zhang et al [2004a,b] introduced an extra integration in the processing formula. This additional integration computes the average of outputs over a small patch of opening and azimuth angles at the image point. Each angle pair at the image point corresponds to a different source point at the upper surface. Thus, integrating over an angular patch on the unit sphere of directions at the image point is equivalent to integrating over a collection of source points–different common-source data sets–at the upper surface. Of course, such a transformation of coordinates has a Jacobian that arises in a standard manner in the computation. That Jacobian is closely related to the ray Jacobian associated with the amplitude propagation from the source point to the image point. These authors went a step further, however, and rewrote the Jacobian in terms of the ray theoretic amplitude to which it is related. As a consequence, the deconvolution imaging condition of true amplitude WEM was transformed into a correlation-type imaging condition for true amplitude WEM. The correlation-type imaging condition is attractive because it does not involve division by the amplitude of a Green's function.

This observation that we can transform a deconvolution-type imaging formula into a correlation-type imaging formula has motivated a re-examination of the above-noted results for Kirchhoff MI; namely, to derive similar correlation-type processes for Kirchhoff imaging and inversion processes. This leads to Kirchhoff MI formulas as sums over sources and receivers at the upper surface, while retaining two important features of the formulas for integration over image point angular coordinates: (i) multi-pathing of rays is fully accounted for in all but a few pathological cases and (ii) the Beylkin determinant is easy to calculate. This last observation is most important when one considers the difficulties in computing the Beylkin determinant for common-offset data in 3D.

The transformed inversion is a sum over all sources and receivers, with the output sorted into common scattering angle–both opening angle and azimuth angle–panels.

Separately, Bleistein [2003], proposed a method for extending Kirchhoff MI to MI's with other than ray-theoretic Green's functions. The Green's functions in this MI are fairly arbitrary; they could be Gaussian beams or true-amplitude one-way Green's functions or full waveform Green's functions derived from the two-way wave equation. Each improvement in Green's function type will produce a corresponding improvement in the quality of the image while retaining the same level of amplitude fidelity of the original multi-arrival Kirchhoff inversion. No matter what choice of Green's function is used, we still need specific ray-theoretic information, namely, the WKBJ Green's function amplitudes and the Beylkin determinant. These arise from asymptotically calculating a normalization factor of the final Kirchhoff inversion with the chosen Green's functions. If we were not to use this approximation, then we would need to carry out a point-wise six-fold integration for the purposes of normalizing the amplitude of the kernel.

An important concept about true-amplitude processing is at work here. "True amplitude" as applied to the output of an inversion algorithm refers to estimation of plane wave reflection coefficients. For anything but plane-wave reflection from planar reflectors in a homogeneous medium, this is a WKBJ-approximate estimate and has little meaning in the context of full waveform solutions of the wave equation. Thus, although we image better with better Green's functions, reflection coefficients are still estimated via ray-theoretic asymptotic solutions. Hence, the normalization factors need be no better than what is provided by ray theory, while the general Green's functions used in the extended algorithm are expected to be numerically close to the WKBJ Green's function away from caustics and other anomalies. Thus, we contend that there is both heuristic and physical justification for this simplification. To summarize, if we start with a Kirchhoff MI in migration dip coordinates, the transformation to surface coordinates can be applied. As in the more classical Kirchhoff MI, this will produce an output that is separated into common-opening-angle/azimuth gathers. The result of that transformation on this Kirchhoff MI with more general Green's functions is stated here as well.

Kirchhoff inversion of data gathered from parallel lines of multi-streamer data is particularly elusive. In 3D, the classical Kirchhoff inversion applies to data sets defined by two spatial parameters that characterize the source/receiver distribution. For example, one might think of common-offset data in which the two parameters define the midpoint between source and receiver, or one might think of common-shot data in which the source point is fixed and the two parameters describe the receiver location. Each shot of a multi-streamer survey "looks like" a common-shot data set, except that the data acquisition in the orthogonal direction to the streamer set is too narrow for fully accurate common-shot 3D MI. Thus, it is necessary to use data from all of the shots to generate an inversion output. That means that we must work with a data set with our spatial parameters: two parameters for each source location and two parameters for each receiver location. Kirchhoff inversion performed as sums over all sources and receivers, as presented here, provides such an inversion.

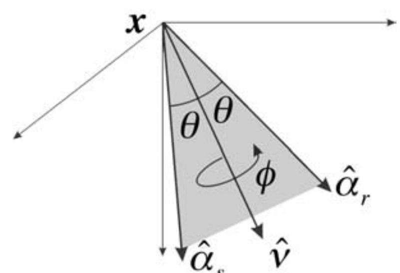

Fig. 2:

Coordinates of the 2D inversion process. $\boldsymbol{x}$; the image point. $\boldsymbol{x}_s$ and $\boldsymbol{x}_r$; source and specular receiver, respectively. θ; incident specular angle of the source ray, also the reflection angle with respect to the normal. $\hat{\boldsymbol{\alpha}}_s$ and $\hat{\boldsymbol{\alpha}}_r$; unit vectors along the specular rays from the image point to the source and receiver, respectively. $\hat{\boldsymbol{\nu}}$, migration dip; also at specular, unit normal to the reflector. α_s, α_r, ν; angles with respect to the vertical of the vectors $\hat{\boldsymbol{\alpha}}_s$, $\hat{\boldsymbol{\alpha}}_r$, $\hat{\boldsymbol{\nu}}$.

Theory: 3D Kirchhoff inversion in dip angle recast as an integral in source/receiver coordinates

Here we present the theory for transforming a basic Kirchhoff inversion as integration in image point variables into an integration in source/receiver coordinates. The relevant coordinate system at the image point is shown in Figure 2. $\hat{\boldsymbol{\nu}}$ defines the migration dip direction; its polar angles are the initial variables of integration. $\hat{\boldsymbol{\alpha}}_s$ and $\hat{\boldsymbol{\alpha}}_r$ are unit vectors in the directions of the rays from source and receiver respectively. θ is the half-opening angle in the plane of these three vectors as indicated, and ϕ is the azimuthal angle of this plane.

Our starting point is the Kirchhoff inversion formula presented in Bleistein and Gray [2002]:

$$R(\boldsymbol{x},\theta,\phi) = \frac{1}{4\pi^2}\frac{2\cos\theta}{\mathrm{v}(\boldsymbol{x})}\int \frac{D_3(\boldsymbol{x}.\boldsymbol{x}_r,\boldsymbol{x}_s)}{A(\boldsymbol{x},\boldsymbol{x}_s)A(\boldsymbol{x},\boldsymbol{x}_r)}\sin\nu_1 d\nu_1 d\nu_2. \tag{1}$$

In this equation, $\boldsymbol{x}$, θ and ϕ are as defined in the caption of Figure 1 and R is the reflectivity function we seek; $\mathrm{v}(\boldsymbol{x})$ is the wave speed. The A's are the amplitudes of the WKBJ or ray-theoretic approximation of the Green's functions for propagation between source point and image point or receiver point and image point as indicated by the pair of arguments in each function. Equation (1) is a deconvolution processing formula for the reflectivity function. The output provides a reflector map—mathematically, a band limited one-dimensional Dirac delta function that peaks on each reflector in the subsurface. The peak value is the product of a plane-wave reflection coefficient at a specular value of the angle θ and the area under the frequency domain source signature of the excitation wave(s). Furthermore, D_3 denotes the preprocessed input data including the appropriate $i\omega$ filter for three dimensional inversion; that is,

$$D_3(\boldsymbol{x}.\boldsymbol{x}_r, \boldsymbol{x}_s) = \frac{1}{2\pi}\int i\omega u(\boldsymbol{x}_r, \boldsymbol{x}_s, \omega) \cdot e^{i\omega(\tau(\boldsymbol{x}.\boldsymbol{x}_r,\boldsymbol{x}_s) + i\kappa(\boldsymbol{x},\hat{\boldsymbol{\nu}},\theta,\phi)\mathrm{sgn}(\omega)\pi/2} d\omega. \tag{2}$$

Here, $u(\boldsymbol{x}_r, \boldsymbol{x}_s, \omega)$ is the observed data on the input trace with source $\boldsymbol{x}_s$, receiver $\boldsymbol{x}_r$, and frequency ω. We start with a fixed θ and ϕ. For each dip direction defined by the polar angles ν_1 and ν_2 defining a direction of the unit migration dip vector $\hat{\boldsymbol{\nu}}$, we trace rays to the upper surface to find the source and receiver points, thereby defining the input trace for this integral. Further, τ is the total travel time from source to image point to receiver, and κ is the KMAH index for the travel path, a count of the number of caustics that the rays pass through on the total ray path.

The integral formula for reflectivity in Equation (1) is really an integral in the polar coordinates of $\boldsymbol{k}$. To see this, note from Figure 1 and the definition of k that

$$\boldsymbol{k} = \omega\nabla\tau = \frac{2\omega\cos\theta}{\mathrm{v}(\boldsymbol{x})}\hat{\boldsymbol{\nu}}. \tag{3}$$

Thus, $\sin\nu_1 d\nu_1 d\nu_2$ is the differential surface area element on the unit sphere of directions of $\boldsymbol{k}$ and $\omega^2 d\omega[\cos\theta/\mathrm{v}(\boldsymbol{x})]^3 = k^2 dk$ provides the differential component in the radial direction and the factor necessary to rescale the unit sphere to the sphere of radius k. Combined with other factors of the inversion formula, only some of the factors of this differential volume in polar coordinates appear in the final formula.

Our objective is to recast the reflectivity of Equation (1) into an integral over the source and receiver points. That would be an integral over four variables of integration, whereas Equation (1) is an integral over two variables. Thus, we need two additional variables of integration. The trick for doing this is to introduce integrations over θ and ϕ as follows.

$$R(\boldsymbol{x}, \theta, \phi) = \frac{1}{4\pi^2}\frac{2\cos\theta'}{\mathrm{v}(\boldsymbol{x})}\int \frac{D_3(\boldsymbol{x}.\boldsymbol{x}_r, \boldsymbol{x}_s)}{A(\boldsymbol{x}, \boldsymbol{x}_s)A(\boldsymbol{x}, \boldsymbol{x}_r)} \cdot\delta(\theta' - \theta)\delta(\phi' - \phi)\sin\nu_1 d\nu_1 d\nu_2 d\theta' d\phi'. \tag{4}$$

Now we are prepared to recast this integral in image point coordinates as an integral in source/receiver coordinates. We describe the process for achieving the transformation we seek. First, we introduce a transformation of coordinates of integration to the pair of polar angles of $\hat{\boldsymbol{\alpha}}_s$ and the pair of polar angles of $\hat{\boldsymbol{\alpha}}_r$. That introduces a Jacobian of transformations among these various angles. As a second step, we transform from the angles of $\hat{\boldsymbol{\alpha}}_s$ to the source coordinates via ray tracing and we transform from the angles of $\hat{\boldsymbol{\alpha}}_r$ to the receiver coordinates, again via ray tracing. The Jacobian of each of these last two changes of variables are just Jacobians used in defining the amplitude of the respective WKBJ Greens functions.

In Bleistein and Gray [2002], we wrote a final representation for the reflectivity using these transformations as described. In this form, the inversion formula still has deconvolution form with additional ray Jacobians. However, Zhang et al [2004a,b], starting from true-amplitude wave equation migration, confronted a similar deconvolution-type reflectivity formula derived from the standard imaging condition, albeit, in their case, in an extension of wave equation migration. They went a step further and replaced the ray Jacobian by its WKBJ amplitude. The result was to recast the deconvolution imaging condition as a correlation imaging condition with all amplitude factors in the numerator. As noted earlier, this leads to more stable computational schemes. We have now added this further step to the sequence of transformations described above. The result is the following representation of the reflectivity function originally given by Equation (1) and revised into Equation (4).

$$R(\boldsymbol{x}, \theta, \phi) = \frac{1}{4\pi^2}\int \frac{\cos\beta_s}{\mathrm{v}(\boldsymbol{x}_s)}\frac{\cos\beta_r}{\mathrm{v}(\boldsymbol{x}_r)} D_3(\boldsymbol{x}.\boldsymbol{x}_r, \boldsymbol{x}_s) \cdot A^*(\boldsymbol{x}, \boldsymbol{x}_s)A^*(\boldsymbol{x}, \boldsymbol{x}_r) \cdot\delta(\theta' - \theta)\delta(\sin\theta'(\phi' - \phi))dx_{s1}dx_{s2}dx_{r1}dx_{r2}. \tag{5}$$

In this equation, the v's are the wave speeds at the source and receiver point, while the angles β_s and β_r are the angles that the emerging rays make with respect to the acquisition surface normal at the source and receiver points, respectively. Note that now the WKBJ Greens function amplitudes appear in the numerator instead of in the dominator, so that this is a *correlation-type* processing formula for the reflectivity.

The logic underlying this formula has also changed. We start from a source/receiver pair. This defines an input trace. We identify the rays traced to an image point. This defines the values of β_s and β_r, θ and ϕ. Discretizing the delta functions would provide a binning procedure in the latter two angles, and then this process produces a suite of reflectivity functions defined by common opening angle and common azimuthal angle at the image point with "common" being within the resolution of the bin size in each angle. With this separation in place, AVA analysis becomes more direct than from procedures based on common offset/common azimuth processing on the upper surface.

The formula in Equation (5) is ideally suited for data gathered from parallel lines of multi-streamer data. In 3D, the classical Kirchhoff inversion applies to data sets defined by two spatial parameters that characterize the source/receiver distribution. Data of this type is really a function of four variables, with no subset of data involving two variables being sufficient for imaging and inversion at decent fidelity. We know of no other processing formula that would achieve the inversion of this type of data acquisition.

Full wave form Kirchhoff inversion

The extension to full waveform Kirchhoff inversion can be formally deduced from this last result. To do so, we combine the WKBJ Greens function amplitudes in the integrand in Equation (5) with the phases produced by the two separate travel times implicit in the formula for D_3 in Equation (2). Each such pairing of amplitude, travel time and KMAH index is a WKBJ approximation of a corresponding Greens function. If we replace each such approximation by a Greens function, the result is

$$R(\boldsymbol{x}, \theta, \phi) = 16\pi \int \mathrm{v}(\boldsymbol{x}) \frac{\cos\beta_s}{\mathrm{v}(\boldsymbol{x}_s)} \frac{\cos\beta_r}{\mathrm{v}(\boldsymbol{x}_r)} i\omega u(\boldsymbol{x}_r, \boldsymbol{x}_s, \omega)$$

$$\cdot G^*(\boldsymbol{x}, \boldsymbol{x}_s, \omega) G^*(\boldsymbol{x}, \boldsymbol{x}_r, \omega) \tag{6}$$

$$\cdot \delta(\theta' - \theta)\delta((\sin\theta'(\phi' - \phi)) dx_{s1} dx_{s2} dx_{r1} dx_{r2}.$$

Here the G's are the Greens functions. When these are exact solutions of the wave equation or they are solutions of the true-amplitude one-way wave equations of Zhang et al [2003], they cannot be separated into a frequency independent amplitude and Fourier-type phase, such as is the case for the WKBJ Greens function. Thus, the inversion formula of Equation (6) will not have the structure of preprocessed Fourier filtered traces evaluated at travel times as in the Kirchhoff inversion defined by Equations (1) and (2). However, better quality Greens functions will lead to better quality images.

The output remains "true-amplitude" in the same sense as the original Kirchhoff inversion. An important principle is at work here as discussed in the introduction. The binning described above produces a suite of reflectivities each associated with a single scattering angle pair. The peak amplitude of the reflectivity is asymptotically equal to the amplitude derived from the original Kirchhoff inversion with WKBJ-approximate Greens functions. Hence, the amplitude is again the product of the area under the source signature and the plane-wave reflection coefficient at a specular incidence angle.

We remark that this last result could be derived more rigorously. We could start from the Kirchhoff forward modeling formula, replacing the WKBJ Green's functions in that formula by the more general Green's functions discussed above. Then, it is a matter of deriving the pseudo-inverse of that modeling operator. This leads to the same result as presented here in Equation (6).

Conclusions

We have reported on a process that transforms deconvolution-type Kirchhoff inversion integrals into correlation-type integrals. The resulting processing formulas are much simpler and simultaneously provide a suite of outputs at common opening angle and common azimuth.

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