Small-angle AVO response of PS-waves in tilted TI media

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Summary

Field records for small source-receiver offsets often contain intensive reflected PS-waves, which are difficult to model without introducing velocity anisotropy on either side of the reflector. Here, we study the small-angle reflection coefficients of the split converted PS_1- and PS_2-waves (R_{PS_1} and R_{PS_2}) for a horizontal interface separating two transversely isotropic media with arbitrary orientations of the symmetry axis.

The normal-incidence reflection coefficients $R_{PS_1}(0)$ and $R_{PS_2}(0)$ vanish when both halfspaces have a horizontal symmetry plane, which happens if the symmetry axis is vertical or horizontal (i.e., if the medium is VTI or HTI). For a tilted symmetry axis in either medium, however, the magnitude of the reflection coefficients can reach substantial values close to 0.1, even if the strength of anisotropy is moderate. To analyze the influence of the symmetry-axis orientation and anisotropy parameters, we develop concise weak-contrast, weak-anisotropy approximations for the PS-wave reflection coefficients and compare them with exact numerical results.

Because of their substantial amplitude, small-angle PS reflections in TI media contain valuable information for anisotropic AVO (amplitude variation with offset) inversion of multicomponent data. Our analytic solutions provide a foundation for linear AVO-inversion algorithms and can be used to guide nonlinear inversion based on the exact reflection coefficients.

Introduction

Converted-wave energy observed on small-offset data (e.g., Thomsen, 2002) can be explained by several factors, including heterogeneity and nongeometrical phenomena. However, the most plausible reason for normal-incidence PS-wave reflections in laterally homogeneous models is velocity anisotropy without a horizontal symmetry plane (Jílek, 2001; Artola et al., 2003). The simplest realistic anisotropic model that can generate P-to-S conversions at zero offset is transverse isotropy with a tilted symmetry axis (TTI medium).

Here, we employ both exact and linearized solutions to identify the parameter combinations responsible for the normal-incidence reflection coefficients and azimuthally varying AVO gradients of PS-waves at isotropic/TTI and TTI/TTI interfaces. Following the approach of Vavryčuk and Pšenčík (1998), and Jílek (2001), we linearize the boundary conditions for TTI media by assuming a weak contrast in the elastic parameters across the interface and weak anisotropy in both halfspaces. To apply the perturbation approach to anisotropic reflection coefficients of

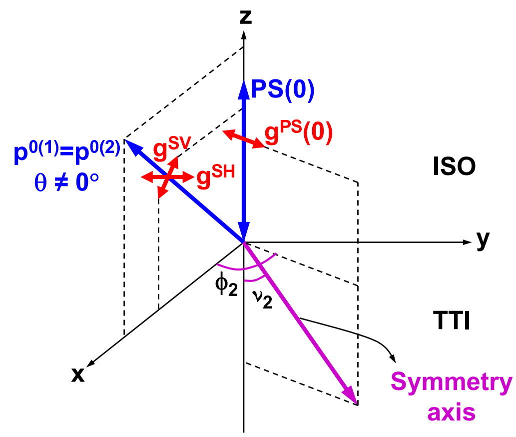

Fig. 1: PS-wave reflected from an interface between isotropic and tilted TI media. At normal incidence, the PS reflection is polarized (vector $\mathbf{g}^{PS}(0)$) in the symmetry-axis plane. For oblique incidence, we analyze the two components of the PS-wave (PSV and PSH) separately.

shear waves, the S-wave polarization vectors in the background isotropic medium have to be close to the actual polarization directions. Hence, the background shear-wave polarizations have to be rotated by the so-called "polarization angle" that depends on the medium parameters (Jech and Pšenčík, 1989). Since the polarization angle is neither a linear function of the perturbations nor is it necessarily small, its presence complicates the derivation of the approximate PS-wave reflection coefficients at oblique incidence (e.g., the derivation of the AVO gradient). The constraints imposed by the treatment of the polarization angle prevented us from obtaining the linearized PS-wave AVO gradients for an incident TTI halfspace with the symmetry axis deviating from the incidence plane.

Normal-incidence reflection coefficient

The general linearized equation for the small-angle PS-wave reflection coefficient as a function of the phase incidence angle θ can be written as (Jílek, 2001; Thomsen, 2002)

$$R_{PS}(\theta) \quad = \quad R_{PS}(0) + G\,\sin\theta\,, \qquad (1)$$

where $R_{PS}(0)$ is the normal-incidence reflection coefficient (AVO intercept) and G is the AVO gradient.

Isotropic/TTI interface

Consider an incidence isotropic halfspace overlying a reflecting TTI halfspace. The normal-incidence PS-wave in this case is polarized in the symmetry-axis plane (i.e.,

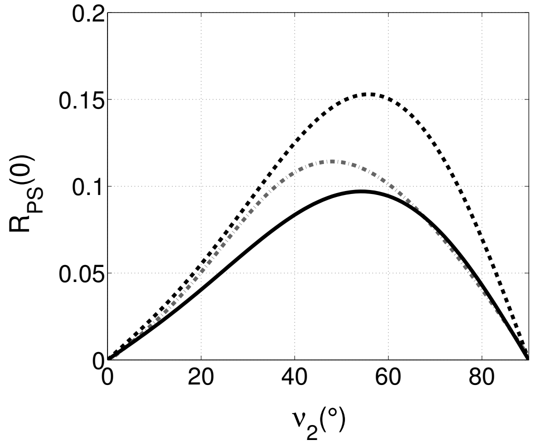

Fig. 2: Normal-incidence PS-wave reflection coefficient for an isotropic/TTI interface as a function of the tilt of the symmetry axis. The solid line is the exact $R_{PS}(0)$, the gray dash-dotted line is computed from equation (3) with the exact first derivative of the P-wave phase velocity, and the dashed black line marks the fully linearized approximation (2). The parameters of the incidence isotropic halfspace are $V_{P,1} = 2.9$ km/s, $V_{S,1} = 1.5$ km/s, and $\rho_1 = 2$ gm/cm^3. The parameters of the reflecting TTI halfspace are $V_{P0,2} = 3.3$ km/s, $V_{S0,2} = 1.8$ km/s, $\rho_2 = 2.2$ gm/cm^3, $\epsilon_2 = 0.4$, $\delta_2 = 0.2$, and $\gamma_2 = 0.11$.

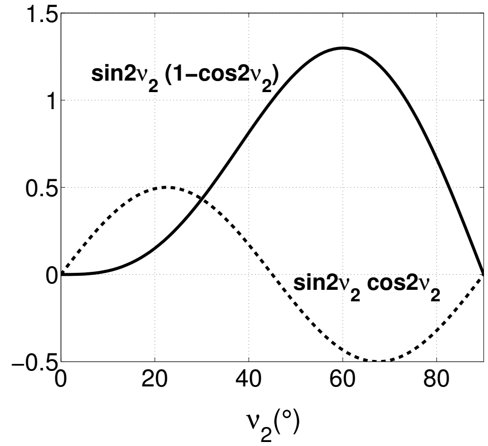

Fig. 3: Functions of ν_2 multiplied with ϵ_2 (solid line) and δ_2 (dashed line) in equation (2). These curves help to explain the influence of ϵ_2 and δ_2 on $R_{PS}(0)$ for different tilts ν_2 in Figure 4.

in the vertical plane that contains the symmetry axis) of the reflecting medium (Figure 1). We describe transverse isotropy by the symmetry-direction velocities of the P- and S-waves (V_{P0} and V_{S0}), Thomsen anisotropy parameters ϵ, δ, and γ, and the tilt ν and azimuth ϕ of the symmetry axis.

The linearized PS-wave normal-incidence reflection coefficient is given by

$$R_{PS}(0) = \frac{g^2 \sin 2\nu_2 \left[\cos 2\nu_2 \left(\delta_2 - \epsilon_2\right) + \epsilon_2\right]}{4(1+g)} \qquad (2)$$

$$= \frac{g^2}{4(1+g)} \frac{1}{V_{P0,2}} \left.\frac{dV_{P,2}(\theta)}{d\theta}\right|_{\theta=0}, \qquad (3)$$

where the subscript "2" denotes the reflecting halfspace, V_P is the P-wave phase velocity, and g is the average P-to-S velocity ratio in the model. It is interesting that the normal-incidence PS-wave reflection coefficient is proportional to the first derivative of the P-wave phase velocity in the reflecting halfspace computed at $\theta = 0$. Although this derivative is supposed to be linearized to make equation (3) equivalent to equation (2), the accuracy of the weak-contrast, weak-anisotropy approximation can be increased by using the exact value of this derivative in equation (3) (Figure 2).

The very existence of the normal-incidence PS reflection is caused by the tilt of the symmetry axis away from the vertical and horizontal directions. Therefore, $R_{PS}(0)$ goes to zero for both VTI ($\nu_2 = 0°$) and HTI ($\nu_2 = 90°$) media. For the model in Figure 4, $R_{PS}(0)$ attains values as high as 0.1 for $\epsilon_2 = 0.4$ at intermediate tilts.

The coefficient $R_{PS}(0)$ is independent of the parameter γ_2 because the P-wave at normal incidence does not excite SH-waves (governed by γ_2) in the reflecting medium. The influence of ϵ_2 and δ_2 on $R_{PS}(0)$ is controlled by the tilt ν_2 of the symmetry axis (Figure 3). If the function of ν_2 multiplied with ϵ_2 and δ_2 in equation (2) becomes zero, the corresponding anisotropy parameter makes no contribution to $R_{PS}(0)$. For example, according to approximation (2), δ_2 has no influence on $R_{PS}(0)$ at $\nu_2 = 45°$. This result is generally supported by the computations of the exact reflection coefficient in Figure 4, although the curves corresponding to different δ_2-values do not intersect exactly at the same point.

For small tilts ν_2, δ_2 has a greater influence on $R_{PS}(0)$ than does ϵ_2, while for larger ν_2, ϵ_2 largely determines the value of $R_{PS}(0)$ (Figures 3 and 4). This is explained by the well-known behavior of the P-wave phase-velocity function in TI media (e.g., Tsvankin, 2001).

TTI/TTI interface

In isotropic media, an incident P-wave excites a single converted (PSV) mode because the symmetry prohibits the generation of PSH conversions. When the incidence halfspace is anisotropic, the PS reflection splits into the PS_1 and PS_2 modes that have different normal-incidence reflection coefficients [$R_{PS_1}(0)$ and $R_{PS_2}(0)$] and AVO gradients. According to our convention, the polarization vector of the PS_1-wave lies in the plane formed by the slowness vector and the symmetry axis (i.e., it would be the SV mode, if the symmetry axis were vertical), while the PS_2-wave is polarized perpendicular to this plane (SH mode). The linearized approximations for $R_{PS_1}(0)$ and $R_{PS_2}(0)$ have the form

$$R_{PS_1}(0) = \frac{g^2}{4(1+g)} \{-\sin 2\nu_1 \left[\cos 2\nu_1 \left(\delta_1 - \epsilon_1\right) + \epsilon_1\right] + \cos(\phi_2 - \phi_1) \sin 2\nu_2 \left[\cos 2\nu_2 \left(\delta_2 - \epsilon_2\right) + \epsilon_2\right]\}, \qquad (4)$$

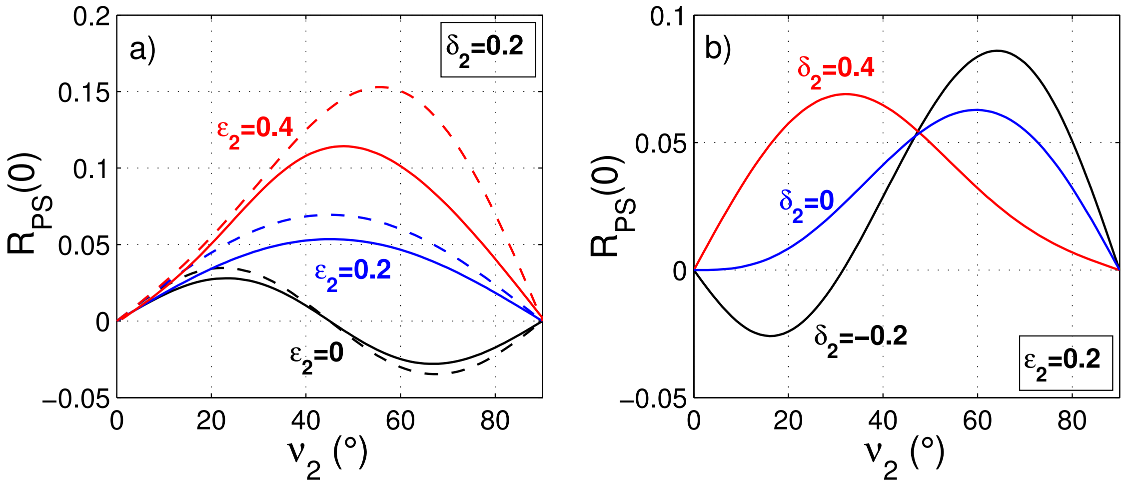

Fig. 4: Dependence of the normal-incidence PS-wave reflection coefficient for an isotropic/TTI interface on the parameters (a) ϵ_2 and (b) δ_2. The solid lines mark the exact $R_{PS}(0)$, the dashed lines represent the linearized approximation (2). The parameters of the incidence isotropic halfspace are $V_{P,1} = 2.9$ km/s, $V_{S,1} = 1.5$ km/s, and $\rho_1 = 2$ gm/cm^3. The parameters of the reflecting TTI halfspace are $V_{P0,2} = 3.3$ km/s, $V_{S0,2} = 1.8$ km/s, $\rho_2 = 2.2$ gm/cm^3, and $\gamma_2 = 0.11$.

$$R_{PS_2}(0) = \frac{g^2}{4(1+g)}\{\sin(\phi_2 - \phi_1)\,\sin 2\nu_2 \\ [\cos 2\nu_2\,(\delta_2 - \epsilon_2) + \epsilon_2]\}\,. \quad (5)$$

It is clear from the symmetry of the model that the normal-incidence reflection coefficients should depend just on the difference between the azimuths of the symmetry axes ($\phi_2 - \phi_1$), which is confirmed by equations (4) and (5). Indeed, a simultaneous azimuthal rotation of both symmetry axes can only change the azimuthal direction of the polarization vectors of the PS-waves. When the vertical symmetry planes of the two TI halfspaces coincide ($\phi_1 = \phi_2$), the P-wave at normal incidence excites only one (PS_1 or PSV) wave polarized in the symmetry-axis plane.

Note that the terms involving the tilt of the symmetry axis and the anisotropy parameters in equations (4) and (5) have the same form as the corresponding terms for the simpler isotropic/TTI model examined above. Therefore, the conclusions drawn above for ν_2, ϵ_2, and δ_2 (Figure 3) apply to ν_1, ϵ_1, and δ_1 as well. If both TI halfspaces have the same orientation of the symmetry axes and the same parameters ϵ and δ, the linearized $R_{PS_1}(0)$ and $R_{PS_2}(0)$ [equations (4) and (5)] vanish, even though there may be a significant jump in the other parameters across the interface. This result is confirmed by the numerical test in Figure 5, where the exact coefficient $R_{PS_1}(0) \approx 0$ for $\phi_1 = \phi_2$. On the other hand, if the symmetry axes are tilted in the opposite directions from the vertical ($\nu_1 = \nu_2$ and $\phi_1 - \phi_2 = 180°$), the normal-incidence reflection coefficient may exceed 0.1 even for moderately anisotropic models (Figure 5).

AVO gradients

The AVO gradients of the split PS-waves can be computed numerically by estimating the best-fit initial slope of the exact reflection coefficient expressed as a function of $\sin\theta$. In the linearized weak-anisotropy, weak-contrast approximation, the gradient G is obtained explicitly as the multiplier of $\sin\theta$ [equation (1)].

Isotropic/TTI interface

If the incidence halfspace is isotropic, only the gradient G_{PS_1} (G_{PSV}) contains both isotropic and anisotropic terms, while G_{PS_2} (G_{PSH}) is purely anisotropic. Numerical testing shows that the gradient G_{PS_1} is not significantly distorted by the anisotropy for common values of the velocity ratio g (Figure 6a). The influence of the anisotropy in the reflecting halfspace changes primarily the normal-incidence coefficient $R_{PS}(0)$ that goes to zero in the isotropic model. Although the AVO gradients of both PS-waves vary with azimuth, their average values are close to those for isotropic media, and the magnitude of the azimuthal variations is typically small.

The wave PS_2 (PSH) vanishes for a reflecting VTI halfspace when the reflected PS-wave is polarized in the incidence plane. Therefore, the gradient G_{PS_2} increases as the symmetry axis in the reflecting medium deviates from the vertical. G_{PS_2} also goes to zero when the symmetry axis is confined to the incidence plane or is perpendicular to it.

TTI/TTI interface

If the incidence halfspace is TTI with the symmetry axis not confined to the incidence plane, then there are no purely isotropic terms in either AVO gradient. The influence of the anisotropy in the upper halfspace leads to the splitting of the reflected PS-wave and a

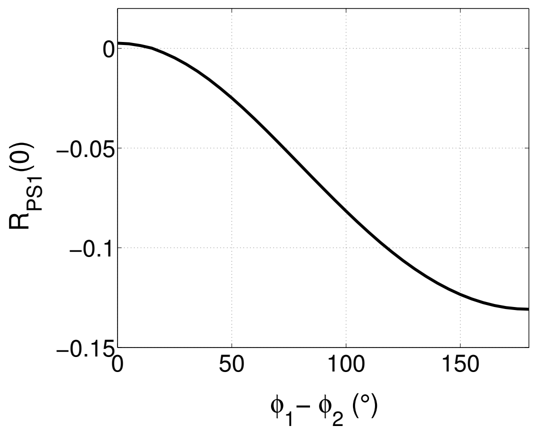

Fig. 5: Exact $R_{PS_1}(0)$ for a TTI/TTI interface as a function of the difference between the azimuths of the symmetry axes. The symmetry-direction velocities and densities are $V_{P0,1} = 2.9$ km/s, $V_{S0,1} = 1.5$ km/s, $\rho_1 = 2$ gm/cm^3, $V_{P0,2} = 3.3$ km/s, $V_{S0,2} = 1.8$ km/s, $\rho_2 = 2.2$ gm/cm^3. The tilt of symmetry axis is the same in both halfspaces, $\nu_1 = \nu_2 = 60°$. Also, $\epsilon_1 = \epsilon_2 = 0.2$, $\delta_1 = \delta_2 = -0.1$, and $\gamma_1 = \gamma_2 = 0.1$. Note that the tilt is measured from the vertical toward the radius-vector with the azimuth ϕ.

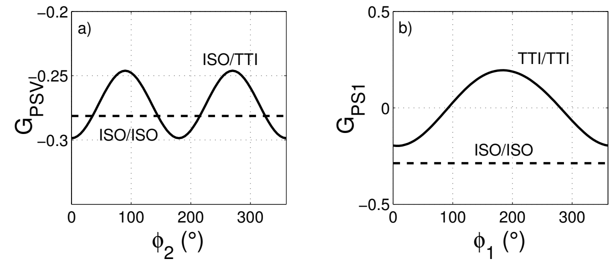

Fig. 6: Exact AVO gradients of the (a) PSV-wave as a function of the azimuth ϕ_2 for isotropic/TTI and isotropic/isotropic interfaces and (b) PS$_1$-wave as a function of the azimuth ϕ_1 for TTI/TTI and isotropic/isotropic interfaces. The parameters of the incidence isotropic medium are $V_{P,1} = 2.9$ km/s, $V_{S,1} = 1.5$ km/s, and $\rho_1 = 2$ gm/cm^3 and those of the reflecting isotropic medium are $V_{P,2} = 3.3$ km/s, $V_{S,2} = 1.8$ km/s, and $\rho_2 = 2.2$ gm/cm^3. The parameters of the incidence TTI medium [in (b)] are $V_{P0,1} = 2.9$ km/s, $V_{S0,1} = 1.5$ km/s, $\rho_1 = 2$ gm/cm^3, $\epsilon_1 = 0.2, \delta_1 = 0.1, \gamma_1 = 0.1$, and $\nu_1 = 60°$; for the reflecting TTI medium, $V_{P0,2} = 3.3$ km/s, $V_{S0,2} = 1.8$ km/s, $\rho_2 = 2.2$ gm/cm^3, $\epsilon_2 = 0.3, \delta_2 = 0.15, \gamma_2 = 0.11, \nu_2 = 30°$, and [in (b)] $\phi_2 = 30°$.

substantial deviation of the gradients from those for the corresponding isotropic model (Figure 6b).

The anisotropy parameters ϵ_1 and δ_1 of the incidence halfspace influence only G_{PS_1}, while γ_1 influences only G_{PS_2}. To explain this result valid for an arbitrary orientation of the symmetry axis of the incidence halfspace, note that the PS$_1$-wave would be the PSV mode if the symmetry axis were vertical, and the PS$_2$-wave would be the PSH (transversely polarized) mode. The P- and SV-wave propagation in TI media is governed only by ϵ and δ, while the SH-wave velocity is controlled by γ.

Conclusions

We studied the small-angle PS-wave AVO response for the most common type of anisotropy: transverse isotropy with a tilted symmetry axis (TTI medium). If the reflector does not coincide with a symmetry plane in either halfspace, a P-wave at normal incidence always generates PS-waves, with the reflection coefficient that can exceed 0.1 for moderate values of the anisotropy parameters typical for shale formations. When the incidence halfspace is isotropic, the reflected PS-wave at normal incidence is polarized in the symmetry-axis plane of the reflecting TTI medium.

Our analytic solutions show that the contributions of the Thomsen parameters ϵ and δ to the normal-incidence reflection coefficients of the split PS-waves [$R_{PS_1}(0)$ and $R_{PS_2}(0)$] are governed by simple functions of the symmetry-axis tilt ν, which have the same form for both halfspaces. When the symmetry-axis orientation and anisotropy parameters do not change across the interface, the normal-incidence reflection coefficients are insignificant, regardless of the strength of the velocity and density contrast. The AVO (amplitude variation with offset) gradients of the PS-waves are mostly influenced by the anisotropy of the incidence medium that causes shear-wave splitting and determines the partitioning of energy between the PS$_1$ and PS$_2$ modes.

The linearized approximations developed here not only provide physical insight into the behavior of the PS-wave reflection coefficients, but can be also used to quickly evaluate the small-angle PS-wave amplitudes and incorporate them into AVO analysis.

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