Plane-wave propagation and radiation patterns in attenuative TI media

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Summary

We develop a consistent analytic treatment of plane-wave properties and point-source radiation for transversely isotropic (TI) media with attenuation anisotropy. To characterize TI attenuation, we follow the idea of Thomsen notation for velocity anisotropy and replace the quality-factor components Q_{ij} by two reference isotropic quantities and three dimensionless anisotropic parameters ϵ_Q, δ_Q, and γ_Q. Assuming weak anisotropy (for both velocity and attenuation) as well as small attenuation allows us to obtain simple linearized attenuation coefficients expressed through the Thomsen-style parameters. These approximations not only provide analytic insight into the behavior of the attenuation coefficients, they also remain accurate for the practically important range of small and moderate anisotropic coefficients, in particular for near-vertical and near-horizontal propagation directions.

We also employ the stationary-phase method to derive the far-field Green's function for arbitrarily anisotropic media with TI attenuation. The influence of the attenuation on the radiation patterns is absorbed by an exponential term that depends on the "group" attenuation coefficient along the raypath. The relationship between the group and phase (plane-wave) attenuation coefficients involves just the group and phase angles and can be used to estimate the Thomsen-style parameters from wide-angle attenuation measurements.

Introduction

Physical-modeling experiments show that attenuation in anisotropic rocks can be directionally dependent, and the attenuation anisotropy is sometimes more significant than the velocity anisotropy (Hosten et al., 1987; Arts and Rasolofosaon, 1992). The influence of anisotropic attenuation on the amplitudes of reflected waves may cause errors in AVO (amplitude variation with offset) analysis. A comprehensive discussion of wave propagation in anisotropic attenuative media is given by Carcione (2001). His treatment, however, is generally based on the stiffness coefficients c_{ij}, and the results are not expressed in a form amenable to data-processing applications.

The first part of this paper is devoted to plane-wave signatures in TI media with TI attenuation. We introduce Thomsen-style parameters that describe the angle-dependent Q for TI attenuation and demonstrate the advantages of the new notation by analyzing the attenuation coefficient as a function of phase angle. Then, extending the formalism of Tsvankin and Chesnokov (1990) to attenuative media, we derive a closed-form expression for the radiation patterns in the presence of TI attenuation and illustrate the results with numerical simulation of SH-wave propagation.

Definition of the Q factor for TI media

The parameters used in attenuation measurements include the quality factor Q, attenuation coefficient, logarithmic decrement of amplitude, and complex modulus (Johnston and Toksöz, 1981). All those parameters, however, were originally designed for isotropic attenuation and need to be generalized for anisotropic materials. Here, we follow Carcione (2001, p. 58) in defining Q as $Q_{ij} \equiv c_{ij}/c^I_{ij}$, where c_{ij} and c^I_{ij} are the real and the imaginary parts, respectively, of the stiffness coefficient $\tilde{c}_{ij} = c_{ij} + i c^I_{ij}$ (no summation over i and j is implied). Evidently, the matrix formed by the Q_{ij} components inherits the structure of the stiffness matrix. For the case of VTI (TI with a vertical symmetry axis) media with VTI attenuation, the $\mathbf{Q}$ matrix has the form

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & Q_{13} & 0 & 0 & 0 \\ Q_{13} & Q_{13} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}, \quad (1)$$

where $Q_{12} = Q_{11} \dfrac{c_{11} - 2c_{66}}{c_{11} - 2c_{66}\, Q_{11}/Q_{66}}$. Although the symmetry axis is assumed to be vertical, the results below are derived for a homogeneous medium and can be readily adapted to TI models with any symmetry-axis orientation.

The discussion here is based on the assumption of a frequency-independent Q, which is often valid in the seismic frequency band. Treatment of velocity dispersion is also outside the scope of this paper.

Thomsen-style notation for TI attenuation

The description of seismic signatures in the presence of velocity anisotropy can be substantially simplified by using Thomsen (1986) notation. The advantages of Thomsen parameters in the analysis of seismic velocities and amplitudes for TI media are discussed in detail by Tsvankin (2001). Here, we extend the idea of Thomsen notation to the directionally dependent attenuation coefficient in TI media. It is convenient to introduce the normalized attenuation coefficient $\mathcal{A}$ that defines the rate of amplitude decay per wavelength:

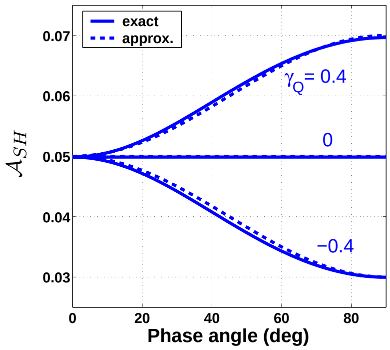

Fig. 1: Attenuation coefficient $\mathcal{A}_{SH}$ for SH-waves in a VTI medium with $\gamma = 0.1$, $Q_{55} = 10$, and $\gamma_Q = -0.4$, 0, and 0.4. The approximate $\mathcal{A}_{SH}$ (dashed lines) is obtained from equation (4), and the exact $\mathcal{A}_{SH}$ (solid) from the Christoffel equation.

$$\mathcal{A} \equiv \frac{k^I}{k}, \tag{2}$$

where k and k^I are the real and imaginary parts (respectively) of the wavenumber. We obtain the coefficient $\mathcal{A}$ by solving the Christoffel equation for plane waves under the assumption of *homogeneous* wave propagation (i.e., the real and imaginary part of the wave vector are taken parallel to each other).

The matrix $\mathbf{Q}$ for models with TI attenuation contains five independent elements, which can be replaced by two reference (isotropic) parameters and three dimensionless coefficients (ϵ_Q, δ_Q, and γ_Q) responsible for the attenuation anisotropy. Since the attenuation coefficient is inversely proportional to the quality factor, we define the Thomsen-style parameters through quantities $\frac{1}{Q_{ij}}$. To maintain close similarity with Thomsen notation for velocity anisotropy, we choose the P- and SV-wave attenuation coefficients in the symmetry direction ($\frac{1}{2Q_{33}}$ and $\frac{1}{2Q_{55}}$) as the reference values.

The attenuation anisotropy parameter γ_Q is defined by analogy with the Thomsen parameter γ as the fractional difference between the attenuation coefficients of SH-waves in the horizontal and vertical directions:

$$\gamma_Q \equiv \frac{1/Q_{66} - 1/Q_{55}}{1/Q_{55}} = \frac{Q_{55} - Q_{66}}{Q_{66}}. \tag{3}$$

The parameter γ_Q controls the magnitude of the SH-wave attenuation anisotropy; for isotropic Q, $\gamma_Q = 0$. When both γ_Q and γ are small ($|\gamma| \ll 1$, $|\gamma_Q| \ll 1$), the SH-wave attenuation coefficient is given by

$$\mathcal{A}_{SH} = \frac{1}{2Q_{55}}(1 + \gamma_Q \sin^2\theta), \tag{4}$$

where θ is the phase angle with the symmetry (vertical) direction. It is clear from equation (4) that γ_Q determines the rate and sign of the variation of $\mathcal{A}_{SH}(\theta)$ away from the symmetry direction (Figure 1).

The parameter ϵ_Q, analogous to Thomsen's ϵ, is equal to the fractional difference between the P-wave attenuation coefficients in the horizontal and vertical directions:

$$\epsilon_Q \equiv \frac{1/Q_{11} - 1/Q_{33}}{1/Q_{33}} = \frac{Q_{33} - Q_{11}}{Q_{11}}. \tag{5}$$

Another attenuation-anisotropy parameter, δ_Q, can be obtained from the second derivative of the P-wave attenuation coefficient: $\left.\frac{d^2\mathcal{A}_P}{d\theta^2}\right|_{\theta=0} = 2\mathcal{A}_P|_{\theta=0}\,\delta_Q$. The role of δ_Q in describing the P-wave attenuation anisotropy is similar to that of δ in the P-wave phase-velocity equation (Thomsen, 1986; Tsvankin, 2001). Since the first derivative of $\mathcal{A}_P$ for $\theta = 0$ is equal to zero, δ_Q is responsible for the near-vertical variation of the P-wave attenuation. Assuming that both the attenuation and attenuation anisotropy are weak, δ_Q can be expressed as

$$\delta_Q \equiv \frac{\frac{Q_{33}-Q_{55}}{Q_{55}} c_{55} \frac{(c_{13}+c_{33})^2}{(c_{33}-c_{55})} + 2\frac{Q_{33}-Q_{13}}{Q_{13}} c_{13}(c_{13}+c_{55})}{c_{33}(c_{33}-c_{55})}. \tag{6}$$

Equation (6) indicates that the attenuation anisotropy is influenced by the velocity anisotropy. By representing the stiffnesses c_{ij} in terms of Thomsen parameters, δ_Q can be expressed as a function of δ and the squared velocity ratio $g \equiv V_{S0}^2/V_{P0}^2 = c_{55}/c_{33}$. In contrast to the velocity anisotropy parameter δ, the absolute value of δ_Q may be large (even greater than unity).

Approximate attenuation coefficients for P- and SV-waves

In combination with the Thomsen parameters for the velocity function, the parameters $\frac{1}{2Q_{33}}$, $\frac{1}{2Q_{55}}$, ϵ_Q, δ_Q, and γ_Q fully characterize the attenuation of P-, SV- and SH-waves. Because of the complexity of the exact equations for the P- and SV-wave attenuation coefficients, here we obtain approximate expressions for $\mathcal{A}$ by assuming simultaneously 1) weak attenuation ($\frac{1}{Q_{ij}} \ll 1$); 2) weak attenuation anisotropy ($|\epsilon_Q| \ll 1$, $|\delta_Q| \ll 1$); and 3) weak velocity anisotropy ($|\epsilon| \ll 1$, $|\delta| \ll 1$).

Linearizing the attenuation coefficients of P- and SV-waves in the small parameters yields equations that have the same form as Thomsen's (1986) weak-anisotropy approximation for the corresponding phase velocities. For P-waves, we find

$$\mathcal{A}_P = \frac{1}{2Q_{33}}(1 + \delta_Q \sin^2\theta\cos^2\theta + \epsilon_Q \sin^4\theta). \tag{7}$$

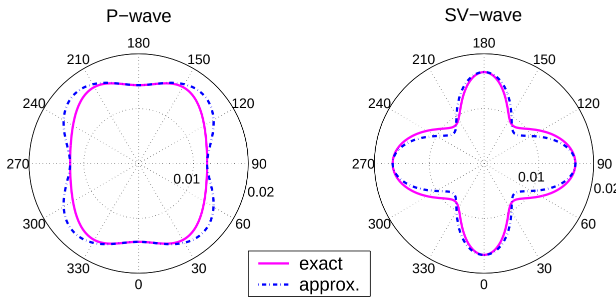

Fig. 2: Attenuation coefficients of P-waves (left) and SV-waves (right) as functions of the phase angle. The solid curves are the exact coefficients; the dashed curves are the approximations from equations (7) and (8). The model parameters are V_{P0}=2.42 km/s, V_{S0}=1.4 km/s, $\epsilon = 0.4$, $\delta = 0.15$, $Q_{33} = 35$, $Q_{55} = 30$, $\epsilon_Q = -0.125$, and $\delta_Q = 0.94$.

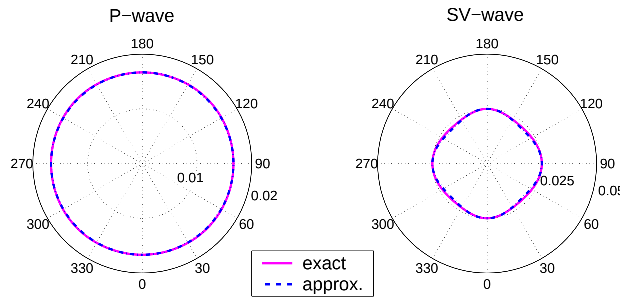

Fig. 3: Same as Figure 2, but for $\epsilon_Q = \delta_Q = 0$.

If both ϵ_Q and δ_Q go to zero, the approximate $\mathcal{A}_P$ is independent of angle. The linearized SV-wave attenuation coefficient is given by

$$\mathcal{A}_{SV} = \frac{1}{2Q_{55}} \left(1 + \sigma_Q \sin^2\theta \cos^2\theta\right) , \tag{8}$$

where $\sigma_Q \equiv \dfrac{(1-g-g_Q)(\epsilon-\delta)+(1-g)(\epsilon_Q-\delta_Q)}{g\, g_Q}$ determines the second derivative of the SV-wave attenuation coefficient $\mathcal{A}_{SV}$ in the symmetry direction, and $g_Q \equiv \dfrac{Q_{33}}{Q_{55}}$.

The accuracy of the approximate solutions (7) and (8) is illustrated by the numerical tests in Figures 2 and 3. The P-wave attenuation coefficient in Figure 2 has an extremum (a maximum) near 43° because ϵ_Q and δ_Q have different signs; otherwise, $\mathcal{A}_P$ varies monotonically between the vertical and horizontal directions. The large negative value of $\sigma_Q = -2.13$ results in the concave shape of the $\mathcal{A}_{SV}$-curve at intermediate angles. The approximations for both attenuation coefficients give accurate results for near-vertical propagation directions with angles θ up to about 30°. The error becomes noticeable for intermediate angles $30° < \theta < 75°$ and then decreases again near the horizontal plane. Note that both the velocity and attenuation anisotropy for the model from Figure 2 is not weak, and the values of σ_Q and $\sigma = 0.75$ are particularly large.

Figure 3 displays the attenuation coefficients for a TI medium with $\epsilon_Q = \delta_Q = 0$. In agreement with equation (7), the P-wave attenuation is almost isotropic, but the attenuation coefficient of SV-waves deviates from a circle because of the contribution of the velocity anisotropy in equation (8).

Radiation patterns in attenuative TI media

Here, we discuss the influence of attenuation on the radiation patterns of body waves excited by a point force in a homogeneous anisotropic medium. To obtain the Green's function, we apply the stationary-phase method to the Weyl-type integral for point-source radiation following the analytic results of Tsvankin and Chesnokov (1990) and Tsvankin (2001, Chapter 2). For an anisotropic medium with TI attenuation, the far-field spectrum of the radiation pattern can be determined by multiplying the spectrum in the reference purely elastic medium with the following factor:

$$\beta_Q^{\nu} = (1 - i\mathcal{A}^{\nu})^2 \exp\left(-\mathcal{A}^{\nu} \frac{\omega R}{V_G^{\nu}}\right) , \tag{9}$$

where ω is the angular frequency, R is the distance between the source and receiver, V_G^{ν} is the group velocity, and ν denotes the wave type (i.e., P-, SV-, or SH-waves). Since $\mathcal{A}^{\nu}\omega R/V_G^{\nu} = k^I R \cos(\psi - \theta)$ (ψ is the group angle), the group attenuation coefficient k_G^I responsible for the amplitude decay along the source-receiver ray can be found as

$$k_G^I = k^I \cos(\psi - \theta) . \tag{10}$$

Equation (10) is similar to the relationship between phase and group velocities in anisotropic media, but the phase attenuation is multiplied (rather than divided, as is the case for the velocities) with $\cos(\psi - \theta)$.

To illustrate the relationship between the group and phase attenuation, we model SH-wave propagation using a 2D finite-difference solution of the wave equation (Figure 4). The influence of attenuation is particularly convenient to study for SH-waves, because in purely elastic TI media the amplitude along the elliptical SH-wavefront is constant (Tsvankin, 2001), as illustrated by Figure 5. Since the quality factor increases away from the symmetry axis, the largest attenuation in Figure 4 is observed in the vertical direction.

The modeling results can be used to measure the attenuation coefficient for a range of group angles ψ (crosses in Figure 6). To verify the relationship between the group and phase attenuation coefficients, we first compute the phase coefficient $k^I(\theta)$ (solid line) from equation (4). Then we calculate the group angle ψ from the phase angle θ and obtain the group coefficient $k_G^I(\psi)$ (dashed line)

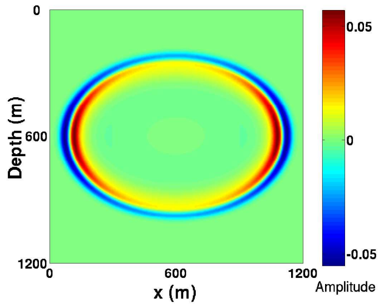

Fig. 4: 2D snapshot of the SH-wavefront (particle displacement) propagating in a VTI medium with VTI Q. The model parameters are $V_{S0} = 1.83$ km/s, $\gamma = 0.44$, $Q_{55} = 5$, and $\gamma_Q = -0.5$; the time t=0.25 s.

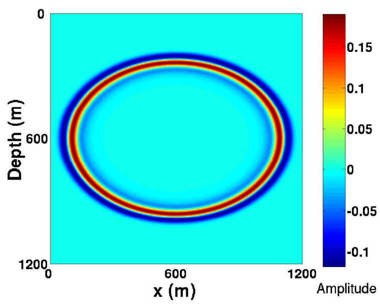

Fig. 5: Same as Figure 4, but without attenuation ($Q_{55} = Q_{66} = \infty$).

from equation (10). This analytic coefficient k_G^I is sufficiently close to the measured value, which corroborates the validity of equation (10). The above results suggest that the attenuation-anisotropy parameter γ_Q can be estimated from the group attenuation coefficient k_G^I, if the velocity function has been reconstructed from traveltime measurements.

Conclusions

The main goal of this paper is to build a practical analytic framework for describing amplitude distortions in attenuative TI media. To facilitate the description of the anisotropic attenuation, we introduce Thomsen-style parameters responsible for the directionally dependent attenuation coefficients of P-, SV-, and SH-waves. The approximate attenuation coefficients for all three modes (P, SV, SH), obtained in the limit of weak attenuation and weak velocity and attenuation anisotropy, have the same form as the corresponding linearized phase-velocity functions. The expressions for the Thomsen-style parameters, however, reflect the coupling between the attenuation and velocity anisotropy. It should be emphasized that in the presence of velocity anisotropy the attenuation may be directionally dependent even in media with a purely isotropic matrix $\mathbf{Q}$.

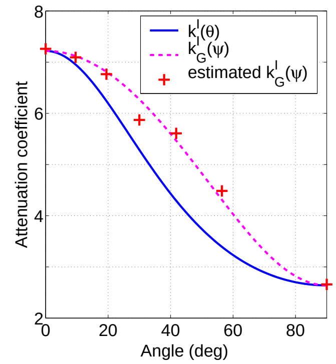

Fig. 6: Comparison of the measured and computed group attenuation coefficients for the model from Figure 4. The solid curve is the phase attenuation coefficient k^I, the dashed curve is the group attenuation coefficient k_G^I computed from equation (10), and crosses are the values of k_G^I measured from the synthetic data. The horizontal axis represents the phase angle for k^I and the group angle for k_G^I.

Asymptotic analysis of the Green's function helps to describe the influence of attenuation anisotropy on point-source radiation and find a simple expression for the effective attenuation along the raypath. These results provide a basis for estimating the attenuation-anisotropy parameters from field data and correcting the AVO response for angle-dependent attenuation.

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