How to obtain true amplitude common-angle gathers from one-way wave equation migration?

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Abstract

True amplitude wave equation migration (WEM) provides the quality image of wave equation migration along with proper weighting of the output for estimation of an angularly dependent reflection coefficient, similar to the output of Kirchhoff inversion. In an earlier paper, Zhang et al. (2003) presented a true amplitude WEM for common-shot data in heterogeneous media. Here we present two additional true amplitude heterogeneous media WEM techniques to produce common-angle gathers. The first is based on the same common-shot image gathers, while the second is based on a modified double-square-root one-way wave equation. True amplitude common-shot migration was based on an imaging condition using the ratio, p_U/p_D. In contrast, common-angle gathers require the *product* $p_U p_D^*$ in the imaging condition. We demonstrate this new method with a simple synthetic example.

Introduction

True amplitude migration treats seismic imaging as an inverse problem of acoustic wave modelling. Among different migration methods, we prefer the true amplitude one because its migrated amplitude does not suffer geometrical spreading loss and gives a direct measurement to the angle dependent reflectivity. True amplitude migration theory was firstly developed for Kirchhoff migration (Beylkin, 1985; Bleistein, 1987; Schleicher et al., 1993), and in usual practice is applied in common-offset domain. Recently developments show that common-angle domain is a natural way of handling true amplitude issue and unfolding multivalued ray fields for complex velocity models (Xu et al., 2001).

As cheaper and faster computers becoming available, the demands of imaging complex geological structures have led to growing popularity of prestack migrations based on one-way wavefield extrapolation. It has been found that the original one-way wave equations used for wave equation migration (WEM) (Claerbout, 1971) were designed to produce accurate traveltimes, but were not intended to produce accurate amplitudes, even at the level of leading order asymptotic WKBJ or ray-theoretic amplitudes. Relying on true amplitude one-way wave equations introduced by G. Zhang (1993), Zhang et al. (2003) developed true amplitude common-shot migration for heterogeneous media. In this abstract, we take a further step to show how to apply true amplitude one-way wave equations to produce true amplitude common-angle gathers from WEM. We will design two true amplitude migration algorithms, one based on common-shot migration and the other based on a modified double-square-root one-way wave equation. Although true amplitude shot gathers require using p_U/p_D in the imaging condition, we will prove that $p_U p_D^*$ turns out to be the right product in the imaging condition for producing true amplitude common reflection angle gathers. The proposed methods have good potential in practice, they are numerically more stable, produce less migration artifacts and give better migrated amplitude versus angle (AVA) performance.

True amplitude common-angle gathers based on common-shot migration

We begin by considering two-dimensional common-shot migration using true amplitude one-way wave equations (Zhang et al., 2003):

$$\begin{cases} \left(\frac{\partial}{\partial z} + \Lambda - \Gamma\right) p_D(\vec{x}; x_s; \omega) = 0, \\ p_D(x, 0; x_s; \omega) = \delta(x - x_s), \end{cases} \tag{1}$$

and

$$\begin{cases} \left(\frac{\partial}{\partial z} - \Lambda - \Gamma\right) p_U(\vec{x}; x_s; \omega) = 0, \\ p_U(x, 0; x_s; \omega) = Q(x; x_s; \omega), \end{cases} \tag{2}$$

with Λ and Γ being pseudo-differential operators with symbols λ and γ given by

$$\begin{aligned} \lambda &= ik_z = \frac{i\omega}{v(\vec{x})}\sqrt{1 - \frac{(v(\vec{x})\bar{k})^2}{\omega^2}}, \quad \vec{x} = (x, z), \\ \gamma &= \frac{v_z(\vec{x})}{2v(\vec{x})}\left(1 + \frac{(v(\vec{x})k_x)^2}{\omega^2 - (v(\vec{x})k_x)^2}\right). \end{aligned} \tag{3}$$

In equations (1) and (2) $\vec{x}_s = (x_s, 0)$, p_D is the downgoing response to the impulsive boundary condition at $z = 0$ and p_U is the upgoing wave that must equal the observed data, Q at the upper surface. To obtain the image, we propose following imaging condition

$$I(\vec{x}; x_s) = 4\int -i \cdot sgn(\omega) p_U(\vec{x}; x_s; \omega) p_D^*(\vec{x}; x_s; \omega) d\omega. \tag{4}$$

The multiplier $-i \cdot sgn(\omega)$ is introduced to obtain the correct inversion phase in 2D (Bleistein et al., 2001). In 3D this term should be modified to $1/(i\omega)$. Relying on the amplitude preserving property of one-way wave equations (1) and (2), we derive the asymptotic form of (4) in terms of the traveltimes and amplitudes of the full wave equation

$$\begin{aligned} I(\vec{x}; x_s) = \int 4|\omega| \; & \frac{\cos\alpha_{s0}\cos\alpha_{r0}}{v_{s0}v_{r0}} A_s A_r \\ & e^{i\omega(\tau_s + \tau_r)} Q(x_r; x_s) dx_r d\omega. \end{aligned} \tag{5}$$

Here, v_{s0} and v_{r0} are the wave speeds at the shot and receiver point and α_{s0} and α_{r0} are the emergence angles of the ray from the image point to the shot and receiver points, respectively. $A_s = A(\vec{x}_s; \vec{x})$, $A_r = A(\vec{x}; \vec{x}_r)$, $\tau_s = \tau(\vec{x}_s; \vec{x})$ $\tau_r = \tau(\vec{x}; \vec{x}_r)$. $A(\vec{x}; \vec{y})$ and $\tau(\vec{x}; \vec{y})$ are amplitude and traveltime functions with source at $\vec{x}$ and observation point at $\vec{y}$, which are solutions of the transport and eikonal equations for the full wave equation, respectively.

To understand the physical meaning of $I(\vec{x};x_s)$ defined in (5), we recall the following true amplitude common-shot inversion formula (Keho and Beydoun, 1988; Zhang et al., 2003) in 2D

$$R(\vec{x};x_s)=\frac{1}{\pi}\int|\omega|\frac{\cos\alpha_{r0}}{v_{r0}}\frac{A_r}{A_s}e^{i\omega(\tau_s+\tau_r)}d\vec{x}_r d\omega. \quad (6)$$

where $R(\vec{x};x_s)$ is the reflectivity function reconstructed from seismic reflection inspired by shot $\vec{x}_s$, which is also a function depending on subsurface reflection angle $\theta(x_s)$. Now let us fix a reflection angle θ and an angle increment $\triangle$, and take an average of the reflectivity function

$$\begin{aligned}\bar{R}(\vec{x};\theta) &= \frac{1}{\triangle}\int_{\theta-\triangle/2}^{\theta+\triangle/2}R(\vec{x};\theta)d\theta \\ &= \frac{1}{\triangle}\int_S R(\vec{x};x_s)\left|\frac{d\theta}{dx_s}\right|dx_s,\end{aligned} \quad (7)$$

where $S=S(x_s;\theta)$ represents the collection of shots such that the specular reflection angle at imaging location $\vec{x}$ is within the angle range $[\theta-\triangle/2,\theta+\triangle/2]$. Since

$$\left|\frac{d\theta}{dx_s}\right|=\left|\frac{dx_s}{d\theta}\right|^{-1}=4\pi\frac{\cos\alpha_{s0}}{v_{s0}}A_s^2 \quad (8)$$

Substituting (8) and (6) into (7) and considering (5), we have

$$\begin{aligned}\bar{R}(\vec{x};\theta) &= \frac{4\pi}{\triangle}\int_S\frac{\cos\alpha_{s0}}{v_{s0}}A_s^2R(\vec{x};x_s)dx_s \\ &= \frac{1}{\triangle}\int_S I(\vec{x};x_s)dx_s,\end{aligned} \quad (9)$$

or

$$\begin{aligned}\bar{R}(\vec{x};\theta)=\int_S dx_s\int\frac{4|\omega|}{\triangle}\frac{\cos\alpha_{s0}\cos\alpha_{r0}}{v_{s0}v_{r0}} \\ A_sA_re^{i\omega(\tau_s+\tau_r)}Q(x_r;x_s)d\omega dx_r.\end{aligned} \quad (10)$$

The above analysis leads us to the following true amplitude common-angle wave equation migration algorithm

1. Downward extrapolate downgoing and upgoing waves p_D and p_U using true amplitude one-way wave equations (1) and (2);
2. Use the imaging condition (4) to output common-shot gathers $I(\vec{x};x_s)$;
3. Stack shot gathers $I(\vec{x};x_s)$ within bins of different reflection angle ranges. According to (9), for a fixed small angle increment $\triangle$, we obtain the angularly dependent reflectivity output $\bar{R}(\vec{x};\theta)$ at each output location.

In 3D, the reflectivity function depends on both the reflection angle θ and azimuthal angle φ. Therefore in 3D, (7) should be modified as a double integral on a surface element S centered at (θ,φ) with an angular area of $\triangle$

$$\begin{aligned}\bar{R}(\vec{x};\theta,\varphi) &= \frac{1}{\triangle}\int_{S(\theta,\varphi)}R(\vec{x};\theta,\varphi)\sin\theta d\theta d\varphi \\ &= \frac{1}{\triangle}\int_{S(\vec{x})}R(\vec{x};x_s,y_s)\left|\frac{\partial(\theta,\varphi)}{\partial(x_s,y_s)}\right|\sin\theta dx_sdy_s \\ &= \frac{16\pi^2v(\vec{x})}{\triangle}\int_{S(\vec{x}_s)}R(\vec{x};x_s,y_s)\frac{\cos\alpha_{s0}}{v_{s0}}A_s^2dx_sdy_s.\end{aligned} \quad (11)$$

Similar to 2D, if we define following 3D imaging condition

$$I(\vec{x};\vec{x}_s)=\int\frac{8\pi v(\vec{x})}{i\omega}p_U(\vec{x};\vec{x}_s;\omega)p_D^*(\vec{x};\vec{x}_s;\omega)d\omega, \quad (12)$$

then

$$\bar{R}(\vec{x};\theta,\varphi)=\frac{1}{\triangle}\int_{S(x_s,y_s)}I(\vec{x};\vec{x}_s)dx_sdy_s. \quad (13)$$

True amplitude double-square-root migration

The equivalence of double-square-root (DSR) migration and common-shot migration with a cross-correlation imaging condition has been discussed in the geophysics literature for conventional one-way wave equations; see for example, (Wapenaar and Birkhout, 1987; Biondi, 2003). A similar equivalence holds for true amplitude one-way wave equation and can be used to produce true amplitude common-angle gathers from DSR migration.

Define the wavefield

$$p(x_s,x_r,z;\omega)=\int p_U(x_r,z;x;\omega)p_D^*(x_s,z;x;\omega)dx,$$

where p_U and p_D satisfy the one-way wave equations (1) and (2). Since

$$\frac{\partial p}{\partial z}=\int\left(\frac{\partial p_U}{\partial z}p_D^*+p_U\frac{\partial p_D^*}{\partial z}\right)dx, \quad (14)$$

substituting (1) and (2) into (14) we find that p actually satisfies the following true amplitude DSR one-way wave equation

$$\left(\frac{\partial}{\partial z}-\Lambda_s-\Lambda_r-\Gamma_s-\Gamma_r\right)p=0, \quad (15)$$

with initial condition

$$p(x_s,x_r,z=0;\omega)=Q(x_r,x_s;\omega). \quad (16)$$

Symbolically Λ_s and Γ_s in (15) are defined as

$$\Lambda_s=\frac{i\omega}{v_s}\sqrt{1-\frac{(v_s\partial_{x_s})^2}{\omega^2}},\quad v_s=v(\vec{x}_s), \quad (17)$$

and

$$\Gamma_s=\frac{\partial_zv_s}{2v_s}\left[1+\frac{(v_s\partial_{x_s})^2}{\omega^2-(v_s\partial_{x_s})^2}\right]. \quad (18)$$

Λ_r and Γ_r are defined similarly.

For DSR migration (15) and (16), we apply the following imaging condition

$$J(\vec{x})=4\int -i\cdot sgn(\omega)p(x,x,z,\omega)d\omega. \quad (19)$$

By asymptotic analysis, we have

$$\begin{aligned}J(\vec{x})= \int 4|\omega|\frac{\cos\alpha_{s0}\cos\alpha_{r0}}{v_{s0}v_{r0}} \\ A_sA_re^{i\omega(\tau_s+\tau_r)}Q(x_r;x_s),dx_rdx_s.\end{aligned} \quad (20)$$

According to Sava and Fomel (2003), we need subsurface offset gathers to produce angle gathers in 2D

$$J(\vec{x},h)=4\int -i\cdot sgn(\omega)p(x-\frac{h}{2},x+\frac{h}{2},z,\omega)d\omega. \quad (21)$$

Comparing (20) with (12), we have the following true amplitude common-angle DSR migration algorithm

1. Downward extrapolate wave filed p using the true amplitude DSR equation (15) with initial condition (16);
2. Use the imaging condition (19) and (21)to output image and subsurface offset gathers;
3. Use the information of $J(\vec{x},h)$ to produce migrated angle gathers by a $\tau-p$ transform (Sava and Fomel, 2003).

Numerical tests

To show how true amplitude common-angle migration works, we apply it to a 2-D horizontal reflector model in a medium with velocity $v = 2000+0.3\cdot z$. Figure 1 shows a single shot record over four horizontal reflectors. The shot is in the center of the section and the receivers cover the surface in an aperture of 10000m on each side. The amplitude variation across travel time and lateral distance is only due to the geometrical spreading loss. Figure 2 left shows a migrated shot record using the common-shot migration algorithm with the imaging condition (4). The normalized peak amplitudes along the four migrated reflectors are shown in Figure 2 right. The migrated amplitudes are poor from a true-amplitude migration point of view, which illustrates that the migrated amplitude has incorrect angular dependence at each point due to the incorrect imaging condition for common-shot migration.

At an image location, after we stack all the migrated common image shot gathers according to different subsurface reflection angles with an angular bin size of 5°, we obtain a common-angle gather shown in Figure 3 left. For such a simple model, the conversion from shot locations to reflection angles is guided by simple ray tracing. The decreased resolution of the image with increasing reflection angle is clearly demonstrated by this output. The normalized peak amplitudes along the reflectors in the angle domain are shown in Figure 3 right. It is clear that the amplitude in the angle domain recovers the reflectivity accurately over a large angular range, aside from edge effects.

Conclusions

We demonstrate how to obtain true amplitude common-angle image gathers from both common-shot WEM and double-square-root WEM. Common-angle migration has certain advantages over other domain migrations, its output directly provides useful information for AVA analysis and migration-based velocity updating. Our analysis indicate that the $p_U p_D^*$ imaging condition is a good candidate to produce true amplitude common-angle gathers. Compared to the p_U/p_D imaging condition which is required for true amplitude common-shot migration, the methods described here suggest a more stable way of doing inversion.

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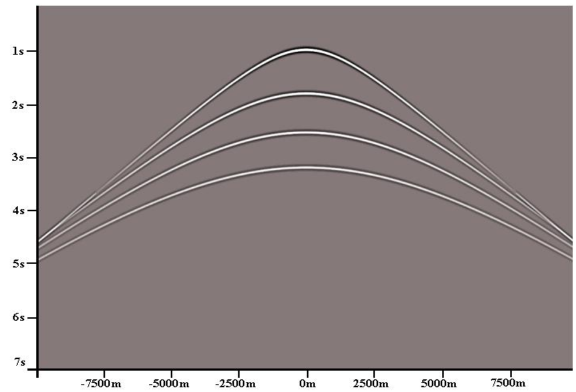

Fig. 1: 2-D shot record from four horizontal reflectors in a medium with velocity $v = 2000 + 0.3 \cdot z$.

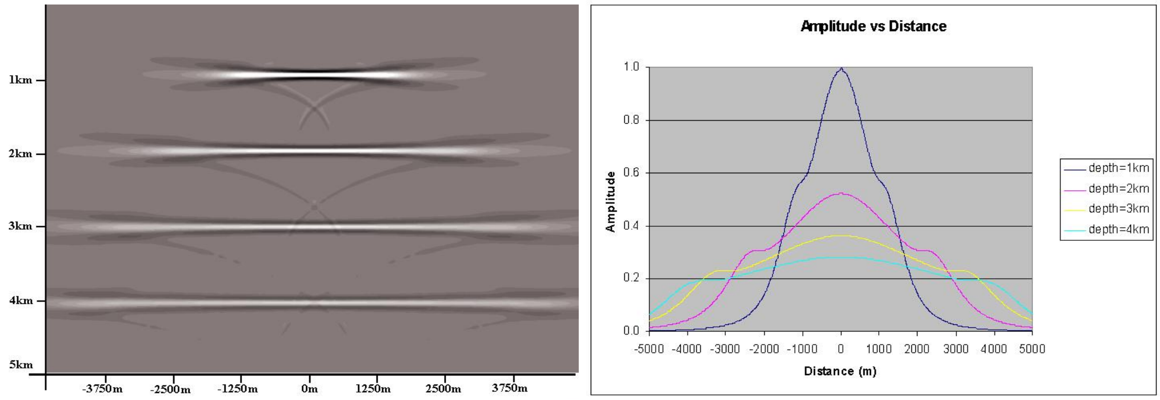

Fig. 2: Left: a migrated shot record using algorithm (1- 4). Right: Normalized peak amplitudes along the migrated reflectors in the left plot.

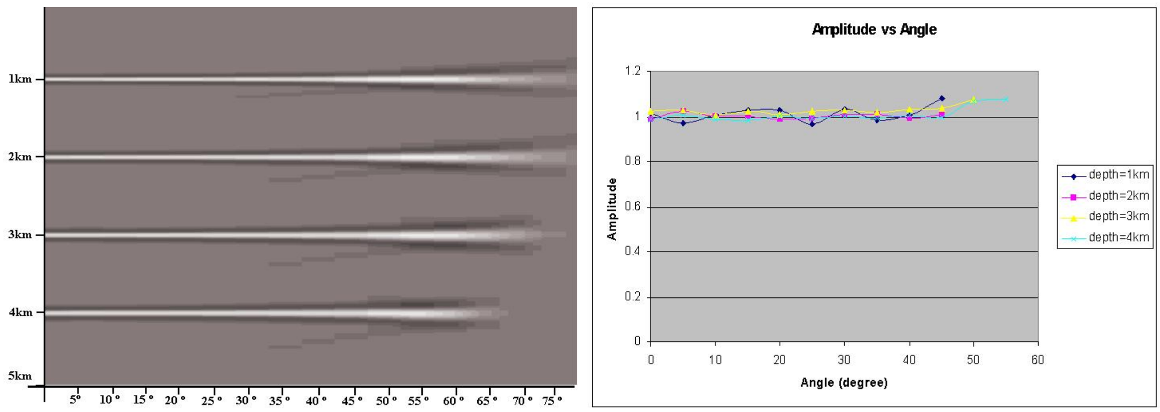

Fig. 3: Left: converted angle gather from migrated shots . Right: Normalized peak amplitudes along the migrated reflectors in the left plot.