Geometrical-spreading correction for P-waves in layered azimuthally anisotropic media

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Summary

Compensation for the geometrical spreading along the raypath is an essential step in AVO (amplitude variation with offset) analysis, in particular for wide-azimuth surveys. Here, we propose an efficient methodology to correct long-spread reflection data for the geometrical spreading in stratified azimuthally anisotropic media. The geometrical spreading is expressed through the reflection traveltime described by the nonhyperbolic moveout equation that has the same form as that in VTI (transversely isotropic with a vertical symmetry axis) media.

The parameters in the moveout equation are estimated from a 3D nonhyperbolic semblance algorithm that operates simultaneously with traces at all offsets and azimuths. Numerical tests for models composed of orthorhombic layers with strong, depth-varying velocity anisotropy confirm the high accuracy of our travetime-fitting procedure and, therefore, of the geometrical-spreading correction. In the presence of azimuthal anisotropy above the reflector, the azimuthal variation of the geometrical spreading is often comparable to that of the reflection coefficient.

The algorithm is applied to 3D data collected at Weyburn field (Canada) to evaluate the geometrical spreading for wide-azimuth P-wave reflections. The geometrical spreading for the reflection from the top of the fractured reservoir is clearly influenced by the azimuthal anisotropy in the overburden, which should cause distortions in the azimuthal AVO attributes. Since our geometrical-spreading correction is entirely based on the kinematics of reflected arrivals, it can be readily incorporated into the processing flow of azimuthal AVO analysis.

Introduction

Analysis of prestack amplitude variation with offset and azimuth (AVO) represents one of the most effective tools for characterization of naturally fractured (i.e., azimuthally anisotropic) reservoirs. An essential step of the AVO analysis is the removal of the geometrical spreading from the measured amplitude. In particular, if the overburden is azimuthally anisotropic, it acts like a 3-D focusing lens that significantly distort the amplitude distribution along the wavefront and, therefore, causes azimuthally varying geometrical spreading (Tsvankin, 2001).

Xu et al. (2003; hereafter referred to as Paper I) express the geometrical-spreading factor L as the following function of traveltime T:

$$L(x,\alpha) = \frac{\sqrt{\cos\phi^s \cos\phi^r}}{V_g} \left[\frac{\partial^2 T}{\partial x^2}\frac{\partial T}{\partial x}\frac{1}{x} + \frac{\partial^2 T}{\partial x^2}\frac{\partial^2 T}{\partial \alpha^2}\frac{1}{x^2} - \left(\frac{\partial T}{\partial \alpha}\right)^2 \frac{1}{x^4} \right]^{-1/2} \quad (1)$$

where x is the source-receiver offset, α is the azimuth of the source-receiver line with respect to the x_1-axis, V_g is the group velocity at the source location, and ϕ^s and ϕ^r are the angles between the ray and the vertical at the source and receiver locations, respectively. Equation (1) is valid for arbitrarily anisotropic, heterogeneous media if the wavefield can be described within the framework of ray theory.

Here, we propose an efficient methodology to compute the geometrical spreading (equation 1) for layered azimuthally anisotropic media and demonstrate its high accuracy by comparing it with a standard dynamic ray-tracing method. Application of our algorithm to both synthetic and field data shows that the recovery of the reflection coefficient from the azimuthal AVO response is impossible without correction for the azimuthally varying geometrical spreading.

Traveltime fitting for layered orthorhombic media

Models with uniform symmetry-plane orientation

It is clear from equation (1) that the key issue in computing the geometrical spreading from surface data is to find a smooth approximation for measured reflection traveltimes. To that end, we need an appropriate moveout equation that adequately describes reflection traveltimes over a wide range of offsets and azimuths.

Numerical testing in Paper I proves that the VTI moveout equation of Alkhalifah and Tsvankin (1995) can be adapted for an orthorhombic layer by introducing an azimuthally varying normal-moveout velocity $V_{\mathrm{nmo}}(\alpha)$ (Grechka and Tsvankin, 1998) and anellipticity coefficient $\eta(\alpha)$ (Pech and Tsvankin, 2003):

$$T^2(x,\alpha) = T_0^2 + \frac{x^2}{V_{\mathrm{nmo}}^2(\alpha)} - \frac{2\eta(\alpha)\, x^4}{V_{\mathrm{nmo}}^2(\alpha)\left[T_0^2\, V_{\mathrm{nmo}}^2(\alpha) + (1+2\eta(\alpha))\, x^2\right]}, \quad (2)$$

Layer Number	1	2	3	4
symmetry type	ISO	ORTH	ORTH	ORTH
V_{P0} (km/s)	1.5	2.437	3.0	3.2
thickness (km)	0.2	0.9	0.9	0.5
$\epsilon^{(1)}$	0	0.329	0.25	0
$\epsilon^{(2)}$	0	0.258	0.15	0
$\delta^{(1)}$	0	0.083	0.05	0
$\delta^{(2)}$	0	-0.078	-0.1	0
$\delta^{(3)}$	0	-0.106	0.15	0

Table 1: Parameters of a four-layer model (model 1) that includes two orthorhombic layers with aligned vertical symmetry planes $\alpha = 0°$ and $\alpha = 90°$.

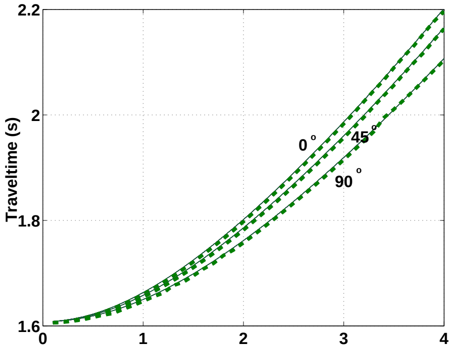

Fig. 1: Accuracy of the traveltime fitting for the azimuths $\alpha = 0°$, $45°$, and $90°$ ($\alpha = 0°$ corresponds to one of the symmetry planes). The dashed line is the exact ray-traced traveltime, the solid line is the traveltime computed from equation (2) with the following estimated moveout parameters: $\phi = 90°$, $V_{\rm nmo}^{(1)} = 2.307$ km/s, $V_{\rm nmo}^{(2)} = 2.675$ km/s, $\eta^{(1)} = 0.305$, $\eta^{(2)} = 0.222$, and $\eta^{(3)} = -0.006$. The model parameters are listed in Table 1; the zero-offset reflection traveltime is $t_0 = 1.608$ s.

$$V_{\rm nmo}^{-2}(\alpha) = \frac{\sin^2(\alpha - \phi)}{V_{\rm nmo}^{(1)}} + \frac{\cos^2(\alpha - \phi)}{V_{\rm nmo}^{(2)}}\,, \qquad (3)$$

$$\begin{aligned}\eta(\alpha) &= \eta^{(1)}\sin^2(\alpha-\phi) + \eta^{(2)}\cos^2(\alpha-\phi) \\ &- \eta^{(3)}\sin^2(\alpha-\phi)\,\cos^2(\alpha-\phi). \qquad (4)\end{aligned}$$

The results of Paper I indicate that equation (2) parameterized by the best-fit values of $V_{\rm nmo}$ and η should be sufficiently accurate for layered orthorhombic models with aligned symmetry planes.

To estimate the effective moveout parameters in equation (2), we employ the 3D nonhyperbolic semblance algorithm of Vasconcelos and Tsvankin (2004). Wide-azimuth synthetic data are generated using ANRAY, the 3D anisotropic ray-tracing code developed by Gajewski and Pšenčík (1987). Vasconcelos and Tsvankin (2004) developed a three-step inversion procedure designed to make the multiparameter semblance search more efficient. First, conventional-spread data are used to reconstruct the NMO ellipse and evaluate the azimuth ϕ and the NMO velocities $V_{\rm nmo}^{(1)}$ and $V_{\rm nmo}^{(2)}$. Second, the anellipticity parameters $\eta^{(1)}$ and $\eta^{(2)}$, which are defined in the vertical symmetry planes, are estimated from the VTI nonhyperbolic semblance analysis in narrow sectors centered at the symmetry-plane directions. Third, the initial values of the parameters ϕ, $V_{\rm nmo}^{(1)}$, $V_{\rm nmo}^{(2)}$, $\eta^{(1)}$, and $\eta^{(2)}$ are used to specify the starting model for nonhyperbolic semblance search based on equations (2) through (4). Application of the algorithm to the four-layer model with the parameters listed in Table 1 confirms that equation (2) accurately describes long-spread moveout for the full range of offsets and azimuths (see Figure 1). The error of equation (2) does not exceed 0.3% of the zero-offset traveltime; similar results were obtained for a wide range of plausible orthorhombic models.

Models with misaligned symmetry planes

For media without throughgoing vertical symmetry planes, the azimuthal variation of the quartic moveout coefficient becomes more complicated and is described by five different trigonometric functions of the azimuth α (Al-Dajani et al., 1998). This implies that equation (4) for the azimuthally varying parameter η may no longer be accurate. However, extensive testing that we performed for a range of orthorhombic models with misaligned symmetry planes shows that errors of equation (2) seldom exceed 0.5% of the zero-offset time. Apparently, the magnitude of the additional terms in the azimuthal dependence of η is relatively small, and the moveout inversion algorithm compensates for these missing terms by adjusting the best-fit parameters $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$.

For an extreme model containing two orthorhombic layers with uncommonly strong polar and azimuthal anisotropy and misaligned vertical symmetry planes, the normalized errors of equation (2) reach 1%, which may not be acceptable for purposes of geometrical-spreading correction. Indeed errors in the traveltime function will be amplified during the computation of the traveltime derivatives that govern the geometrical-spreading factor [equation (1)].

Although models similar to this extreme model is not typical, equation (2) can be modified in a relatively straightforward way to improve time fitting for multilayered media with misaligned symmetry planes. To introduce this modification, we analyze the effective parameter $\eta(\alpha)$ for a stack of horizontal orthorhombic layers by applying the VTI averaging equation (Tsvankin, 2001, equation 4.47) for each azimuth. Figure 2 shows a comparison between the parameter η computed from the VTI averaging equation (solid curve) and estimated by the moveout-inversion algorithm (dashed) for a two-layer orthorhombic model with misaligned symmetry planes. The shape of the two curves is quite similar, which explains the relatively low magnitude of the time residuals produced by equation (2)˜(4). The misalignment of the symmetry planes, however, causes a rotation of the estimated η-curve with respect to the one calculated from the VTI averaging equation. The moveout-inversion algorithm cannot accommo-

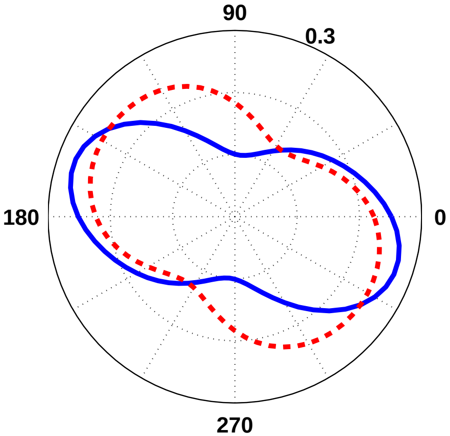

Fig. 2: Comparison of the effective parameter $\eta(\alpha)$ computed from the VTI averaging equation (solid curve) and estimated by the inversion algorithm (dashed). The model is composed of two orthorhombic layers; for the top layer, $\phi = 15°$, $V_{P0} = 2.5$, $\epsilon^{(1)} = 0.2$, $\epsilon^{(2)} = 0.15$, $\delta^{(1)} = -0.1$, $\delta^{(2)} = 0.15$, and $\delta^{(3)} = 0.15$; for the bottom layer, $\phi = 0°$, $V_{P0} = 3.0$, $\epsilon^{(1)} = 0.15$, $\epsilon^{(2)} = 0.2$, $\delta^{(1)} = 0.15$, $\delta^{(2)} = -0.1$, $\delta^{(3)} = -0.15$.

date this rotation because the "principal axes" of the azimuthal variation of $\eta(\alpha)$ in equation (4) are parallel to the axes of the NMO ellipse [equation (3)]. Therefore, the traveltime fitting at far offsets can be improved by decoupling the nonhyperbolic moveout term from the NMO ellipse and replacing ϕ in equation (4) with a different azimuth ϕ_1. Based on the above analysis, the last step of the moveout-inversion algorithm is modified to search for the angle ϕ_1 and the other parameters while fix the azimuth of the semi-major axis ϕ obtained from the first step. Application of this modified algorithm to the extreme model results in a greatly improved time fitting and a 15% increase in the semblance value for the best-fit model.

Geometrical-spreading correction

The traveltime derivatives in the geometrical-spreading equation (1) can be computed from the best-fit moveout parameters in equation (2). Explicit expressions for these derivatives are given in Paper I. The geometrical-spreading factor also depends on the group angles at the source (ϕ^s) and receiver (ϕ^r) locations, which are equal to each other for models with a horizontal symmetry plane. In most cases, the subsurface layer containing the source can be treated as isotropic and has a known P-wave velocity V. Then the angle ϕ^s can be computed using the ray parameter p estimated from the traveltime derivative dT/dx: $\sin\phi^s = pV$ (Ursin and Hokstad, 2003).

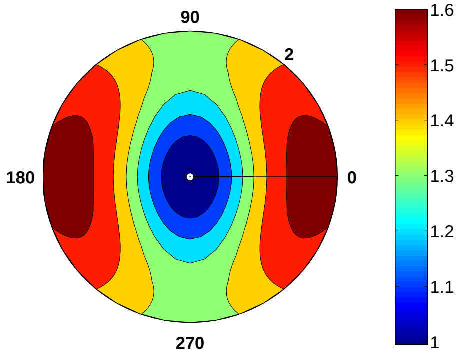

Fig. 3: Plan view of the geometrical spreading for the reflection from the bottom of layer 3 in model 1 (Table 1). The factor L is normalized by its value in the reference isotropic homogeneous medium with $V_{\rm nmo} = (V^{(1)}_{\rm nmo} + V^{(2)}_{\rm nmo})/2$. The maximum offset-to-depth ratio is two.

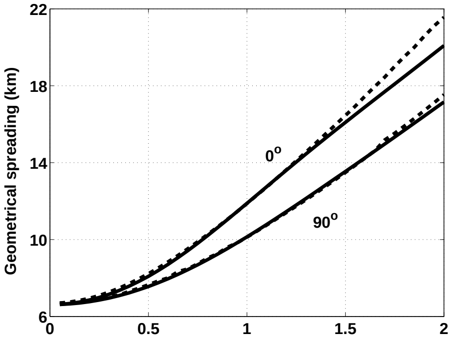

Fig. 4: Accuracy of our method for the reflection from layer 3 in model 1; the azimuths are $\alpha = 0°$ and $90°$. The factor L is computed from dynamic ray tracing (dashed line) and our algorithm (solid).

Synthetic example

Using equation (2) with the inverted moveout parameters, we computed the geometrical-spreading factor for model 1. As was the case for a homogeneous orthorhombic medium discussed in Paper I, the influence of anisotropy leads to pronounced, azimuthally-dependent distortions of the geometrical spreading (Figure 3). For example, the factor L for the reflection from the bottom of layer 3 decreases by 17% between the azimuths $\alpha = 0°$ and $90°$ (the offset is close to the reflector depth). The reflection coefficient for this event, however, increases by only 12.6% over the same azimuthal range. Clearly, if the anisotropic geometrical spreading is unaccounted for, it can completely compromise the azimuthal AVO signature for this model. The high accuracy of our algorithm is verified by the comparison with the results of dynamic ray tracing (using code ANRAY) for model 1 in Figure 4. The geometrical-spreading factors computed by the two methods are almost identical for offset-to-depth ratios less than 1.5, and only slightly diverge at longer offsets. The deviation of our result from that of the ray tracing, which reaches 8% for an azimuth of 0°, can be explained by small errors in the traveltime fitting. Nevertheless, our method produces a sufficiently close approximation to the ray-traced geometrical-spreading factor for a wide range of offsets and azimuths.

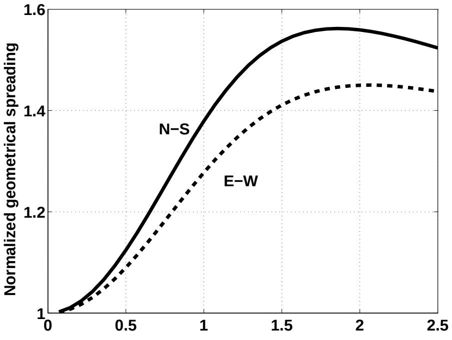

Fig. 5: Normalized geometrical spreading for the reflection from the Mississippian formation computed in the east-west and north-south directions for CMP 10829.

Field-data application

To demonstrate the influence of azimuthal anisotropy on the geometrical spreading for field data, we applied the algorithm to wide-azimuth reflection events acquired above a fractured reservoir at Weyburn field in Canada by the Reservoir Characterization Project (a research consortium at CSM). Vasconcelos and Tsvankin (2004) carried out nonhyperbolic moveout inversion for P-wave reflections from several interfaces in the overburden and obtained relatively large values of the parameters $\eta^{(1,2,3)}$ reaching 0.25. They also concluded that at least the shallow part of the overburden exhibits non-negligible azimuthal anisotropy.

Jenner (2001) found that the P-wave AVO attributes at the reservoir level vary with azimuth. His amplitude processing, however, included only the conventional geometrical-spreading correction for isotropic media. To evaluate possible distortions of the AVO response caused by the influence of anisotropy on the geometrical spreading, we applied our algorithm to the reflection from the top of the reservoir. The moveout parameters were obtained by Vasconcelos and Tsvankin (2004) using the original equation (2) with a single azimuthal angle ϕ.

The influence of anisotropy causes a dramatic 50% distortion in the geometrical spreading for offset-to-depth ratios close to two. The magnitude of the azimuthal variation of the factor L at far offsets reaches 10% (Figure 5). Such a difference between the geometrical spreading in the east-west and north-south directions is sufficiently large to cause distortions in the azimuthal variation of the AVO gradient studied by Jenner (2001).

Discussion and conclusions

The formalism suggested in Paper I for describing the geometrical spreading of reflected waves is used here to develop a practical methodology for the P-wave geometrical-spreading correction in layered azimuthally anisotropic media. The correction, which involves the spatial derivatives of the reflection traveltime and the group-velocity vector at the source location, does not require knowledge of the velocity model. If the layer containing the source is isotropic, the group angle can be estimated in a straightforward way from the slope of the traveltime curve. Hence, the main issue in computing the geometrical-spreading factor from surface data is to find a sufficiently accurate, smooth approximation for wide-azimuth, long-offset reflection moveout, which we obtain by using the nonhyperbolic moveout equation that has the same form as that in VTI media.

The importance of correcting wide-azimuth data for geometrical spreading prior to AVO analysis was highlighted by applying the algorithm to field data acquired at Weyburn field in Canada. The geometrical-spreading factor for the reflection from the top of the fractured reservoir is influenced by the ellipticity of the NMO-velocity function and, especially, by the large values (exceeding 0.2) of the effective parameters $\eta^{(1,2,3)}$. The reliability of the AVO attributes studied by Jenner (2001) can be improved by taking into account the variation of the geometrical spreading between the east-west and north-south directions.

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