Application of PS-wave moveout asymmetry in parameter estimation for tilted TI media

Pawan Dewangan and Ilya Tsvankin, Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines*

Summary

One of the distinctive features of mode-converted waves is their asymmetric moveout (i.e., PS-wave traveltime may not stay the same if the source and receiver are interchanged) caused by lateral heterogeneity or elastic anisotropy. If the medium is anisotropic, the moveout asymmetry contains valuable information for parameter estimation that cannot be obtained from pure reflection modes. Here, we generalize the so-called "PP+PS=SS" method, which is designed to replace reflected PS modes in velocity analysis with pure (non-converted) SS-waves, by supplementing the output SS traces with the moveout asymmetry attributes of the PS-waves.

The critical importance of including the PS-wave asymmetry attributes in anisotropic velocity analysis is demonstrated for two typical transversely isotropic models with a tilted symmetry axis (TTI). The first model includes a system of obliquely dipping cracks embedded in a horizontal, otherwise isotropic layer, and the second is that of a dipping TI layer with the symmetry axis orthogonal to the bedding. The inversion algorithm combines the asymmetry attributes of the PSV-wave with the hyperbolic moveout of the pure PP- and SS-waves in the symmetry-axis plane (i.e., the vertical plane that contains the symmetry axis). The numerical examples show that 2D measurements of the PS-wave asymmetry attributes can help to reconstruct the TTI model in depth using only surface reflection data.

In addition to providing an improved velocity model for imaging beneath TTI beds, our algorithm can yield valuable information for lithology discrimination and structural interpretation. If the TTI model is formed by obliquely dipping fractures, the estimated anisotropic parameters can be inverted further for the fracture orientation and compliances.

Introduction

Building anisotropic models for depth imaging usually requires supplementing P-waves with mode conversions or S-waves (e.g., Tsvankin, 2001). Although the benefits of using PS-waves in various applications are well documented in the literature (e.g., Thomsen, 1999), processing of mode conversions is complicated by several factors related to the asymmetry of their raypath. The difficulties in adjusting seismic processing algorithms for PS data prompted Grechka and Tsvankin (2002) and Grechka and Dewangan (2003) to develop the so-called "PP+PS=SS" method designed to construct "quasi-SS" reflections from PP and PS data without precise knowledge of the velocity model.

While replacing PS-waves with pure SS reflections is advantageous from the processing viewpoint, the PP+PS=SS method does not preserve the information about the asymmetry of PS moveout. If the medium is either laterally heterogeneous or anisotropic without a horizontal symmetry plane, the traveltime of PS-waves does not remain the same when the source and receiver are interchanged (Tsvankin and Grechka, 2000). This moveout asymmetry of PS reflections was shown by Tsvankin and Grechka (2000, 2002) to provide important attributes for parameter estimation in transversely isotropic media with a vertical symmetry axis (VTI).

Here, we demonstrate that supplementing the output of the PP+PS=SS method (i.e., PP and SS data) with the moveout asymmetry attributes of PS-waves makes it possible to estimate the parameters of two typical TTI models without *a priori* information. First, we consider a horizontal TTI layer that describes, for example, obliquely dipping, rotationally invariant fractures embedded in isotropic host rock. The possibility of using the traveltime asymmetry of PS-waves in detecting dipping fractures was demonstrated on field data by Angerer et al. (2002). The second model, which includes a dipping TTI layer with the symmetry axis orthogonal the layer's bottom, is typical for dipping shale beds near salt domes and in the fold-and-thrust region of the Canadian Foothills. For both TTI models, we derive analytic expressions for the PS-wave asymmetry attributes and develop 2D inversion algorithms that operate with the pure and converted reflection modes in the vertical plane that contains the symmetry axis (the symmetry-axis plane).

Methodology

The construction of SS-waves with the correct kinematics (but not amplitudes) does not require explicit information about the velocity field, but it is necessary to correlate PP and PS arrivals and identify the events reflected from the same interface. The original version of the PP+PS=SS method described by Grechka and Tsvankin (2002) operates with PP and PS traveltimes picked on prestack data. As illustrated in Figure 1, matching the reflection slopes on common-receiver gathers makes it possible to find two PS rays (recorded at points $x^{(3)}$ and $x^{(4)}$) with the same reflection point as the PP reflection $x^{(1)}Rx^{(2)}$. Then the traveltime of the SS-wave is determined from

$$\tau_{SS}(x^{(3)}, x^{(4)}) = t_{PS}(x^{(1)}, x^{(3)}) + t_{PS}(x^{(2)}, x^{(4)}) - t_{PP}(x^{(1)}, x^{(2)}) \,. \quad (1)$$

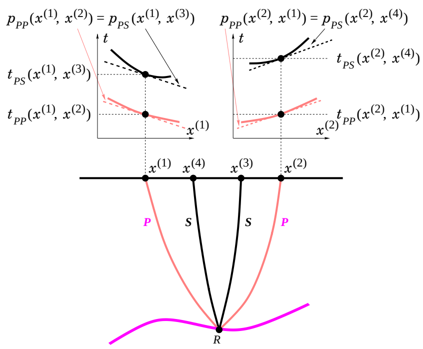

Fig. 1: 2D ray diagram of the PP+PS=SS method (after Grechka and Tsvankin, 2002). The reflected PP ray from $x^{(1)}$ to $x^{(2)}$ and the PS rays from $x^{(1)}$ to $x^{(3)}$ and $x^{(2)}$ to $x^{(4)}$ have the same reflection point R.

A generalized version of the PP+PS=SS method was developed by Grechka and Dewangan (2003) who apply a particular convolution of PP and PS traces to produce seismograms of the corresponding SS-waves.

To preserve information about the moveout asymmetry of the recorded PS-wave, we suggest to generate an *asymmetry gather* in addition to the SS data. When using the PP+PS=SS method, it is natural to define the asymmetry through the difference between the two PS traveltimes corresponding to the same reflection point (Figure 1):

$$\Delta t_{PS}(x^{(3)}, x^{(4)}) = t_{PS}(x^{(1)}, x^{(3)}) - t_{PS}(x^{(2)}, x^{(4)}) . \quad (2)$$

We modified the algorithm of Dewangan and Grechka (2003) to compute the asymmetry factor from equation (2). If we denote the slowness projections onto the dip and strike directions of the reflector at the conversion point by $p_{\rm int1}$ and $p_{\rm int2}$ (both $p_{\rm int1}$ and $p_{\rm int2}$ are confined to the reflector plane), the PS traveltime asymmetry can be represented as

$$\Delta t_{PS} = t_{PS}(p_{\rm int1}, p_{\rm int2}) - t_{PS}(-p_{\rm int1}, -p_{\rm int2}) \quad (3)$$

Similar to time asymmetry, we can also define the measure of asymmetry in the offset vector $\boldsymbol{x}$ as

$$\boldsymbol{\Delta x}_{PS} = \boldsymbol{x}_{PS}(p_{\rm int1}, p_{\rm int2}) + \boldsymbol{x}_{PS}(-p_{\rm int1}, -p_{\rm int2}) . \quad (4)$$

Using equations (3) and (4), we derived analytic expressions for the asymmetry attributes in the two TTI models described in the introduction. The TTI medium is parameterized by the velocities of P- and S-waves in the symmetry direction (V_{P0} and V_{S0}, respectively), the tilt of the symmetry axis from the vertical (ν) and Thomsen's anisotropic parameters ϵ and δ defined in the coordinate system associated with the symmetry axis (Tsvankin, 2001).

Model 1: Horizontal TTI layer

For laterally homogeneous models such as a horizontal TTI layer, equations (3) and (4) define the asymmetry of PS-waves in the slowness domain, since the projection of the slowness vector onto the reflector is equal to the horizontal slowness (ray parameter p). To analyze the dependence of the time asymmetry on the TTI parameters, equation (3) can be linearized in ϵ and δ under the assumption of weak anisotropy. The approximate factor Δt_{PS} of the PSV-wave in the symmetry-axis plane is given by

$$\Delta t_{PS}(p) = -8\,\eta\,z\,V_{P0}^2\;p^3\;\sin 4\nu\,, \quad (5)$$

where $\eta \equiv (\epsilon - \delta)/(1 + 2\delta) \approx \epsilon - \delta$ is the Alkhalifah-Tsvankin anellipticity parameter responsible for time processing of P-wave data in VTI media, and z is the reflector depth. The horizontal slowness p can be obtained from the derivative of the reflection traveltime with respect to the offset x. The sign of the time difference in equation (5) is specified by assuming that the symmetry axis is dipping in the positive x_1-direction.

According to equation (5), the asymmetry factor vanishes for VTI ($\nu = 0^\circ$) and HTI ($\nu = 90^\circ$) media because these two models have a horizontal symmetry plane. It depends on the *difference* $\epsilon - \delta$ and vanishes if the anisotropy is elliptical ($\epsilon = \delta$). The magnitude of the asymmetry factor reaches its maximum for the tilts $\nu = 22.5^\circ$ and $\nu = 67.5^\circ$. Therefore, Δt_{PS} is quite sensitive to the deviation of the symmetry axis from the vertical and horizontal directions. Similarly, linearizing the offset asymmetry factor [equation (4)] yields

$$\Delta x_{PS}(p) = 2\,x_0 + 12\,\eta\,z\,V_{P0}^2\;p^2\;\sin 4\nu\,, \quad (6)$$

where x_0 is the offset corresponding to the PS-wave traveltime minimum. Note that the p^2-term in equation (6) does not provide independent information for the inversion because it depends on the same parameter combination as the factor Δt_{PS}.

Parameter estimation

Effective application of the PP+PS=SS method requires acquisition of long-offset (i.e., offsets should reach at least twice the reflector depth) PP and PS data. If the offset-to-depth ratio for the recorded arrivals is less than two, the range of offsets for the constructed SS data is insufficient for obtaining a reliable estimate of the S-wave stacking velocity.

The numerical tests below prove that for a wide range of models, the inversion can be performed using 2D data in the symmetry-axis plane. Full-azimuth acquisition, however, is necessary to find the orientation of this plane unless it is known, for example, from geological information. Conventional 2D hyperbolic velocity analysis of the PP data yields their stacking velocity ($V_{\rm nmo,P}$) and zero-offset reflection traveltime (t_{P0}). Then, application of the PP+PS=SS method to the PP and PS records produces traces of "quasi-shear" waves that have the kinematics of the pure SS (SVSV) reflections (Grechka and Tsvankin, 2002; Grechka and Dewangan, 2003). Therefore, processing of the constructed

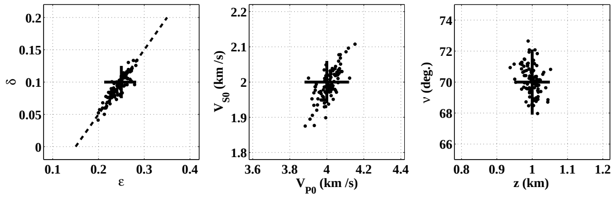

Fig. 2: Inverted parameters (dots) of a horizontal TTI layer obtained from 2D PP and PS data in the symmetry-axis plane. The correct model parameters ($V_{P0} = 4$ km/s, $V_{S0} = 2$ km/s, $\epsilon = 0.25$, $\delta = 0.1$, $\nu = 70°$, $z = 1$ km) are marked by the crosses. The dashed line on the [ϵ, δ] plot corresponds to the correct value of the difference $(\epsilon-\delta)$. The input data were contaminated by Gaussian noise with the standard deviations of 2% for the NMO velocities, 0.5% for the zero-offset traveltimes, and 2% for the PS-wave asymmetry attributes.

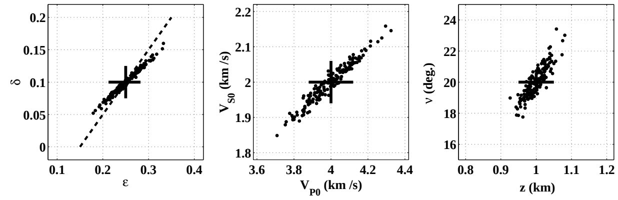

Fig. 3: Inversion results for a model with the same parameters as those in Figure 2 except for the tilt $\nu = 20°$.

SS arrivals can be used to estimate the stacking velocity ($V_{\rm nmo,S}$) and zero-offset traveltime (t_{S0}) of the SS-waves that are not physically excited in the survey. Here, we supplement the pure-mode signatures in parameter estimation with the PS-wave asymmetry attributes obtained from the PP+PS=SS method to form the data vector $\boldsymbol{d}$ [$V_{\rm nmo,P}$, t_{P0}, $V_{\rm nmo,S}$, t_{S0}, $\Delta t_{PS}(p)$, x_0], which is governed by the model vector $\boldsymbol{m}$ [V_{P0} , V_{S0} , ϵ , δ , ν, z]. To estimate the vector $\boldsymbol{m}$, we applied nonlinear inversion (the Gauss-Newton method) based on exact equations for all components of the data vector.

Figure 2 displays the results of inverting noise-contaminated input data for a model with a large tilt ν. Such models can be considered typical for dipping fracture sets because the fracture planes seldom deviate far from the vertical. The inversion results are unbiased, and the noise is not amplified by the parameter-estimation procedure. The standard deviations are close to 0.02 for ϵ and δ, 1% for V_{P0}, 2% for V_{S0} and z, and 1° for ν. The maximum offset-to-depth ratio for PP and PS data in all the tests is close to two. The best-constrained parameter combination is the difference $(\epsilon - \delta)$, which controls the asymmetry in the slowness domain [equation (5)] and has a strong influence on the NMO velocity of the constructed SS-waves. It should be emphasized that the parameter estimation is feasible only if the asymmetry information of the PS-wave is included in the inversion algorithm.

Models with mild tilts ν are not plausible if the anisotropy is caused by dipping fractures, but they may be adequate for effective TTI media formed by progradational sequences. For a tilt of 20° (Figure 3), the scatter in the inversion results is slightly higher than that for large tilts, but the standard deviations are less than 0.03 for ϵ and δ, 3% for V_{P0}, V_{S0} and z, and 2° for ν (Figure 3). As expected, the parameter estimation breaks down when the model approaches VTI, and the tilt ν becomes smaller than 10°.

Model 2: Dipping TTI layer

Another possible reason for the PS-wave moveout asymmetry is lateral heterogeneity in the form of lateral velocity variations or non-horizontal reflectors (e.g., Tsvankin and Grechka, 2000). In active tectonic areas, originally horizontal TI layers are often rotated in such a way that the symmetry axis remains orthogonal to the layer boundaries. Hence, here we assume that the tilt of the symmetry axis is equal to the dip of the reflector. The approximate asymmetry attributes in the symmetry-axis (dip) plane are convenient to express through the SS-wave offset x_{SS}. The leading terms in x_{SS}, however, depend just on parameters that can be obtained from the pure (PP and SS) modes, such as the anisotropic coefficient $\chi \equiv (\sigma - \delta)/(1 + 2\sigma)$, where $\sigma \equiv V_{P0}^2/V_{S0}^2\,(\epsilon - \delta)$ (Grechka and Dewangan, 2003). Therefore, we have to rely on long-offset PS data in constraining the medium parameters.

Parameter estimation

Processing the pure-mode reflections in the symmetry-axis plane yields their NMO velocities, zero-offset traveltimes and time slopes (horizontal slownesses p) on the zero-offset section. The data vector (not including the asymmetry attributes) then has six elements ($V_{\rm nmo,P}$, t_{P0}, p_{P0}, $V_{\rm nmo,S}$, t_{S0}, and p_{S0}) but only five of them are independent because of the constraint $p_{S0}/p_{P0} = t_{S0}/t_{P0}$. They are governed by the same model vector as that for a horizontal layer, with the tilt (ν) equal to the reflector dip and the depth z replaced by the normal distance z_d between the common midpoint and the reflector. Since the model vector includes six parameters, the 2D inversion cannot be carried out without additional information, such as the asymmetry attributes of the PS-wave.

We propose the following algorithm to invert the 2D multicomponent data in the symmetry-axis plane. For each value of the tilt ν from 0° to 90°, we find the model vector from the pure-mode data. The range of plausible tilts can be restricted by putting reasonable constraints on the parameter ϵ ($0 \leq \epsilon \leq 1$). Taking into account errors in the measured quantities, we compute the range of models that fit the pure-mode data within the noise level for each plausible value of tilt. Then, for all models obtained at the previous step, we compute the asymmetry attributes Δt_{PS} and Δx_{PS} from the exact equations (3) and (4) and minimize the misfit function for Δt_{PS} and Δx_{PS} over the full range of offsets.

The inversion for large tilts, which are typical for shale layers in fold-and-thrust belts, produces unbiased results

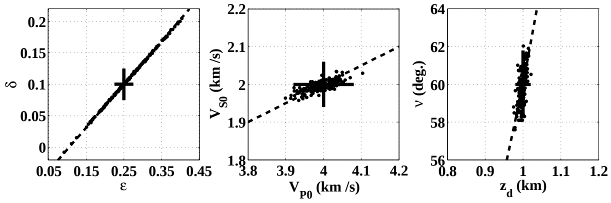

Fig. 4: Inverted parameters (dots) of a dipping TTI layer with $\nu = 60°$ obtained from 2D PP and PS data in the symmetry-axis plane. The correct model parameters are marked by the crosses. The dashed lines on the [ϵ, δ], [V_{P0} , V_{S0}], and [ν , z_d] plots correspond to the correct values of χ, V_{P0}/V_{S0}, and $\sin\nu/z_d$, respectively. The input data were contaminated by Gaussian noise with the same standard deviations as in the caption of Figure 2.

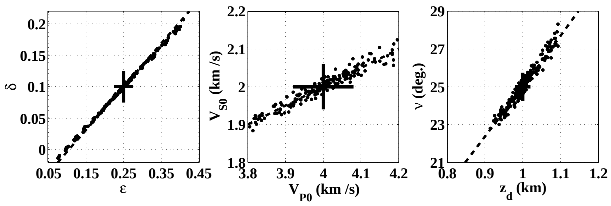

Fig. 5: Inversion results for a model with the same parameters as those in Figure 4 except for the tilt $\nu = 25°$. The standard deviations in the asymmetry attributes are increased to 6%.

(Figure 4). If we assume that the error in the asymmetry attributes is 2%, the parameters V_{P0}, V_{S0}, and z_d are well constrained (the standard deviation is less than 1%), and the standard deviation in ν is only 1°(Figure 4). However, the inverted anisotropic parameters ϵ and δ exhibit more scatter, with the standard deviation reaching 0.06 and 0.04, respectively. Also, the estimates of ϵ and δ degrade rapidly as the error in the asymmetry attributes increases, while the deviations in V_{P0}, V_{S0}, and z_d remain small. It is clear from Figure 4 that the best-constrained parameter combinations are χ, $\sin\nu/z_d$, and V_{P0}/V_{S0}. In principle, ϵ and δ for large tilts can be obtained with sufficient accuracy from wide-azimuth PP and SS data, as demonstrated by Grechka et al. (2002).

If the tilt for the model from Figure 4 is reduced from 60° to 25° (Figure 5), the uncertainty in Δt_{PS} and Δx_{PS} should increase because of the smaller magnitude of the moveout asymmetry. If the error in the asymmetry attributes is set to 6%, the standard deviation of the tilt ν is almost the same (1°) as that in Figure 4. Despite the high resolution in ν, the standard deviations in V_{P0}, V_{S0}, and z_d increase to about 4%; the deviations in ϵ and δ are also substantial (0.08 and 0.05, respectively). The inversion breaks down for quasi-VTI models with mild tilts $\nu < 15 - 20°$.

Conclusions

We modified the PP+PS=SS method of Grechka and Tsvankin (2002) to supplement the computed quasi-SS data with the time (Δt_{PS}) and offset (Δx_{PS}) asymmetry attributes of the converted waves. The new algorithm was applied to the inversion of multicomponent data acquired over horizontal or dipping TI layers with a tilted symmetry axis.

The weak-anisotropy approximation helped to obtain concise expressions for the asymmetry attributes of PSV-waves in terms of the tilt ν of the symmetry axis and Thomsen's anisotropic parameters. We combined the factors Δt_{PS} and Δx_{PS} with the NMO velocities, zero-offset traveltimes and time slopes of PP-waves and the constructed SS-waves in a nonlinear parameter-estimation algorithm. Although it is desirable to have a wide range of source-receiver azimuths, the parameter estimation for a wide range of ν can be performed using just 2D data in the symmetry-axis plane. The orientation of this plane, however, has to be determined beforehand from either the pure-mode NMO ellipses (using wide-azimuth data) or from the PS-wave polarization at small offsets.

For a horizontal TTI layer, including the PS-wave moveout asymmetry makes the 2D inversion sufficiently stable if the symmetry axis deviates by at least 10° from the vertical (VTI) and horizontal (HTI) directions. In contrast, the parameters of a dipping TTI layer with the symmetry axis orthogonal to the layer's bottom are constrained only if $\nu > 25°$, and the accuracy of the inverted parameter ϵ is marginal even for large ν (i.e., for steep dips).

References

Angerer, E., Horne, S. A., Gaiser, J. E., Walters, R., Bagala, S. and Vetri, L., 2002, Characterization of dipping fractures using PS mode-converted data: 72th Ann. Internat. Mtg., Soc. of Expl. Geophys., Expanded Abstracts, 1010–1013.

Grechka, V., and Dewangan, P., 2003, Generation and processing of pseudo shear-wave data: Theory and case study: Geophysics, **68**, 1807–1816.

Grechka, V., and Tsvankin, I., 2002, PP+PS=SS: Geophysics, **67**, 1961–1971.

Grechka, V., Pech, A., and Tsvankin, I., 2002, Multicomponent stacking-velocity tomography for transversely isotropic media: Geophysics, **67**, 1564–1574.

Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous, anisotropic media: Geophysics, **64**, 678–690.

Tsvankin, I., 2001, Seismic signatures and analysis of reflection data in anisotropic media: Elsevier Science Publ. Co., Inc.

Tsvankin, I., and Grechka, V., 2000, Dip moveout of converted waves and parameter estimation in transversely isotropic media: Geophys. Prosp., **48**, 257–292.

Tsvankin, I., and Grechka, V., 2002, 3D description and inversion of reflection moveout of PS-waves in anisotropic media: Geophys. Prosp., **50**, 301–316.