Image Gathers of SV-waves in Homogeneous and Factorized VTI Media

Ramzy Al-Zayer and Ilya Tsvankin
Colorado School of Mines, Center for Wave Phenomena, Department of Geophysics

Summary

One of the main problems in the velocity analysis of P-wave data for VTI (transversely isotropic with a vertical symmetry axis) media is the need for *a priori* information in building a model for depth imaging. Including SV-wave moveout in the parameter-estimation procedure, either alone or in combination with P-waves, can help in positioning the reflectors at the correct depth using only reflection traveltimes. Here, in order to develop a foundation for shear-wave migration velocity analysis (MVA) in VTI media, we study SV-wave image gathers obtained after prestack depth migration.

For purposes of the moveout inversion of SV-waves, it is convenient to parameterize the model in terms of the normal-moveout (NMO) velocity $V_{\rm nmo}$ of horizontal SV events, the anisotropic parameter σ, which largely controls SV-wave anisotropy, and Thomsen parameters ϵ and δ. The moveout of horizontal events on image gathers is close to hyperbolic and depends just on $V_{\rm nmo}$ out to relatively large offset-to-depth ratios of about 1.7. The influence of the parameter σ on the migrated depth rapidly increases as the offset-to-depth ratio approaches two. However, estimation of σ (and, therefore, the vertical velocity) from SV-wave moveout is hampered by the tradeoff between σ and ϵ on long-spread gathers.

For factorized $v(z)$ VTI media with a constant SV-wave vertical-velocity gradient k_{zs}, flattening of both horizontal and dipping events requires the correct NMO velocity at the surface, the gradient k_{zs}, and the parameters σ and ϵ. On the whole, the ambiguity in the estimation of σ and reflector depth from SV-wave moveout highlights the need to combine P- and SV-wave data in migration velocity analysis for VTI media.

Introduction

Velocity model-building for seismic imaging is usually implemented as an iterative process that includes migration followed by velocity analysis and model updating. Most existing migration velocity analysis (MVA) algorithms are designed for P-waves in heterogeneous isotropic media (e.g., Liu, 1997). An MVA method for P-wave data in VTI media was presented by Sarkar and Tsvankin (2003, 2004) who identified the parameter combinations needed to flatten P-wave events in migrated (image) gathers and place them at the correct depth. In addition to homogeneous models, they studied factorized $v(x,z)$ VTI media in which the P-wave vertical velocity V_{P0} varies linearly in both vertical and horizontal directions but the anisotropic parameters and the V_{P0}/V_{S0} ratio (V_{S0} is the S-wave vertical velocity) are constant. For VTI models, however, P-wave reflection traveltimes alone usually are insufficient for estimating reflector depth and Thomsen anisotropic parameters ϵ and δ (Alkhalifah and Tsvankin, 1995).

To overcome the ambiguity in the inversion of reflection data, Tsvankin and Thomsen (1995) suggested to combine long-spread P- and SV-wave traveltimes from horizontal interfaces. They demonstrated that the strongly nonhyperbolic SV-wave moveout in directions close to the velocity maximum (i.e., near 45° incidence angle) helps to constrain the velocity V_{S0} and reconstruct the vertical scale of the model.

The goal of this work is to analyze the information contained in the moveout of long-spread SV events in image gathers. Since correlating P and SV reflection events on field data is not an easy task, it is important to learn if SV-waves alone can be used to determine reflector depth in VTI media. By employing analytic expressions and performing actual depth migration, we establish the combinations of model parameters needed to accurately image shear-wave data.

Homogeneous VTI medium

SV-wave propagation in VTI media is controlled by four Thomsen parameters: V_{P0}, V_{S0}, ϵ, and δ. For purposes of SV-wave moveout analysis, it is convenient to replace the two vertical velocities by the SV-wave NMO velocity $V_{\rm nmo}$ for horizontal interfaces and the parameter σ:

$$V_{\rm nmo} = V_{S0}\sqrt{1+2\sigma}\,, \qquad (1)$$

$$\sigma \equiv \frac{V_{P0}^2}{V_{S0}^2}\,(\epsilon-\delta)\,. \qquad (2)$$

Therefore, the parameter set used in the tests below includes $V_{\rm nmo}$, σ, ϵ, and δ.

Horizontal events

The weak-anisotropy approximation for the residual moveout moveout of horizontal SV events in image gathers can be obtained by adapting the P-wave result of Sarkar and Tsvankin [2003, equation (5)]:

$$\begin{aligned} z_M^2(h) \;\approx\; & r^2\, z_T^2 \,+\, h^2\, V_{S0,M}^2 \left(\frac{1}{V_{{\rm nmo},T}^2} - \frac{1}{V_{{\rm nmo},M}^2}\right) \\ & -\; \frac{2\,h^4}{h^2+z_T^2}\left(\sigma_M\,\frac{V_{{\rm nmo},T}^2}{V_{{\rm nmo},M}^2} - \sigma_T\,\frac{V_{{\rm nmo},M}^2}{V_{{\rm nmo},T}^2}\right), \end{aligned} \qquad (3)$$

where the subscript T refers to the true model and M to the migration model, z_M is the migrated depth, z_T is

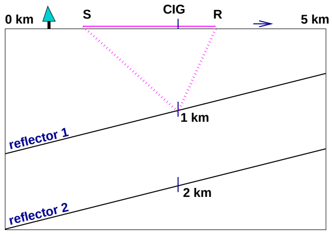

Fig. 1: Geometry of the model used in the numerical modeling. Two parallel plane reflectors are embedded in a VTI medium, with the dip varying for different models between 0° and 40°. The common-image gathers (CIG) in all subsequent tests are displayed at the location where the depths of the two reflectors are 1 km and 2 km; the maximum offset is 3 km.

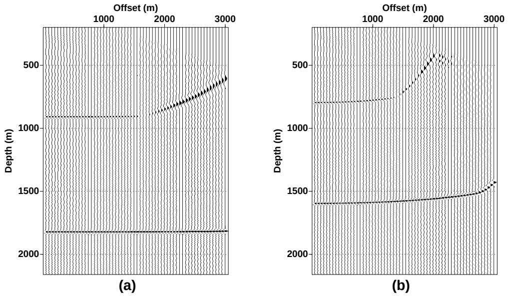

Fig. 2: Influence of errors in $V_{\rm nmo}$ and σ on common-image gathers of horizontal SV events. In section (a), $V_{\rm nmo}$ is correct but σ is erroneous ($\sigma_M = 0.5$); in section (b), σ is correct but $V_{\rm nmo}$ is erroneous ($V_{{\rm nmo},M} = 1.936$ m/s). The correct parameters are $V_{\rm nmo,T} = 2.42$ km/s and $\sigma_T = 0.333$.

the true depth, h is half the source-receiver offset, and $r \equiv V_{S0,M}/V_{S0,T}$ is the ratio of migration and true vertical velocities. The residual moveout is described by the quadratic term that depends on the NMO velocity and the quartic (nonhyperbolic) term influenced by both $V_{\rm nmo}$ and σ.

According to equation (3), using the correct $V_{\rm nmo}$ and σ in the migration process not only removes residual moveout but also positions the reflector at the true depth. Indeed, it is clear from equation (1) that a model with the correct values σ and $V_{\rm nmo}$ must have the correct vertical velocity V_{S0} as well. In contrast, flattening P-wave image gathers in VTI media does not guarantee the correct depth scale of the section (Sarkar and Tsvankin, 2003).

To study common-image gathers of SV-waves, we carry out prestack depth migration for the model in Figure 1. Migration was performed using a Kirchhoff code developed by Liu (1997) for isotropic media in Seismic Unix (SU). The only change required to migrate SV data from VTI media was in generating the appropriate traveltime table by anisotropic ray tracing. For computing synthetic seismograms and performing ray tracing, we adapted for SV-waves the SU codes written by Alkhalifah (1995) for P-waves in factorized VTI media.

The influence of errors in the two main SV-wave kinematic parameters, $V_{\rm nmo}$ and σ, is illustrated in Figure 2. The correct velocity $V_{\rm nmo}$ flattens the events up to a sizable offset-to-depth ratio of about 1.7, whether or not the value of σ is accurate (Figure 2a). For the shallow reflector, the residual moveout in Figure 2a rapidly increases at larger offsets because of the erroneous value of σ. Figure 2b, in contrast, shows that the residual moveout caused by errors in $V_{\rm nmo}$ influences the entire offset range. Notice that neither event in Figure 2 is imaged at the correct depth (1 km and 2 km) because r in equation (3) is not equal to unity.

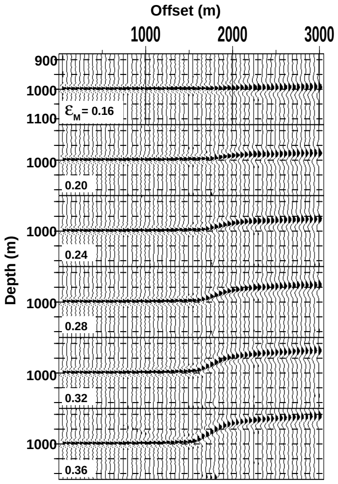

Fig. 3: Influence of errors in ϵ on the residual moveout of a horizontal SV event. The gathers were generated for a range of ϵ_M values ($\epsilon_T = 0.16$) and the correct parameters $V_{\rm nmo,T} = 2.420$ km/s, $\sigma_T = 0.6$, and $\delta_T = 0.1$.

The absence of the parameters ϵ and δ in equation (3) is a consequence of using the weak-anisotropy approximation. While variations of δ within a realistic range do not produce measurable changes in the migrated depths, the influence of ϵ on the residual moveout is not negligible (Figure 3) and may cause errors in estimating the parameter σ.

Since $V_{\rm nmo}$ can be obtained with high accuracy from conventional-spread SV data, reliable evaluation of σ would make it possible to determine the vertical velocity V_{S0} and reflector depth [see equation (1)]. Suppose our goal is to estimate σ by flattening long-spread SV-wave moveout in image gathers. Unless we have *a priori* information, the value of ϵ used in the migration would be erroneous (Figure 4). The error in ϵ causes substantial residual moveout on the panel with the correct $\sigma = 0.6$; consequently, the processor would likely try changing σ to flatten the event. In Figure 4, the smallest residual moveout is observed for distorted values of σ between 0.5 and 0.55, which exemplifies a certain degree of interplay between σ and ϵ. Although none of the gathers for $0.5 < \sigma < 0.55$ is perfectly flat, the residual moveout for this range of σ values may not be detectable on field data in the presence of noise, lateral heterogeneity, and near-surface anomalies, especially if the offset range is more

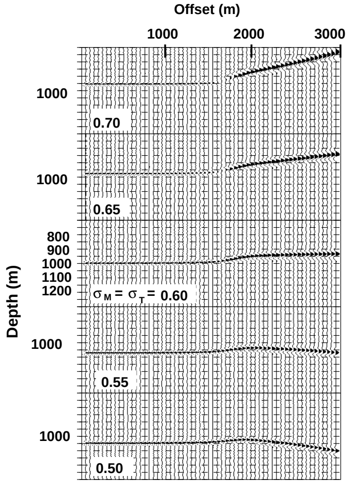

Fig. 4: Tradeoff between the parameters σ and ϵ for long-spread horizontal events. The image gathers are computed for a range of σ values ($\sigma_T = 0.6$) with erroneous $\epsilon_M = 0.36$ ($\epsilon_T = 0.16$). The NMO velocity was fixed at the correct value, $V_{\text{nmo,T}} = 2.420\,\text{km/s}$; $\delta_M = 0.26$ ($\delta_T = 0.1$).

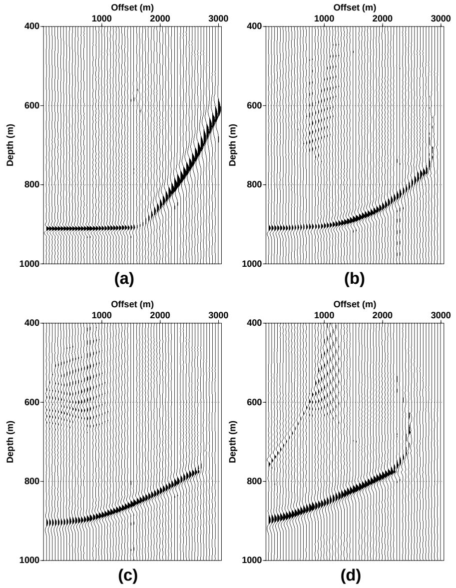

Fig. 5: Influence of errors in σ on the residual moveout of a dipping event. The test in Figure 2a is repeated here for the shallow reflector, this time dipping at (a) 0°; (b) 20°; (c) 30°; and (d) 40°. The parameter $\sigma_M = 0.5$, while the actual value $\sigma_T = 0.333$; the rest of the model parameters are correct ($V_{\text{nmo,T}} = 2.421\,\text{km/s}$, $\epsilon_T = 0.1$, and $\delta_T = -0.1$).

limited than that in Figure 4. Therefore, the tradeoff between σ and ϵ may distort estimates of σ by about 0.1, which would cause unacceptable errors (exceeding 5%) in the vertical velocity and time-to-depth conversion.

Dipping events

Since the NMO velocity of dipping events depends not only on the zero-dip value (V_{nmo}) but also on the anisotropic parameters, errors in σ lead to residual moveout even at small offsets (Figure 5). Similarly, the velocity V_{nmo} influences the entire offset range, as was the case for horizontal events.

With both V_{nmo} and σ contributing to the residual moveout of dipping events over a wide range of offsets, we can expect a degree of tradeoff between these two parameters. Figure 6 confirms that errors in V_{nmo} can indeed be compensated by errors in σ, and a dipping event can be flattened out to an offset-to-depth ratio of $x/z = 2.5$ using a vastly erroneous migration model. This ambiguity, however, can be reduced by putting constraints on the V_{P0}/V_{S0} ratio.

The contribution of ϵ to the residual moveout can cause an additional source of nonuniqueness in the parameter estimation. For dipping events, errors in ϵ can produce depth distortions even at relatively small offsets because the magnitude of the nonhyperbolic moveout for SV-waves becomes extremely large at intermediate dips between 30° and 50°. Therefore, the SV-wave moveout from a single dipping reflector is insufficient for estimating any of the model parameters without *a priori* information.

Our numerical testing shows, however, that the tradeoff between σ and ϵ can be resolved if SV reflections from both horizontal and dipping interfaces are available (e.g., in the presence of fault planes). Note that combining the P-wave NMO velocities of horizontal and dipping events provides a stable way of estimating the parameter η in vertically heterogeneous VTI media (Alkhalifah and Tsvankin, 1995). The main difference between that P-wave DMO inversion algorithm and the moveout analysis of SV-waves discussed here is the need to use *long-spread* horizontal SV events that help to constrain the parameters σ and ϵ.

Factorized vertically heterogeneous VTI medium

In factorized anisotropic media the stiffness coefficients vary in space but their ratios (i.e., the anisotropic coefficients) are held constant. Factorized models provide an efficient framework for simultaneous study of the influence of heterogeneity and anisotropy on seismic signatures (Sarkar and Tsvankin, 2003, 2004). We consider a special type of factorized VTI media in which the vertical velocities V_{P0} and V_{S0} are linear functions of depth z:

$$V_{P0}(z) = V_{P0}(0) + k_{zp}\,z\,; \qquad (4)$$

$$V_{S0}(z) = V_{S0}(0) + k_{zs}\,z\,, \qquad (5)$$

where $V_{P0}(0)$ and $V_{S0}(0)$ are the velocities at the surface ($z = 0$), and k_{zp} and k_{zs} are the vertical-velocity gradients for P-and SV-waves, respectively. According to equations (4) and (5), for the ratio $V_{P0}(z)/V_{S0}(z)$ to be independent of z, the gradients have to satisfy the condition $k_{zp}/k_{zs} = V_{P0}(0)/V_{S0}(0)$.

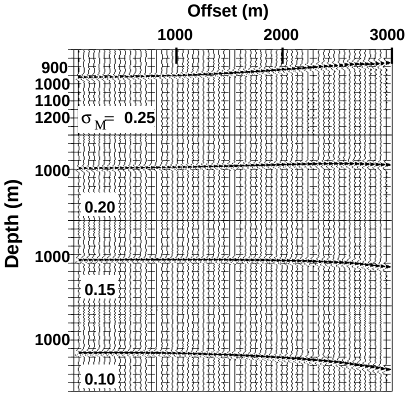

Fig. 6: Image gathers of a dipping event ($\phi = 20°$) obtained using erroneous values of $V_{\rm nmo,M} = 2\,{\rm km/s}$ and σ_M (marked on the plot); the parameters ϵ and δ are correct. The true model parameters are $V_{\rm nmo,T} = 2.420\,{\rm km/s}$, $\sigma_T = 0.6$, $\epsilon_T = 0.16$, and $\delta_T = 0.1$. The gather for $\sigma_M = 0.15$ is practically flat.

The NMO velocity of SV-waves from a horizontal reflector can be found as a function of the vertical reflection time t_0 by adapting the corresponding result for P-waves in Sarkar and Tsvankin (2003, Appendix B):

$$V^2_{\rm nmo}(t_0) = \frac{V^2_{\rm nmo}(t_0 = 0)}{t_0\, k_{zs}} \left(e^{t_0\, k_{zs}} - 1 \right); \qquad (6)$$

$V_{\rm nmo}(t_0 = 0) = V_{S0}(0)\sqrt{1 + 2\sigma}$ is the SV-wave NMO velocity at the surface. We parameterize the factorized medium by the velocity $V_{\rm nmo}(t_0 = 0)$, gradient k_{zs}, and the anisotropic parameters σ, ϵ, and δ.

From equation (6), the correct NMO velocity for a range of vertical times t_0 can be obtained only by setting both the velocity $V_{\rm nmo}(t_0 = 0)$ and gradient k_{zs} in the migration model to the correct values. The influence of the parameters $V_{\rm nmo}(t_0)$, σ, ϵ and δ on the residual moveout in vertically heterogeneous media is similar to that discussed above for homogeneous models. The model parameters needed in the depth migration of SV-waves for factorized $v(z)$ media include $V_{\rm nmo}(t_0 = 0)$, the gradient k_{zs}, and the anisotropic parameters σ and ϵ. The velocity $V_{\rm nmo}(t_0 = 0)$ and gradient k_{zs} can be found from conventional-spread moveout for two reflectors sufficiently separated in depth, as suggested by Sarkar and Tsvankin (2003, 2004) for P-wave data.

Conclusions

SV-wave moveout can provide useful information for migration velocity analysis of multicomponent data in VTI media. Table 1 summarizes the results of our analysis of the influence of anisotropy on the residual moveout of SV reflection events. The parameters needed to flatten horizontal and dipping SV events in image gathers are listed along with their P-wave counterparts. Note that the factorized VTI model may not be realistic for shear-wave analysis because it involves a fixed (and, possibly, artificial) relationship between the velocity gradients of P- and SV-waves.

Medium type	P − wave	SV − wave
Homogeneous VTI	$V_{\rm nmo,P}$ η	$V_{\rm nmo,SV}$ σ ϵ
Factorized $v(z)$ VTI	$V_{\rm nmo,P}(0)$ k_{zp} η	$V_{\rm nmo,SV}(0)$ k_{zs} σ ϵ

Table 1: Model parameters needed to flatten P and SV events in long-spread image gathers.

The main obstacle in the depth-domain velocity analysis of SV data is the tradeoff between the depth-conversion parameter σ and Thomsen parameter ϵ on long-spread gathers of horizontal events. Possible ways to resolve this ambiguity are to combine either horizontal and dipping SV events (the dips should exceed 25°) or long-spread P and SV reflections from horizontal interfaces. Implementation of the latter approach in the migrated domain will require a search for a model that flattens P- and SV-wave image gathers simultaneously and ensures that the P and SV migrated sections are tied in depth.

Acknowledgments

We thank Saudi Aramco and the Center for Wave Phenomena at CSM for supporting this project.

References

Alkhalifah, T., 1995, Efficient synthetic-seismogram generation in transversely isotropic, inhomogeneous media: Geophysics, **60**, 1139–1150.

Alkhalifah, T. and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Grechka, V. and Tsvankin, I., 1999, 3-D moveout inversion in azimuthally anisotropic media with lateral velocity variation: Theory and a case study: Geophysics, **64**, 1202–1218.

Liu, Z., 1997, An analytical approach to migration velocity analysis: Geophysics, **62**, 1238–1249.

Sarkar, D., and Tsvankin, I., 2003, Analysis of image gathers in factorized VTI media: Geophysics, **68**, 2016–2025.

Sarkar, D., and Tsvankin, I., 2004, Migration velocity analysis in factorized VTI media: Geophysics, in print (also CWP Research Report CWP-451).

Tsvankin, I., and Thomsen, L., 1995, Inversion of reflection traveltimes for transverse isotropy: Geophysics, **60**, 1095–1107.