Geometrical Spreading of P-waves in Azimuthally Anisotropic Media

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Summary

Geometrical spreading is highly sensitive to elastic anisotropy and may strongly influence the AVO signature of reflected waves recorded over anisotropic formations. For purposes of processing and inversion of reflection data, it is convenient to express geometrical spreading through the reflection traveltime measured at the earth's surface. Here, we obtain the inverse geometrical-spreading factor L^{-1} for horizontally layered anisotropic media with a horizontal symmetry plane as a simple function of the traveltime derivatives with respect to offset and azimuth. By employing the Tsvankin-Thomsen nonhyperbolic moveout equation, the factor L^{-1} is represented through the moveout coefficients which can be estimated from surface seismic data.

This analytic result is applied to P-wave reflections in an othorhombic layer to evaluate the distortions of the geometrical spreading for a model typical for fractured reservoirs. The weak-anisotropy approximation, verified by numerical tests, shows that azimuthal velocity variations make a significant contribution to the factor L^{-1}, and the existing equations for vertical transverse isotropy cannot be applied even in the symmetry planes. The shape of the azimuthally varying function L^{-1} is close to elliptical for offsets smaller than the reflector depth but becomes more complicated for larger offset-to-depth ratios.

While the developed formulation is helpful in modeling the anisotropic geometrical-spreading factor, its main application is in correcting the wide-azimuth AVO signature for the influence of the anisotropic overburden.

Introduction

Analysis of prestack amplitude variation with offset and azimuth (AVO) represents one of the most effective tools for characterization of azimuthally anisotropic (e.g., naturally fractured) reservoirs. An important element of AVO processing is the removal of the geometrical-spreading factor from the measured amplitude. In particular, if the overburden is anisotropic, it acts like a 3-D focusing lens that significantly changes the amplitude distribution along the wavefront of the reflected wave (Tsvankin, 1995).

For application in AVO analysis, geometrical spreading can be computed from the traveltimes of reflection events. The P-wave geometrical-spreading factor for transversely isotropic media with a vertical symmetry axis (VTI) was represented through reflection traveltime by Ursin and Hokstad (2002). Ettrich et al. (2002) discussed the influence of out-of-plane velocity variations on geomerical spreading in TI and orthorhombic models.

Here, we express geometrical spreading in terms of the reflection traveltime for a stack of horizontal, azimuthally anisotropic layers and show that the geometrical-spreading factor in orthorhombic media is strongly distorted by both polar and azimuthal velocity variations.

Geometrical spreading as a function of reflection traveltime

The geometrical-spreading factor is usually defined through the ratio of the wavefront curvatures at the source and receiver locations. Schleicher et al. (2001) demonstrate that in the generalized ray approximation, geometrical spreading can be conveniently derived as a function of the traveltime along the ray in the "local ray coordinates" associated with the group-velocity vector. Assuming that the incident and reflected rays are close to the vertical incidence plane at the earth's surface, the formalism of Schleicher et al. (2001) can be used to obtain the inverse geometrical-spreading factor L^{-1} of a reflected wave in the following form:

$$L^{-1}(x,\alpha) = [\cos\phi^s\,\cos\phi^r]^{-1/2}\left[\frac{\partial^2 T}{\partial x^2}\frac{\partial T}{\partial x}\frac{1}{x} + \frac{\partial^2 T}{\partial x^2}\frac{\partial^2 T}{\partial \alpha^2}\frac{1}{x^2} - \left(\frac{\partial T}{\partial \alpha}\right)^2\frac{1}{x^4}\right]^{1/2}, \qquad (1)$$

where T is the reflection traveltime expressed through the source-receiver offset x and the azimuth α of the source-receiver line, and ϕ^s and ϕ^r are the group angles with the vertical at the source and receiver locations, respectively. Equation (1) provides an accurate description of the factor L^{-1} for horizontally layered anisotropic models with a horizontal symmetry plane in each layer.

The first ("in-plane") term involving T in equation (1) coincides with the geometrical-spreading factor for VTI media (Ursin and Hokstad, 2002), while the other two ("out-of-plane") terms depend on the azimuthal variation of the traveltime caused by azimuthal anisotropy. Note that in vertical symmetry planes the first derivative $\partial T/\partial\alpha$ vanishes but the second derivative $\partial^2 T/\partial\alpha^2$ does not, which implies that azimuthal anisotropy may distort even symmetry-plane amplitudes (Tsvankin, 2001; Ettrich et al., 2002).

Geometrical spreading in a horizontal orthorhombic layer

Signatures of reflected waves in orthorhombic media are convenient to describe using the notation suggested by Tsvankin (1997, 2001). The anisotropic parameters $\epsilon^{(1)}$, $\delta^{(1)}$, $\gamma^{(1)}$, $\epsilon^{(2)}$, $\delta^{(2)}$, $\gamma^{(2)}$, and $\delta^{(3)}$ play the roles of Thomsen's VTI coefficients in the three mutually orthogonal symmetry planes (Figure 1). The vertical

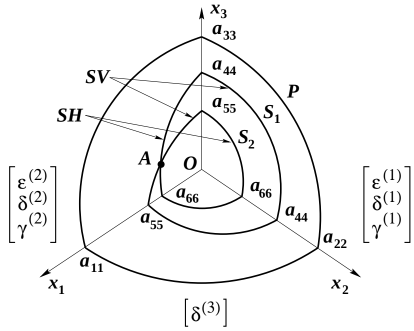

Fig. 1: Sketch of body-wave phase-velocity surfaces in orthorhombic media. Tsvankin's (1997) parameters are defined in the coordinate planes that coincide with the symmetry planes of the model.

P-wave velocity is denoted by V_{P0}, and V_{S0} is the velocity of the vertically propagating S-wave polarized in the x_1-direction. The time-domain signatures of P-waves, including reflection traveltime, depend on five combinations of the above parameters (Grechka and Tsvankin, 1999): the normal-moveout (NMO) velocities of horizontal events in the vertical symmetry planes, $V_{\rm nmo}^{(1)} = V_{P0}\sqrt{1+2\delta^{(1)}}$ and $V_{\rm nmo}^{(2)} = V_{P0}\sqrt{1+2\delta^{(2)}}$, and the anellipticity coefficients $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ defined in the symmetry planes by analogy with the Alkhalifah-Tsvankin (1995) VTI parameter η.

Factor L^{-1} in terms of medium parameters

To express P-wave reflection traveltime and then geometrical spreading through the time-processing parameters of orthorhombic media, we use the nonhyperbolic moveout equation of Tsvankin and Thomsen (1994):

$$T^2(x,\alpha) = T_0^2 + \frac{x^2}{V_{\rm nmo}^2(\alpha)} + \frac{A_4(\alpha)\,x^4}{1+A(\alpha)\,x^2}\,, \tag{2}$$

where T_0 is the zero-offset time, $V_{\rm nmo}(\alpha)$ is the azimuthally varying NMO velocity (i.e., the NMO ellipse), and $A_4(\alpha)$ is the quartic coefficient responsible for nonhyperbolic moveout. The parameter $A(\alpha)$ in the denominator depends on the horizontal group velocity $V_{\rm hor}(\alpha)$ and is designed to make T convergent at offsets $x \to \infty$:

$$A(\alpha) = \frac{A_4(\alpha)}{V_{\rm hor}^{-2}(\alpha) - V_{\rm nmo}^{-2}(\alpha)}\,. \tag{3}$$

The general form of equation (2) makes it sufficiently accurate for P-wave moveout in models with substantial azimuthal anisotropy, provided the azumuthal variation of the moveout coefficients is taken into account (Al-Dajani et al., 1998). For a horizontal orthorhombic layer, the NMO ellipse is given by (Tsvankin, 2001)

$$V_{\rm nmo}^{-2}(\alpha) = [V_{\rm nmo}^{(1)}]^{-2}\sin^2\alpha + [V_{\rm nmo}^{(2)}]^{-2}\cos^2\alpha\,; \tag{4}$$

the azimuth α is defined with respect to the x_1-axis. The quartic coefficient A_4 in an orthorhombic layer was derived by Al-Dajani et al. (1998) as

$$A_4(\alpha) = A_4^{(1)}\sin^4\alpha + A_4^{(2)}\cos^4\alpha + A_4^{(x)}\sin^2\alpha\,\cos^2\alpha\,, \tag{5}$$

where $A_4^{(1)}$ and $A_4^{(2)}$ are the symmetry-plane coefficients and $A_4^{(x)}$ is a cross-term that makes a contribution in off-symmetry directions.

Equation (2) can be used to find the traveltime derivatives needed to obtain the geometrical spreading for a horizontal orthorhombic layer from equation (1). The term $[\cos\phi^s\,\cos\phi^r]^{-1/2} = 1/\cos\phi$ (ϕ is the group angle) can be determined as $\sqrt{x^2+T_0^2\,V_{P0}^2}/(T_0\,V_{P0})$. Substitution of equations (3)–(5) yields the factor L^{-1} as a function of the parameters of orthorhombic media.

The moveout equation can be simplified by employing the kinematic equivalence between the vertical planes of orthorhombic and VTI media valid for weak anisotropy (Tsvankin, 2001, p. 54). Then long-spread P-wave moveout for any azimuth α can be described by the VTI equation of Alkhalifah and Tsvankin (1995):

$$T^2(x,\alpha) = T_0^2 + \frac{x^2}{V_{\rm nmo}^2(\alpha)} - \frac{2\eta(\alpha)\,x^4}{V_{\rm nmo}^2(\alpha)\,[T_0^2\,V_{\rm nmo}^2(\alpha) + (1+2\eta(\alpha))\,x^2]}\,; \tag{6}$$

$$\eta(\alpha) = \eta^{(1)}\sin^2\alpha - \eta^{(3)}\sin^2\alpha\,\cos^2\alpha + \eta^{(2)}\cos^2\alpha\,. \tag{7}$$

The replacement of equation (2) with equation (6) in computing the geometrical spreading does not cause a substantial error even for strongly anisotropic models. Also, linearizing the factor L^{-1} derived from equation (6) in the η coefficients gives the closed-form weak-anisotropy approximation discussed below.

Analysis of the weak-anisotropy approximation

While the full linearized expression for L^{-1} is still rather long, it takes a much more concise form in the vertical symmetry planes. For the $[x_1,x_3]$–plane, we find

$$L^{-1} = \cos^{-1}\phi\,\frac{A+Bx^2+Cx^4}{V_{\rm nmo}^{(1)}\,V_{\rm nmo}^{(2)}\,(T_0^2\,[V_{\rm nmo}^{(2)}]^2+x^2)^3}\,, \tag{8}$$

where

$$A = T_0^5\left[V_{\rm nmo}^{(2)}\right]^6, \tag{9}$$

$$B = T_0^3\left[V_{\rm nmo}^{(2)}\right]^2\Big\{2\left(1-4\eta^{(2)}\right)\left[V_{\rm nmo}^{(2)}\right]^2 + \left(\eta^{(2)}+\eta^{(3)}-\eta^{(1)}\right)\left[V_{\rm nmo}^{(1)}\right]^2\Big\}, \tag{10}$$

$$C = T_0\Big\{\left(1+\eta^{(2)}\right)\left[V_{\rm nmo}^{(2)}\right]^2 + \left(\eta^{(2)}+\eta^{(3)}-\eta^{(1)}\right)\left[V_{\rm nmo}^{(1)}\right]^2\Big\}. \tag{11}$$

At zero offset, the factor L^{-1} becomes $1/(T_0\,V_{\rm nmo}^{(1)}\,V_{\rm nmo}^{(2)})$, which is an exact expression that can be obtained directly from the wavefront curvatures for any strength of the anisotropy. Hence, the anisotropic coefficients $\delta^{(1)}$ and $\delta^{(2)}$ responsible for the NMO ellipse also control the geometrical spreading at vertical incidence. For VTI media,

$V_{\rm nmo}^{(1)} = V_{\rm nmo}^{(2)}$, and $L^{-1}(x=0)$ reduces to $1/(T_0\, V_{\rm nmo}^2)$; for isotropic media, $L^{-1} = 1/(T_0\, V_{P0}^2)$.

The "near-offset" spreading coefficient B and the "far-offset" coefficient C in equation (8) depend on both in-plane and out-of-plane velocity variations. The term $2(1-4\eta^{(2)})\,[V_{\rm nmo}^{(2)}]^2$ in the expression for B represents the in-plane (VTI) contribution, while the term $(\eta^{(2)} + \eta^{(3)} - \eta^{(1)})\,[V_{\rm nmo}^{(1)}]^2$ is entirely due to azimuthal anisotropy. It is interesting that C contains exactly the same out-of-plane term as B. The factor L^{-1} in the $[x_2, x_3]$–plane can be obtained from equations (8)–(11) by switching the superscripts (1) and (2) in the NMO velocities and the coefficients η.

The weak-anisotropy approximation shows that the azimuthal variation of $L^{-1}(\alpha)$ for x = const is controlled by the differences $(\delta^{(1)} - \delta^{(2)})$, $(\eta^{(1)} - \eta^{(2)})$ and, at far offsets, by $\eta^{(3)}$. For small offsets $L^{-1}(\alpha)$ is proportional to $\cos 2\alpha$ and traces out a curve close to an ellipse.

Numerical examples

In the numerical tests we use an orthorhombic model formed by parallel, vertical, penny-shaped cracks embedded in a VTI background (Schoenberg and Helbig, 1997). Although this model has substantial azimuthal velocity variations, it is dominated by the VTI component, with both ϵ coefficients close to 30%. The inverse geometrical spreading factor L^{-1} is computed using both the "exact" formalism (solid lines) and the weak-anisotropy approximation (dashed). The "exact" result is based on combining equation (1) for L^{-1} with the moveout equations (2)–(5). (The word "exact" is put in quotes because the Tsvankin-Thomsen equation is an accurate but not exact representation of P-wave traveltime.)

The factor L^{-1} is normalized by its value in a purely isotropic layer with the velocity V_{P0}. As illustrated by Figure 2, the influence of anisotropy leads to significant distortions of geomertical spreading in a wide range of offsets for both symmetry planes. If the corresponding η coefficient (e.g., $\eta^{(2)}$ in the $[x_1, x_3]$-plane) is positive, the normalized L^{-1} rapidly decreases with offset at near-vertical incidence.

Comparison with the factor L^{-1} for the reference VTI model (which has the vertical velocity V_{P0} and the anisotropic parameters ϵ and δ defined in a given symmetry plane) reveals a significant influence of azimuthal anisotropy even at vertical incidence (Figure 2). The contribution of azimuthal velocity variations to the offset dependence of L^{-1} in the $[x_1, x_3]$ symmetry plane is controlled by the combination $(\eta^{(2)} + \eta^{(3)} - \eta^{(1)})$, which is positive and relatively large (0.38). As a result, the factor L^{-1} in the $[x_1, x_3]$–plane initially decreases with offset slower than in the reference VTI medium (Figure 2a). The offset dependence of L^{-1} in the $[x_2, x_3]$–plane, however, is barely influenced by azimuthal anisotropy because the relevant combination $(\eta^{(1)} + \eta^{(3)} - \eta^{(2)})$ is small (Figure 2b). The weak-anisotropy aproximation correctly predicts the general character of the function $L^{-1}(x)$ but loses accuracy for offsets exceeding the reflector depth.

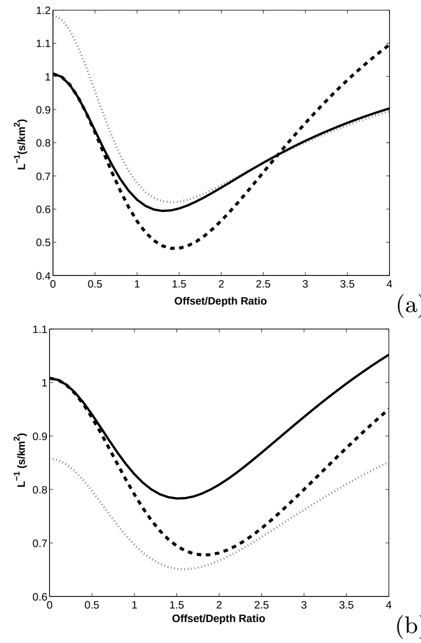

Fig. 2: Normalized inverse geometrical spreading L^{-1} as a function of the offset-to-depth ratio in the symmetry planes $[x_1, x_3]$ (a) and $[x_2, x_3]$ (b) of a horizontal orthorhombic layer. The solid line is the "exact" result, the dashed line is the weak-anisotropy approximation, and the dotted line is L^{-1} in the reference VTI model. The model parameters are $V_{P0} = 2.437$ km/s, $\epsilon^{(1)} = 0.329$, $\epsilon^{(2)} = 0.258$, $\delta^{(1)} = 0.083$, $\delta^{(2)} = -0.078$, and $\delta^{(3)} = -0.106$ ($\eta^{(1)} = 0.211$, $\eta^{(2)} = 0.398$, $\eta^{(3)} = 0.193$).

The azimuthal variation of L^{-1}, which repeats in each quadrant, is almost elliptical for offsets close to the reflector depth (Figure 3a). The shape of the curve $L^{-1}(\alpha)$ becomes more complicated with increasing offset (Figure 3b), but the magnitude of the azimuthal variation of $L^{-1}(\alpha)$ for the model at hand remains almost the same (about 30%). The combined influence of polar and azimuthal anisotropy creates a rather complicated spatial pattern of the normalized factor L^{-1} (Figure 4), with the largest anisotropy-induced distortions of the geometrical spreading reaching 40% near the $[x_1, x_3]$–plane for offset-to-depth ratios of about 1.5.

Discussion and conclusions

Although the geometrical spreading of reflected waves is determined by the medium properties along the whole raypath, it can be obtained from the reflection traveltime and the group angles at the surface. For a stack of horizontal azimuthally anisotropic layers with a horizontal symmetry plane, geometrical spreading can be found as a function of the azimuthally varying moveout coefficients using the Tsvankin-Thomsen (1994) nonhyperbolic moveout equation.

The inverse geometrical-spreading factor L^{-1} in a horizontal orthorhombic layer was related to the medium pa-

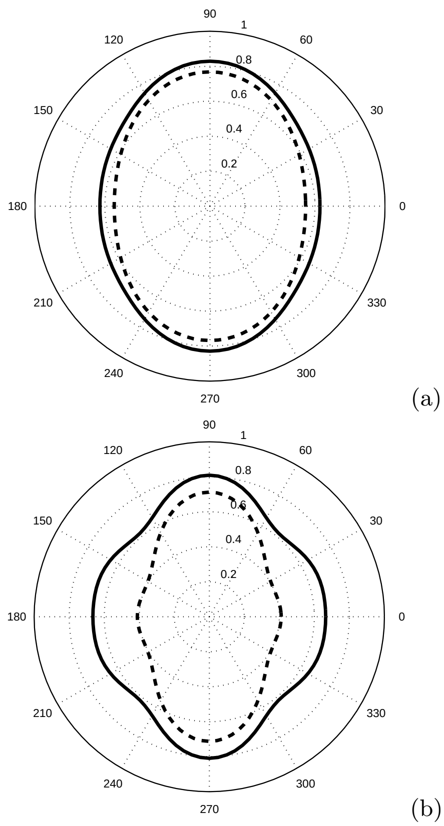

Fig. 3: Azimuthal variation of the normalized factor L^{-1} for offset-to-depth ratios of one (a) and two (b). The azimuth α (numbers on the perimeter) is measured with respect to the x_1-axis. The solid line is the "exact" solution, the dashed line is the weak-anisotropy approximation.

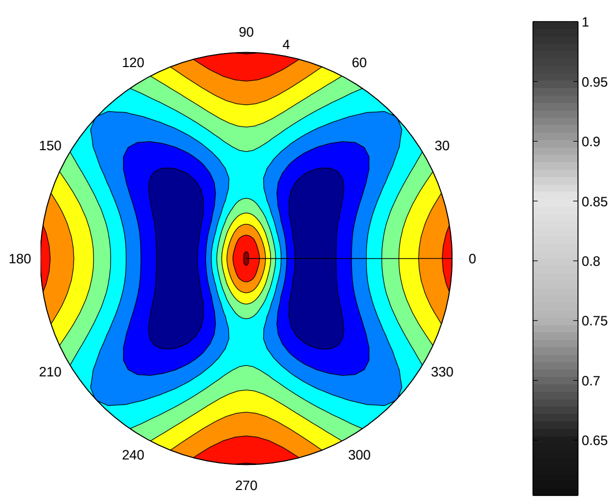

Fig. 4: Map of the "exact" normalized factor L^{-1} as a function of offset and azimuth. The offset-to-depth ratio varies from zero to four.

rameters by using explicit expressions for the NMO ellipse and the quartic moveout coefficient. The traveltime and geometrical spreading of P-waves for this model is governed by the NMO velocities $V_{\rm nmo}^{(1)}$ and $V_{\rm nmo}^{(2)}$ in the vertical symmetry planes and the anellipticity coefficients $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$. At zero offset, the factor L^{-1} is inversely proportional to the product of the symmetry-plane NMO velocities. The offset-dependent part of L^{-1} in the symmetry planes can be separated (in the weak-anisotropy approximation) into the in-plane term computed for the corresponding VTI medium, and the out-of-plane term associated with azimuthal anisotropy and controlled by the η coefficients.

The variation of the geometrical spreading with offset and azimuth has a rather complicated character, and the maximum error of the isotropic expression for L^{-1} reaches 40% for the typical orthorhombic model used here. Clearly, reliable interpretation of the AVO response for media with azimuthally anisotropic overburden requires a geometrical-spreading correction that can be based on the reflection traveltimes.

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