Time-lapse Monitoring with Multiply Scattered Waves

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Summary

Previous formulations of coda wave interferometry make it possible to assess the average change of the medium, but they do not allow for the spatial localization of this change (Snieder *et al.*, 2002; Snieder, 2002). We present an approach for localizing temporal changes in the medium using strongly scattered waves, and test it with numerical models for 2D scalar waves. Using an integral representation for the diffuse wavefield, we derive an expression for the mean traveltime perturbation due to a small perturbation in the slowness. We validate the theory using synthetic seismograms calculated with a finite-difference algorithm. In general, for localized slowness perturbations, the theory predicts the mean traveltime change of the diffuse wavefield in a multiple-scattering medium. The technique presented here can be used in many applications such as medical imaging, non-destructive testing, and reservoir monitoring, to infer temporal changes of the multiple-scattering medium.

Introduction

Coda waves are multiply scattered waves. Little attention has been paid to the problem of *imaging* with coda waves, i.e., localizing changes in the medium. Here, we propose a technique for localizing the temporal change in the medium with multiply-scattered waves. Specifically, we detect changes in the effective medium characterized by strong and random fluctuations of the velocity or slowness.

We take advantage of the fact that the wave transport acquires a diffusive character in a strongly scattering medium to obtain an expression for the mean or average traveltime change of the diffuse wavefield caused by a slowness perturbation. This constitutes the forward problem; the purpose of this work is to test this formalism with synthetic seismograms computed by finite-differences for different perturbations in the slowness. The inverse problem, estimating the temporal change in the slowness given the measured traveltime change between the unperturbed and perturbed wavefield, will be addressed in future work.

Recently, coda waves have been used to study the temperature dependence of the seismic velocity in granite (Snieder *et al.*, 2002) using a technique called *Coda Wave Interferometry*. Snieder *et al.* (2002) found that one can estimate small changes on the order of 1% in medium velocity from the multiple scattered waves by measuring the time shift between the unperturbed and perturbed wavefields. We extend their work with the modifications to the theory needed to spatially localize the change in the medium. We then introduce localized slowness perturbations and use finite-difference simulations in a 2D multiple-scattering to measure the traveltime change of the multiply scattered waves before and after the perturbation. To assess the validity of our theory, we compare the observed and predicted traveltime change. With this technique, it is possible to localize the temporal change of the multiple-scattering medium from measurements of the mean traveltime change of the diffuse wavefield.

Velocities in a Multiple-Scattering Medium

To generate the synthetic seismograms for our study we use a fourth-order 2D acoustic finite-difference code that propagates a finite-duration pulse through a specified velocity model. Following Frankel and Clayton (1984), we model the 2D multiple-scattering velocity field as a constant-background model with added random velocity fluctuations that scatter the waves. The total velocity field is decomposed as

$$v(\mathbf{r}) = v_0 + v_r(\mathbf{r}), \tag{1}$$

where v_0 is the background velocity and v_r are the random velocity fluctuations with a Gaussian autocorrelation function. The autocorrelation function of the velocity fluctuations has the form

$$< v_r(\mathbf{r}'), v_r(\mathbf{r}' + \mathbf{r}) >= \sigma^2 \exp\left[\frac{-r^2}{a^2}\right], \tag{2}$$

where σ is the standard deviation of the velocities, and a is the correlation distance. The values of the correlation distance a and velocity fluctuations σ are set in such a way that multiply-scattered waves are produced as the result of waves propagating in the random medium.

Effective Slowness in a Diffuse Medium

When the scattering is strong, waves propagate with a velocity that is less than the mean or background velocity v_0; this is a consequence of the slowing down of the ballistic or direct pulse, caused by multiple scattering. In a multiple-scattering medium we can define an effective velocity c, which is the speed with which the coherent pulse propagates in the multiple scattering medium. We obtain the effective velocity c in a medium with random velocity fluctuations v_r using the effective medium theory approximation (Tatarski, 1963; Sheng, 1995). Let us start with the Helmholtz equation in 2D:

$$\nabla^2\Psi(\mathbf{r}, w) + k_0^2(1 - \epsilon(\mathbf{r}))^2\Psi(\mathbf{r}, w) = \delta(r), \tag{3}$$

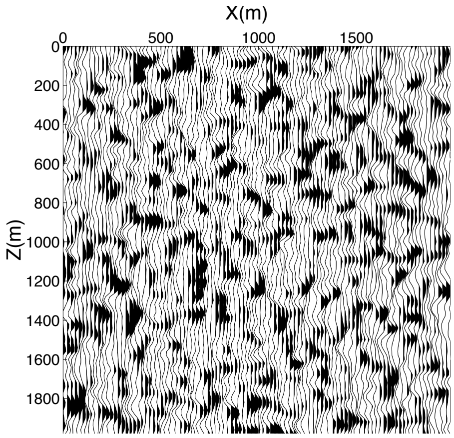

Fig. 1: Representation of a 2D random slowness model with a Gaussian autocorrelation function. The correlation distance a is 40 m.

where k_0=ωs_0 is the wavenumber in the homogeneous reference medium, $\epsilon(\mathbf{r})$ is the normalized random velocity fluctuation, and $\Psi(\mathbf{r})$ is the wavefield propagating through the random medium. Eq. (3) can be formulated as a *random integral equation* (Frisch, 1965). We can obtain a solution to the mean field $\bar{\Psi}$ radiated by a point source in a random medium, which satisfies the equation (Frisch, 1965)

$$(\nabla^2 + {k_{eff}}^2)\bar{\Psi}(\mathbf{r}, w) = \delta(\mathbf{r}), \quad (4)$$

$${k_{eff}}^2 = {k_0}^2 - {k_0}^4 \int_A G_0(\mathbf{r}, w)\Gamma(\mathbf{r})d^2\mathbf{r}, \quad (5)$$

where $G_0(\mathbf{r}, \omega)$ is the free-space Green's function and $\Gamma(\mathbf{r})$ is the autocorrelation function of the velocity fluctuations. Eq. (4) is the Helmholtz equation with an effective wavenumber k_{eff}. From Eq. (5) we can obtain the effective slowness, i.e., the slowness with which the coherent pulse propagates in the multiple-scattering medium.

Perturbations of the Effective Slowness

We calculate the slowness field as the inverse of the fluctuating velocity field v defined in eq. (1). This slowness field represents the unperturbed slowness (see Figure 1). Let us introduce a constant slowness perturbation in our slowness field, creating the perturbed slowness field,

$$s_p = s_{unp} + \delta s_0, \quad (6)$$

where s_{unp} is the unperturbed slowness field, s_p is the perturbed field, and δs_0 is the mean slowness perturbation. The effective slowness perturbation is proportional to the mean slowness perturbation. The constant of proportionality, which can be derived from the effective medium theory, depends on the specific parameters of the random slowness fluctuations.

From eq. (5), the effective wavenumber depends on the mean wavenumber in the following way:

$$k_{eff}^2 = k_0^2 + F(k_0), \quad (7)$$

where $F(k_0)$ is the function that describes the dependence of k_{eff} on k_0. We obtain the perturbed effective wavenumber after adding the effective slowness perturbation to the effective wavenumber, i.e.,

$$k_{eff}^p = k_{eff} + \delta k_{eff} = k_{eff} + \frac{\partial k_{eff}}{\partial k_0}\delta k_0, \quad (8)$$

where k_{eff}^p is the perturbed effective wavenumber. If we subtract from eq. (8) the unperturbed effective wavenumber k_{eff} and divide by frequency ω, we obtain the effective slowness perturbation

$$\delta s_{eff} = \frac{\partial k_{eff}}{\partial k_0}\delta s_0. \quad (9)$$

Traveltime change of the Diffuse Wavefield

In order to localize changes in the scattering medium, we derive an expression relating the mean traveltime change of the diffuse wavefield to the effective slowness perturbation in the medium as a function of position. With this expression we predict the temporal change in the traveltimes of the multiply-scattered waves after introducing a given slowness perturbation. Snieder *et al.* (2002) introduced coda wave interferometry whereby multiply scattered waves are used to detect temporal changes in the medium by using the scattering medium as an interferometer. They found that for a small perturbation in the velocity, estimates of this perturbation can be obtained from multiply scattered waves by a time-windowed crosscorrelation. The assumption used is that when the temporal change is weak, the influence of this change on the geometrical spreading and on the scattering coefficient can be ignored, and the dominant change on the wavefield arises from the change in the traveltime of the wave that travels along each path.

For small and homogeneous perturbations in the effective slowness, the time-windowed crosscorrelation of the unperturbed and perturbed wavefield provides an estimate of the time shift between the two waveforms. With the traveltime change τ, we can estimate the effective slowness perturbation with the equation (Snieder *et al.*, 2002)

$$\frac{\delta s_{eff}}{s_{eff}} = \frac{\tau(t, T)}{t}, \quad (10)$$

where $\tau(t, T)$ is the time shift of the time-windowed crosscorrelation centered at time t with window size T. The traveltime change τ is weighted by the energy of the wave along each multiple-scattering path with total traveltime t from the source to the receiver.

For localized slowness perturbations, eq. (10) is no longer valid and we need a new expression describing the traveltime change of the multiply-scattered waves caused by a localized slowness perturbation. We obtain such an expression using an integral representation of the diffuse wavefield, based on the assumption that in a multiple-scattering medium the diffusion equation describes the propagation of the average intensity (Tourin & Fink, 2000). In that approach, the average intensity is treated as the probability $P(r,t)$ of a wave undergoing a random walk, traveling a net distance r in a time t.

In a two-dimensional medium of infinite extent, and in the long-time limit, the average intensity can be approximated well by the solution of the diffusion equation (Paaschens, 1997),

$$P(r,t) = \frac{1}{4\pi Dt} \exp\left[\frac{-r^2}{4Dt}\right], \qquad (11)$$

where r is the source-receiver separation and D is the diffusion constant. With this representation of the average intensities in a multiple-scattering medium, we arrive at the following equation for the mean traveltime change of the diffuse wavefield caused by an effective slowness perturbation $\delta s_{eff}/s_{eff}$,

$$\tau(t) = \int_A \frac{P(\mathbf{r}',\mathbf{s}) * P(\mathbf{r},\mathbf{r}')}{P(\mathbf{r},\mathbf{s},t)} \frac{\delta s_{eff}}{s_{eff}}(\mathbf{r}') dV(\mathbf{r}'), \qquad (12)$$

where $\mathbf{r}'$ is the location of the slowness perturbation, $P(\mathbf{r}',\mathbf{s}) * P(\mathbf{r},\mathbf{r}')$ is the temporal convolution of the intensity $P(\mathbf{r}',\mathbf{s})$ at location $\mathbf{r}'$ due to an intensity impulse at the source location $\mathbf{s}$ and the intensity $P(\mathbf{r},\mathbf{r}')$ at the receiver location $\mathbf{r}$ due to an intensity impulse at $\mathbf{r}'$; $P(\mathbf{r},\mathbf{s},t)$ is the intensity at the receiver at time t due to an impulse intensity at the source at time t=0.

Eq. (12) describes the mean traveltime change τ of all waves initiated at the source at time t=0, traveling through the multiple-scattering medium on a random walk trajectory, and arriving at the receiver at time t. The estimation of $\delta s_{eff}/s_{eff}$ from the data $\tau(t)$ from Eq. (12) constitutes a standard linear inverse problem.

Examples

Having derived the relation between the mean traveltime change of the diffuse wavefield and the effective slowness perturbation, we test the theory with finite-difference simulations of multiply scattered waves in a medium with random velocity fluctuations as defined in eq. (2) before and after introducing a localized effective slowness perturbation.

We consider a localized perturbation within an area in the shape of a ring, centered on the source location (see Figure 2). The strength of the effective slowness perturbation $\delta s_{eff}/s_{eff}$ is constant within the ring, and a taper is applied outwards from the ring to avoid generating diffractions from the borders. The strength of the effective slowness perturbation is $\delta s_{eff}/s_{eff}$=0.016. We calculate the perturbed and unperturbed finite-difference

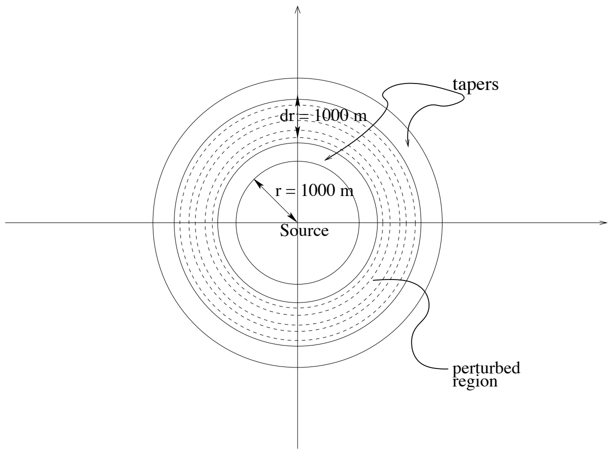

Fig. 2: Localized perturbation in the slowness in the shape of a ring (dotted area) centered on the source location. Tapers with length of 500 m are applied outside the ring to avoid diffractions from the borders of the perturbation.

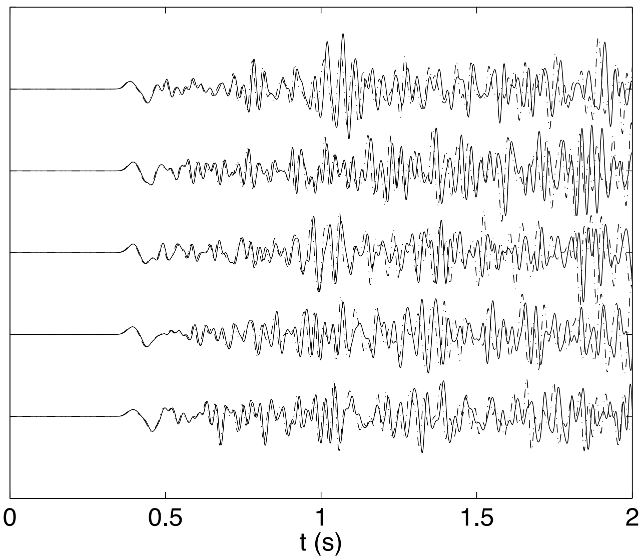

Fig. 3: Finite-difference synthetic seismograms before (solid) and after (dashed) introducing the slowness perturbation in Figure 2 for five different receivers located 2000-m away from the source.

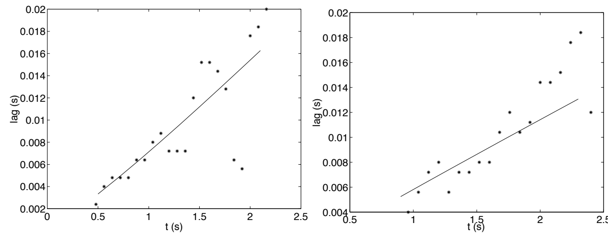

Fig. 4: Theoretical mean traveltime change (solid) versus traveltime change for one receiver (asterisks) located 2000 m (left) and 4000 m (right) away from the source.

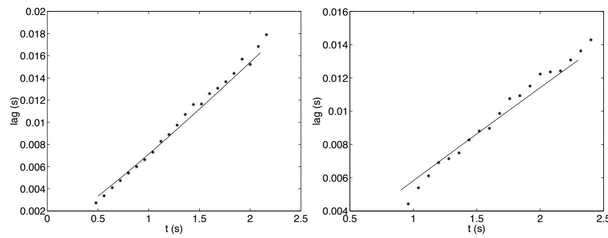

Fig. 5: Theoretical mean (solid) versus estimated averaged (asterisks) traveltime change for a receiver located 2000 m (left) and 4000 m (right) away from the source.

synthetic seismograms for arrays of 100 receivers around the source for two source-receiver separations: 2000 and 4000 m. Figure 3 shows the calculated wavefield for five receivers located 2000-m away from the source. Note the strength of the multiple scattered waves after the highly attenuated ballistic arrival.

Figure 4 shows the traveltime change estimated with coda wave interferometry for one receiver located 2000 m and another located 4000 m away from the source versus the predicted mean traveltime change calculated with eq. (12). Note the fluctuations of the estimated traveltime change, for one receiver, around the theoretical mean traveltime change of the diffuse wavefield. These fluctuations arise because the traveltime change for one realization of the multiply scattered wavefield differs from the mean traveltime change of the diffuse wavefield.

To obtain a better estimate of the mean traveltime change from the synthetic seismograms, we measure the traveltime change for all 100 receivers in the circular array, and average the resulting 100 traveltime-change curves to obtain the averaged traveltime change of the multiply scattered wavefield. Because the random medium is statistically homogeneous and the slowness perturbation is symmetric around the source, the 100 seismograms recorded in the circular array around the source are equivalent to 100 realizations of the multiply scattered wavefield. The result of the averaging of the traveltime change for the receivers located 2000 m and 4000 m away from the source is shown in Figure 5. After averaging, the estimated mean traveltime change shows better agreement with the predicted mean traveltime change.

Conclusions

We have developed a theory that relates the mean traveltime change of the diffuse wavefield to localized perturbations of the effective medium for 2D acoustic waves. This theory was developed by means of the representation of the multiply scattered wavefield as a diffusion process. For the example presented here, we show that our theory accurately predicts the mean traveltime change of the diffuse wavefield in a multiple-scattering medium. In order to detect a localized perturbation in the multiple-scattering medium, we need access to many realizations of the diffuse wavefield propagating through the medium characterized by strong and random velocity fluctuations. For the example shown here, we obtain an estimate of the mean traveltime change by averaging over different receiver locations. In general, many realizations of the experiment are not available. In that case, one can average over different source-receiver pairs. Even tough we have tested the technique only for 2D acoustic waves, the theory is valid for 3D scalar waves. Further work needs to be done to adapt the technique for the more complicated and realistic 3D elastic wave propagation. Application of this technique in geophysics, non-destructive testing, time-lapse monitoring, and other situations where elastic waves are used, require the extension of the present formulation to include mode conversions of the elastic waves propagating through a medium characterized by random fluctuations of its elastic parameters.

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