P077 APPARENT ATTENUATION FROM SHORT-PERIOD MULTIPLES AND INSTRINSIC ABSORPTION IN THE SEISMIC WAVELET MODEL

A. MATEEVA[1], DOUGLAS HART[2] and SCOTT MACKAY[3]

[1] *Colorado School of Mines, Center for Wave Phenomena, Golden, CO 80401 , U.S.A,*

[2] *Regis University*

[3] *WesternGeco*

Abstract

For the purposes of predictive deconvolution one assumes that the seismic trace results from the convolution of a reflectivity series with a wavelet. A fundamental component of the seismic wavelet model is the intrinsic absorption of the earth that causes loss of high frequencies to anelastic processes during propagation. Another potentially important component of the wavelet model is the apparent attenuation caused by short-period multiples. Here we examine whether these two processes act similarly enough to be combined into a single "effective attenuation" operator for signal processing purposes. We conclude that they can be combined except in cyclic depositional environments that contain many high reflection coefficients, when the apparent attenuation operator can no longer be accurately modeled as minimum phase.

Introduction: Can a single "effective attenuation" operator be used in the convolutional model of the trace to account for both intrinsic absorption and the filtering action of thin, horizontal layering? Because the reflection coefficient series describing the subsurface is "blue" (e.g., Saggaf and Robinson, 2000) the transmission response of the layered earth is dispersive and high-frequency deficient even in the absence of anelasticity (O'Doherty and Anstey, 1971.) Additionally, it is minimum-phase (Sherwood and Trorey, 1965, Treitel and Robinson, 1966, Banik *et al.,* 1985). In this sense, the fine layering in the earth acts similarly to absorption on the transmitted signal. However, a seismic trace is more complicated than the transmission response of a stack of layers; it results from a reflection experiment conducted over a half-space that is bounded by a free surface. Consequently, the question posed at the beginning of this section should be approached with care.

The intent of this paper is to investigate whether the intrinsic absorption and apparent attenuation operators can be combined for the purposes of wavelet estimation and deconvolution. First, we derive a convolutional operator accounting for short-period multiples and transmission losses at the interfaces in the earth. We then compare its spectral properties to those of intrinsic absorption. We focus on the phase spectrum and show that the apparent attenuation operator can be significantly non-minimum phase in media characterized by strong reflectivities. The deviation from the minimum-phase property is caused by multiples from the earth's surface and is larger when the short-period multiples in the medium are strong.

Convolutional models for the seismic trace: Consider a horizontally-layered medium characterized by a reflection coefficient series, r. Let r_m denote its reflection impulse response. The sequence r_m includes transmission losses at interfaces and multiples. Next, let w_o be a wavelet, which is defined in more detail below. A noise-free model of the seismic trace results from the convolution, $w_o * r_m$. On the other hand, for signal processing purposes, one models the seismic trace as the convolution of some wavelet, w, with the earth's reflection coefficient series, r. The equivalence of the two models of the seismic trace is expressed in the frequency domain as

$$WR = W_o R_m, \tag{1}$$

where the capital letters stand for the Fourier transforms of the respective time series. This equation implies that

$$W = W_o R_m/R, \tag{2}$$

As an example, suppose that the basic wavelet, W_o , contains the source signature, S, the receiving instrumentation response, I , and the effect of anelasticity, Q. We can then write equation (2) as

$$W = S\, I\, Q\, R_m/R, \tag{3}$$

Thus, multiples and elastic transmission losses can be included in the wavelet model through the apparent attenuation operator, R_m/R. If this operator has the same properties as the intrinsic absorption operator, Q, namely: *exponential decay with frequency* and *minimum phase*, short-period multiples and intrinsic absorption can be combined into an "effective attenuation" operator, Q_{eff}. This would permit us to model the seismic wavelet as simply

$$W = S\, I\, Q_{eff}\,, \tag{4}$$

where S and I are known, and Q_{eff} is measurable from the trace.

Although the amplitude decay with frequency of the apparent attenuation operator is not exactly exponential, it can be modeled as such reasonably well over a limited frequency range.

The minimum phase property is under investigation in this paper. We shall see that the apparent attenuation operator can be significantly non-minimum phase when the geology is characterized by a strong "cyclic" reflectivity. The analysis performed here is for a specific time window, as is common for predictive deconvolution. The non-stationary aspect of the absorption is not directly addressed.

The operator R_m/R: The popular assumption that the stratigraphic filter of the horizontally layered earth is minimum-phase rests on the fact that the transmission through a stack of thin layers is minimum phase. Therefore, to understand the phase of the operator R_m/R, it's useful to relate it to the transmission response of a stack of layers, which we will denote by M.

Below we discuss some weak-reflectivity approximations of R_m/R. Although our analysis is not limited to the weak-reflectivity case, these approximations are useful in predicting when the minimum-phase property of R_m/R might fail. For simplicity, we assume that the reflection coefficient series is stationary, i.e., its spectrum does not change with depth.

Earth model without a free surface: Consider a surface seismic trace that is free of surface-related multiples. As is illustrated in Figure 1, in a short time window starting at two-way time T, the elastic impulse response R_m is approximately

$$R_m \approx M^2 R, \tag{5}$$

This can be seen as a convolution between the series of reflection coefficients describing the subsurface interval reached by the direct arrival at time $T/2$ and the two-way transmission filter of the overburden. Thus, in the absence of surface-related multiples, the operator R_m/R coincides with the two-way transmission response M^2 of the layered overburden, that is minimum-phase, with an amplitude spectrum given by O'Doherty and Anstey's formula:

$$|\, R_m/R \,| \approx e^{-|R|^2 T}, \tag{6}$$

In media characterized by strong reflectivities equation (6) may become inaccurate but, as synthetic tests show, R_m/R remains minimum-phase. Deviations from the minimum phase property can be observed if a long time-window is used for spectral estimation. The reason is that R_m/R, in equation (6), is not stationary. Thus, its estimate from a long time window includes summation over different minimum-phase operators, the result of which is generally not minimum-phase. Establishing that R_m/R is minimum-phase in the absence of surface-related multiples is promising but insufficient. The earth's surface has a very strong influence on the seismic trace and must be taken into account.

Earth model with a free surface: Let $(R_m/R)_0$ denote the operator discussed above for the model without a free surface, and let $(R_m/R)_1$ be that for a model with a free surface. Then,

$$(R_m/R)_1 \approx (R_m/R)_0\; [1 + R(\omega) + R^2(\omega) + \ldots], \tag{7}$$

The terms proportional to powers of $R(\omega)$ in the brackets account for different orders of surface-related multiples. Since these terms have random phases, $(R_m/R)_1$ is not minimum-phase. However, the phase distortion occurs mainly at high frequencies where $|R|$ is large. At the low frequencies, containing most of the power of R_m/R, the phase of $(R_m/R)_1$ will be almost the same as that of $(R_m/R)_0$. This is illustrated in Figure 2a by a synthetic example based on 100 realizations of a strong cyclic reflectivity, similar to that of Well 8 from the papers of Walden and Hosken (1985, 1986). Figure 2b shows that the presence of a free surface whitens the power spectrum of R_m/R, i.e., $|R_m/R|_1$ has a smaller slope than $|R_m/R\,|_0$. Indeed, as is seen from equations (6) and (7), surface-related multiples partially compensate for the high-frequency deficit in $|R_m/R\,|_0$. This whitening effect is more pronounced in strong reflectivities and at low frequencies.

In summary, our observation is that surface-related multiples do not alter significantly the phase of R_m/R. However, they reduce the slope of its amplitude spectrum. Therefore, the minimum-phase equivalent of $|R_m/R|_1$ will underestimate the phase of the apparent attenuation operator. For our strong reflectivity example, Figure 3a shows that the error in the phase of $(R_m/R)_1$ derived under the minimum-phase assumption reaches up to 45 degrees over the lower third of the spectrum, which includes the frequency band of maximum trace power.

Modeling the phase of the apparent attenuation operator: As seen in the previous section, the phase of the apparent attenuation operator $(R_m/R)_1$ is not well predicted by the minimum-phase equivalent of its power spectrum when the subsurface is characterized by a strong, blue reflectivity. A better prediction of the phase of $(R_m/R)_1$ in such cases can be made on the basis of two assumptions. First, that $(R_m/R)_0$ is minimum phase. Second, that the phase of $(R_m/R)_1$ is approximately the phase of $(R_m/R)_0$. Numerical tests suggest that these assumptions hold well even under unfavorable conditions, namely, strong reflectivities and long time windows for spectral estimation. Thus, it seems reasonable to attempt modeling the phase of $(R_m/R)_1$ through the minimum-phase equivalent of $|R_m/R|_0$. The result of such modeling is shown in Figure 3b. The fit is obviously better than that in Figure 3a. Most important is the improvement over the low frequencies, where the power of the trace is concentrated. The phase deviations at higher frequencies are a consequence of the random phase of the reflection coefficient series and the assumption that surface related multiples do not change the phase of R_m/R. Near the Nyquist frequency both spectra in Figure 3b go to zero, which is a characteristic behavior of Hilbert transforms of power spectra.

To achieve the phase match of Figure 3b we need an estimate of $|R_m/R|_0$. This quantity cannot be measured from the seismic trace but can be computed from sonic and density logs. Borehole data can be also used to predict the phase of the effective attenuation Q_{eff} as a whole, rather than the phase of its elastic component alone. For example, we could use well logs to assess the impulse response of the medium with and without surface-related multiples. Then, compute the difference between the minimum phase corresponding to $|R_m/R|_0$ and that corresponding to $|R_m/R|_1$. Adding this difference to the minimum-phase equivalent of the effective Q, measured on the trace, would correct the phase of the wavelet derived under the assumption that the stratigraphic filter is minimum-phase.

This process may be repeated for several time windows. The phase correction will grow with time because, while $|R_m/R|_1$ is stationary (White *et al.*, 1990, equations (7) and (8),) $|R_m/R|_0$ becomes progressively high-frequency deficient, i.e., the minimum-phase equivalents of $|R_m/R|_0$ and $|R_m/R|_1$ diverge.

Finally, it should be pointed out that the phase correction becomes significant only in very strong, blue reflectivities, e.g., thinly interlaced lithologies with contrast impedances. In weak, or even moderately strong reflectivities, the spectral slope of $|R_m/R|_0$ is practically the same as that of $|R_m/R|_1$ and the standard assumption that the stratigraphic filter is minimum-phase works well.

Conclusions: Short-period multiples can be included in the wavelet model through the apparent attenuation operator R_m/R. In most cases, apparent attenuation and intrinsic absorption can be combined in a single effective attenuation operator for the purposes of signal processing. However, this cannot be done in finely layered geologies with strong reflection coefficients. In such cases the apparent attenuation operator becomes significantly non-minimum phase. The minimum-phase equivalent of the observed effective attenuation underestimates the phase-lag of the wavelet.

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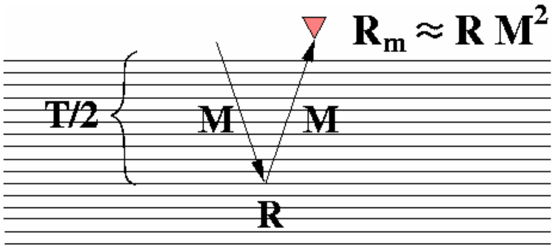
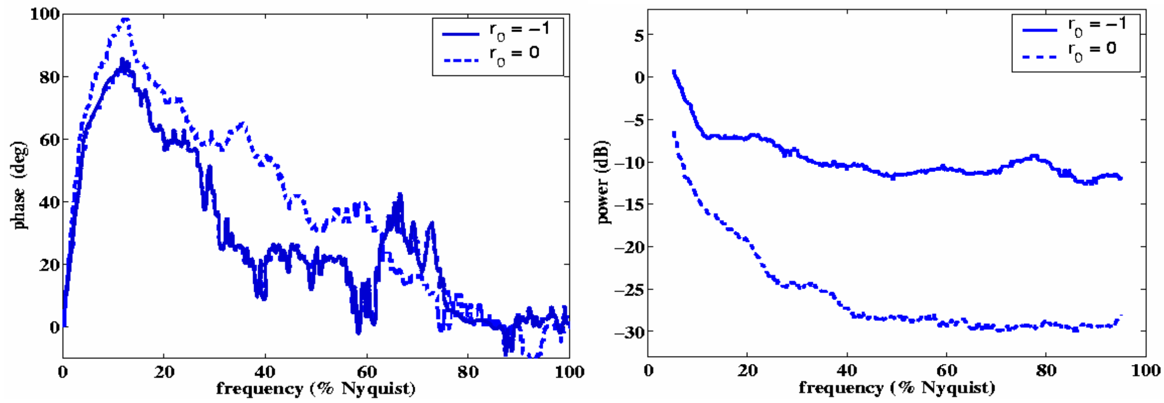
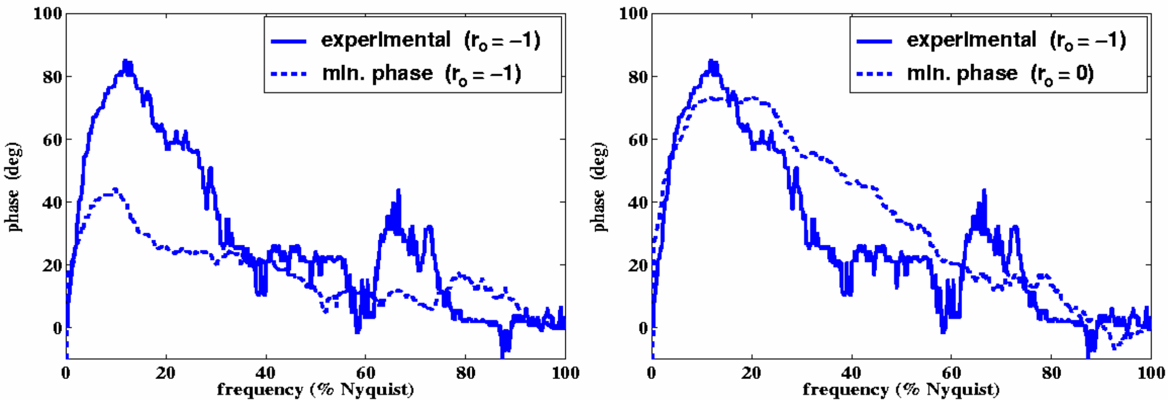


Figure 1. The reflection impulse response R_m of a layered half-space without a free surface: a weak-reflectivity approximation ignoring ray-paths trapped in the shallow subsurface.



(a) (b)

Figure 2. R_m/R with (solid) and without (dashed) surface-related multiples (reflection coefficient of the earth surface set to -1 and 0, respectively): (a) phase spectrum, (b) power spectrum.



(a) (b)

Figure 3. Phase spectrum of R_m/R (solid) and minimum-phase spectrum (dashed) computed from: (a) $|R_m/R|_1$, (b) $|R_m/R|_0$.