Elastic Coda Wave Interferometry, a New Tool for the Instrumented Oilfield

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Summary

Coda wave interferometry is a technique for time-lapse seismics that employs the high sensitivity of multiply scattered waves for changes in the medium to make inferences about tiny changes in the subsurface. The original formulation of coda wave interferometry provides a measurement the velocity change in the medium. In elastic media, there are two wave velocities, v_P and v_S. Here it is shown that coda wave interferometry provides information on a weighted average of the change in v_P and v_S. For realistic values of the elastic wave velocity, this weighted average is dominated by the change in the shear wave velocity v_S. This means that coda wave interferometry can be used to provide highly accurate time-lapse measurments of the change in the shear velocity. Because of this accuracy, this technique can be an important tool for the instrumented oilfield.

Single versus multiple scattering

Seismic imaging is to a large extent based on single scattering theory. Ignoring multiply scattered waves makes migration and other forms of linearized imaging a tractable problem because for a given velocity model, and for a given source-receiver combination, one knows that the scatterer is located somewhere on a surface rather than in a volume. In practice, the restriction of single scattering is imposed by limiting the frequency content of seismic signals to relatively low frequencies.

Multiply scattered waves are harder to analize, especially if the goal is to create deterministic images of the subsurface. However, the multiply scattered waves are much more sensitive to details in the medium than are the single scattered waves. Also, the multiply scattered waves are more sensitive to the small-scale details of the subsurface than the single scattered waves. The wavelength of waves used a seismic experiment (say 30 meters), is much larger than the scale of layering in the Earth, the size of faults and cracks, and the small-scale variations in reservoir rocks. For this reason it is worthwhile to explore what additional information can be obtained from multiply scattered waves.

The sensitivity of multiply scattered waves to the details of the medium is illustrated with the numerical example shown in figure 1. An acoustic source indicated with an asterisk radiates acoustic waves into a 2D medium with isotropic point scatterers randomly placed at locations marked by the filled circles. The wave field recorded at a receiver indicated with a triangle is computed using a numerical implementation (Groenenboom and Snieder, 1985; Snieder 1999) of Foldy's method(Foldy, 1945). This wave field is indicated with the solid wiggle trace; it consists of an extended wave-train of multiple scattered waves. The locations of the scatterers are then randomly perturbed to the locations indicated by the open circles. In figure 1 this displacement relative to the initial position is greatly exaggerated to make it visible. The wave field for this perturbed configuration of scatterers is shown by the dashed wiggle trace in figure 1. The perturbation of the wave field increases with time because the waves that have bounced back and forth more often are more sensitive to changes in the scatterer locations than are the waves that have been scattered only a few times.

Coda wave interferometry

In the example the locations of the scatterers is perturbed to show the sensitivity of the multiply scattered waves to a perturbation of the medium. Multiply scattered waves are not very useful for creating deterministic images of the subsurface. However, the example of figure 1 shows that these waves are highly sensitive to slight perturbations of the medium. This is the basis of coda wave interferometry where the change in multiply scattered waves is used to make inferences about the change in the medium. For the example of figure 1 it is shown by Snieder et al. (2002), that the root-mean-square displacement of the scatterer locations can successfully be retrieved from the change in the multiple scattered waves.

Coda wave interferometry has been used to determine the change of the seismic velocity in a rock sample by using the change in ultrasonic waves that propagate through the sample as the temparature is increased. The waveforms for a temperature increase from 45 to 50 degrees C are shown in figure 2. The inset of the first arrivals shown in the top right shows that these arrivals are virtually identical. The change in the velocity cannot be determined from these first arriving waves. However, the later arriving waves show a distint phase shift. This phase shift has been used by Snieder et al. (2002) to measure a change in the velocity that is about 0.1% with an error that is about 0.02%. More details on this experiment can be found in the abstract of Grêt et al. (2002).

In an elastic medium, the waves propagate both as P and S waves, each with a distinct velocity. Moreover, a multiply scattered elastic wave is repeatedly converted from a P wave to an S wave and vice versa. This means that for

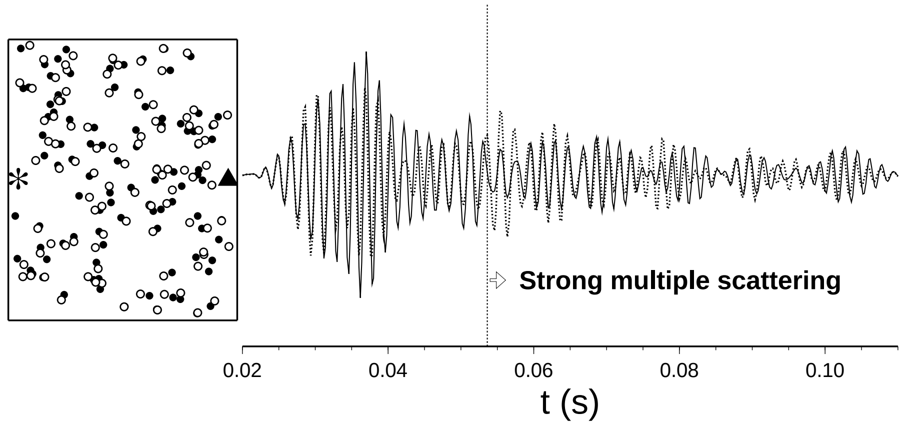

Fig. 1: Left: location of 100 scatterers before and after the perturbation (filled dots and open dots respectively) with the source (asterisk) and receiver location (triangle). For clarity, the perturbation in the locations of the scatterers is exaggerated with a factor 40. The scatterers are placed in an area of 40×80 m. The waveforms recorded before and after the perturbation at the receiver are shown on the right in solid and dotted lines, respectively.

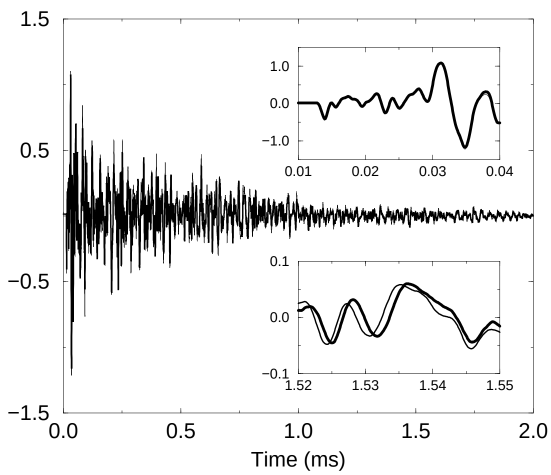

Fig. 2: Ultrasonic waveforms recorded in a granite sample for temperatures of $45°C$ (thin line) and $50°C$ (thick line) respectively. The insets show details of the waveforms around the first arrival (top) and in the late coda (bottom).

an elastic medium there is no unique wave velocity, and one needs to specify how the velocity change measured by Grêt et al. (2002) is related to the change in the change the velocities for P and S waves. Here I address this problem by considering the partitioning between P and S wave energy for elastic wave propagation. The reason for studying the ratio of the P wave energy to the S wave energy is that in coda wave interferometry the change in the medium is inferred from the cross-correlation of the waves before and after the change. This has the result that the contribution from each wave path to the change in the waveforms is weighted by the energy of the waves that propagate along that path (Snieder et al., 2002).

Partitioning of P and S energy

The partitioning of P wave energy to S wave energy has been studied by counting the number of P and S modes (Weaver, 1982), from evolution equations for P and S wave energy (Egle, 1981), from radiative transfer theory (Papanicolaou and Ryzhik, 1999; Trégourès and van Tiggelen, 2002), and from seismological observations(Campillo et al., 1999). The temporal evolution of the P and S wave energies in an elastic medium can be studied with a hierarchy of different methods. In radiative transfer theory (Papanicolaou and Ryzhik, 1999; Chandrasekhar, 1960; Wu, 1985; Ryzhik et al., 1997; Margerin et al., 1998) the spatial distribution of the intensity and the direction of propagation are treated as independent parameters. When the transition to diffusive wave propagation is made, the direction of energy propagation is related to the gradient of the intensity (Morse and Feshbach, 1953; Trégourès and van Tiggelen, 2002).

As an alternative theoretical description I use an even simpler model to account for the equilibration of P and S waves where the information regarding the spatial distribution and direction of the waves is discarded. In this model, the waves move around as balls. A ball is either in a P state, or in the S_1 or S_2 states that represent the two polarizations of S waves. The balls are a metaphore for units of energy. However, the balls should not be confused with the quanta of elastic wave propagation (phonons), they are nothing but a tool for keeping track of the distribution of energy among the different wave modes. The balls in the P state travel with the P wave velocity v_P, while the balls in the S states travel with the S wave velocity v_S. (We assume here that the medium is isotropic.) After each ball has propagated over a distance a, with a

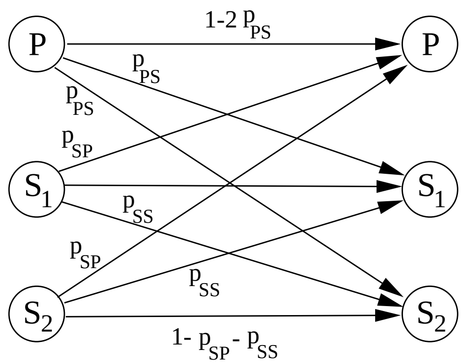

Fig. 3: Diagrammatic representation of the transition probabilities for the conversions of balls among the P state and the two S states.

certain probability it can convert to a ball of another state with transition probabilities as shown in Figure 3.

One can show that the number of balls in the different states for this model satisfy the following system of differential equations

$$\dot{N}_P = \frac{1}{a}\left(p_{SP} v_S N_{S_1} + p_{SP} v_S N_{S_2} - 2 p_{PS} v_P N_P\right)$$
$$\dot{N}_{S_1} = \frac{1}{a}\left(p_{SS} v_S N_{S_2} + p_{PS} v_P N_P - \left(p_{SP} + p_{SS}\right) v_S N_{S_1}\right)$$
$$\dot{N}_{S_2} = \frac{1}{a}\left(p_{SS} v_S N_{S_1} + p_{PS} v_P N_P - \left(p_{SP} + p_{SS}\right) v_S N_{S_2}\right) \tag{1}$$

By adding the three equations it follows that the total number of balls, and hence the total energy, is conserved. After equilibration the time derivatives in the system (1) vanish and the ratio of the P and S balls is given by

$$\left(\frac{N_P}{N_S}\right)_{eq} = \frac{p_{SP} v_S}{2 p_{PS} v_P} \; . \tag{2}$$

The ratio of the number of balls, and hence the ratio of the P and S wave energies depends on the ratio p_{SP}/p_{PS} of the transition probabilities. Aki (1992) gives a simple derivation that is based on reciprocity that explains why these probabilities are different. The transition probabilities are defined in expression (1) where they multiply terms such as $v_P N_P$. When N_P denotes the P wave energy, then $v_P N_P$ describes the energy flux of P waves. In scattering theory the scattering cross section is defined by the change in the energy flux. Therefore, the ratio p_{SP}/p_{PS} is defined by the ratio of the scattering cross sections for PS scattering and SP scattering respectively that is given by Margerin at el. (2000).

$$\frac{p_{SP}}{p_{PS}} = 2\frac{\sigma_{SP}}{\sigma_{PS}} = \left(\frac{v_S}{v_P}\right)^2 \; . \tag{3}$$

The factor 2 in the middle term is due to the fact that the transition probability in (1) is defined for each S wave polarization separately whereas the cross section σ_{SP} is defined for the total S wave energy. Inserting the ratio (3) in (2) and identifying the number of balls with energy, the following conditions follows for the equilibrium value of the P and S wave energy (Weaver, 1982; Egle, 1981; Weaver, 1982; Papanicolaou and Ryzhik, 1999; Trégourès and van Tiggelen, 2002):

$$\left(\frac{E_P}{E_S}\right)_{eq} = \frac{1}{2}\left(\frac{v_S}{v_P}\right)^3 \; . \tag{4}$$

For a Poisson medium, for which $v_P = \sqrt{3} v_S$, this ratio is given by $\left(E_P/E_S\right)_{eq} \approx 0.096$, which indicates that the S wave energy is much larger than the P wave energy. There are three reasons for this result. First, there are two S states compared to only one P state. Second, the P waves propagate faster than do the S waves, so the probability per unit time that a P wave is converted to an S wave is much larger than the reverse conversion. Third, the transition probability for the conversion from P to S is larger than from S to P, see expression (3).

Implication for coda wave interferometry

In coda wave interferometry, the change in the medium is inferred from the cross-correlation of the waveforms before and after the medium has changed (Snieder et al., 2002). Because the cross correlation is used, waveforms apppear quadratically in the final expressions. This means that the change in the waveforms is weighted by the intensity (energy) of these waveforms. After the P and S waves have equilibrated by multiple scattering the ratio of the energy is given by expresion (4).

When coda wave interferometry is used to measure small changes in the velocity, as in the example of figure 2, then one can show that the energy partitioning given by equation (4) leads to a change in the effective velocity of the waves that is given by

$$\left(\frac{\delta v}{v}\right)_{eff} = \frac{v_S^3}{2v_P^3 + v_S^3}\frac{\delta v_P}{v_P} + \frac{2v_P^3}{2v_P^3 + v_S^3}\frac{\delta v_S}{v_S} \; . \tag{5}$$

Coda wave interferometry thus constrains only the weighted average of the P and S velocity perturbation given by expression (5). For a Poisson medium,

$$\left(\frac{\delta v}{v}\right)_{eff} \approx 0.09\frac{\delta v_P}{v_P} + 0.91\frac{\delta v_S}{v_S} \; , \tag{6}$$

so that coda wave interferometry for elastic waves depends much more strongly on the relative perturbation of the S wave velocity than on that of the P wave velocity. This implies that coda wave interferometry provides information on changes in the S velocity rather than the P velocity.

A new tool for the instrumented oilfield

Coda wave interferometry has been applied to the ultrasound waveforms shown in figure 2 that were recorded in the laboratory. However, there is no reason why coda wave interferometry cannot be applied in a hydrocarbon reservoir. Landrø (1999) discusses the repeatability of VSP data and gives numerous examples of complex waveforms recorded in boreholes that show a spectacular repeatability. This repeatability depends on the accuracy of the positioning of the source and receiver for the baseline survey and the repeat measurements, but coda wave interferometry can also be used to determine the relative source and receiver locations at the two measurements (Snieder et al. 2002).

Mjaaland et al. (2001) show that the waves recorded in a borehole are extremely sensitive to small changes in the position of the gas-oil contact. Coda wave interferometry is a tool to quantify such a change in the subsurface from the change in the waveforms.

Presently, time-lapse seismics is mostly used by repeating surface seismic experiments with a time interval that is of the order of years. The limited sensitivity of the single scattered waves that are used to create images with these data ususally does not justify the expense to carry out the repeat survey at earlier times. Coda wave interferometry offers a much higher sensitivity for changes in the subsurface. With permanent down-hole sensors it provides the opportunity to monitor a reservoir for changes in its shear velocity that can be as small as 0.1%. This makes coda wave interferometry a technique that can play an important role in taking full advantage of the intelligent oil field.

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