Analysis of image gathers in factorized VTI media

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Summary

Image gathers generated after prestack depth migration are sensitive to the velocity field and are often used in migration velocity analysis for isotropic media. Here, we present an analytic and numerical study of P-wave image gathers in factorized transversely isotropic media with a vertical symmetry axis (VTI) and establish the conditions for flattening the gathers and positioning them at the true reflector depth. For homogeneous and factorized $v(z)$ media, flattening image gathers for any reflector dip requires accurate values of the zero-dip NMO velocity at the surface ($V_{\rm nmo}$), the vertical velocity gradient k_z, and the anellipticity coefficient η.

For factorized $v(x,z)$ media with a linear velocity variation in the x- and z-directions, the moveout on image gathers is controlled by four parameter combinations: $V_{\rm nmo}$ (x=z=0), k_z, η, and a combination of the horizontal velocity gradient k_x and the Thomsen parameter δ ($k_x\sqrt{1+2\delta}$). If too high a value of any of these four quantities is used in migration, the image gathers curve down (i.e., depth increases with offset) while a negative error causes the opposite sign of residual moveout. Flat image gathers in VTI media, however, do not guarantee the correct depth scale of the model because reflector depth depends on the vertical migration velocity. These results provide a basis for extending to VTI media conventional velocity-analysis methods operating with image gathers.

Introduction

Existing work on the application of image gathers in velocity analysis is largely restricted to isotropic subsurface models (e.g., Al-Yahya, 1987; Stork, 1991; Liu, 1995). For example, Liu (1995) developed an analytic approach for inverting the residual moveout on image gathers and computing corrections (updates) of the velocity model. However, ubiquitous evidence for the strong influence of seismic anisotropy on reflection moveout (e.g., Thomsen, 1986; Alkhalifah, 1996; Tsvankin, 2001) suggests that flattening of image gathers using purely isotropic models can often lead to erroneous velocity fields and distortions in migrated sections.

Here, we analyze P-wave image gathers for the most common anisotropic model – transverse isotropy with a vertical symmetry axis (VTI media). In contrast to a single scalar velocity responsible for isotropic P-wave propagation, the kinematic signatures of P-waves in VTI media are governed by three parameters: the vertical velocity V_{P0} and Thomsen's (1986) anisotropic coefficients ϵ and δ (Tsvankin and Thomsen, 1994; Tsvankin, 2001). The goal of this work is to study the residual moveout on image gathers caused by errors in these parameters and to establish the conditions needed to flatten and correctly position image gathers in homogeneous and factorized [$v(z)$ and $v(x,z)$] VTI media. We present weak-anisotropy approximations for moveout on image gathers and verify them by performing numerical tests for a representative set of VTI models.

Homogeneous VTI media

By applying the weak-anisotropy approximation (i.e., linearization in ϵ and δ), we obtained the following equation of an image gather for a horizontal reflector embedded in a homogeneous VTI medium:

$$z_M^2(h) = \gamma^2 z_T^2 - h^2 V_{P0,M}^2 \left(\frac{1}{V_{{\rm nmo},M}^2} - \frac{1}{V_{{\rm nmo},T}^2} \right) + 2\,\frac{h^4}{h^2+z_T^2} \left(\eta_M \frac{V_{{\rm nmo},M}^2}{V_{{\rm nmo},T}^2} - \eta_T \frac{V_{{\rm nmo},T}^2}{V_{{\rm nmo},M}^2} \right), \quad (1)$$

where the subscript T refers to the true model and M to the model used for migration, $z_M(h)$ is the migrated depth for the half-offset h, z_T is the true depth, $\gamma \equiv V_{P0,M}/V_{P0,T}$ is the ratio of the migration and true vertical velocities, $V_{\rm nmo} = V_{P0}\sqrt{1+2\delta}$ is the zero-dip normal-moveout (NMO) velocity, and $\eta \equiv (\epsilon-\delta)/(1+2\delta)$ is the Alkhalifah–Tsvankin (1995) anellipticity parameter responsible for time processing of P-wave data in VTI media.

Equation (1) shows that the moveout of horizontal events on an image gather is fully controlled by the parameters $V_{\rm nmo}$ and η. If the migration and true values are identical ($V_{{\rm nmo},M} = V_{{\rm nmo},T}$ and $\eta_M = \eta_T$), the migrated depth $z_M(h)$ [equation (1)] is independent of the offset h, and the image gather is flat (Figure 1a). The same conditions ($V_{{\rm nmo},T} = V_{{\rm nmo},M}$ and $\eta_M = \eta_T$) are sufficient to flatten image gathers for dipping reflectors (Figure 1b). However, since the migration was performed with the wrong value of V_{P0}, the migrated depths are scaled by the factor $\gamma \approx 0.90$.

In equation (1), the parameter η contributes only to the quartic moveout term h^4, which is also the case for the P-wave nonhyperbolic reflection moveout equation (Alkhalifah and Tsvankin, 1995; Tsvankin, 2001). Therefore, the influence of η is expected to be substantial only for relatively large offset-to-depth ratios exceeding unity. However, for dipping reflectors the residual moveout caused by an error in η is not confined to the large offset-to-depth ratios because η governs the dip dependence of NMO velocity (Alkhalifah and Tsvankin, 1995). Numerical tests

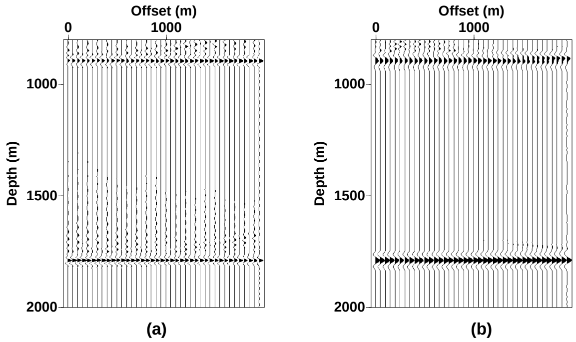

Fig. 1: Image gathers for (a) two horizontal reflectors and (b) two reflectors dipping at 30° embedded in a homogeneous VTI medium. The model parameters are $V_{P0,T} = 2000$ m/s, $\epsilon_T = 0.1$, and $\delta_T = -0.1$. Prestack depth migration was performed for a different model that has the correct $V_{\text{nmo},M} = V_{\text{nmo},T} = 1789$ m/s and $\eta_M = \eta_T = 0.25$ ($V_{P0,M} = 1789$ m/s, $\epsilon_M = 0.25$, and $\delta_M = 0$). In Figures 1-7 the maximum offset-to-depth ratio $x_{\max}/z = 2h_{\max}/z$ in the true model is equal to two for the shallow reflector and one for the deep reflector.

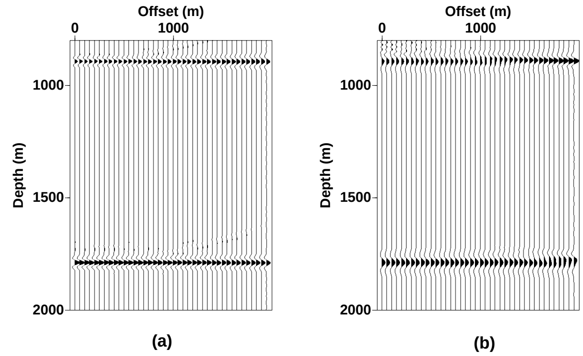

Fig. 2: Image gathers for (a) two horizontal reflectors and (b) two reflectors dipping at 30° embedded in a factorized VTI medium. The model parameters are $V_{P0,T} = 2000$ m/s, $k_{z,T} = 0.6$ 1/s, $\epsilon_T = 0.1$ and $\delta_T = -0.1$. Prestack depth migration was performed for a different model that has the correct $V_{\text{nmo},M} = V_{\text{nmo},T} = 1789$ m/s, $k_{z,M} = k_{z,T} = 0.6$ 1/s, and $\eta_M = \eta_T = 0.25$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.6$ 1/s, $\epsilon_M = 0.25$ and $\delta_M = 0$).

for the same true model and offset range as in Figure 1 show that for a horizontal reflector an error in η of 0.15 results in a depth error of 30 m at the largest offset. The error increases to 50 m for a reflector dipping at 30° and then decreases to 35 m for a dip of 45°.

The NMO velocity in equation (1) not only controls the quadratic term h^2, but also influences the quartic term h^4. Hence, an inaccurate value of V_{nmo} may cause significant residual moveout for the whole offset range. For the model in Figure 1, an error in V_{nmo} of 200 m/s causes a depth error of 80 m for a horizontal reflector, which decreases to 65 m for a 30° reflector, and to 45 m for a dip of 45°.

Factorized $v(z)$ VTI medium

Factorized VTI models have constant values of the anisotropic parameters and spatially varying vertical velocities of P- and S-waves. In this section we study $v(z)$ media in which the kinematic signatures of P-waves are defined by the velocity V_{P0} at the surface ($z = 0$), the constant vertical velocity gradient k_z and the parameters ϵ and δ.

Using the P-wave nonhyperbolic moveout equation for $v(z)$ VTI media (Tsvankin, 2001), it can be shown that reflection traveltimes of horizontal events in this factorized model are governed by the effective values of the NMO velocity and the parameter η:

$$v_{\text{nmo}}^2(t_0) = \frac{V_{P0}^2(1+2\delta)}{t_0\, k_z} \left[e^{k_z\, t_0} - 1\right], \qquad (2)$$

$$\hat{\eta}(t_0) = \frac{1}{8}\left\{\frac{(1+8\eta)\,(e^{2k_z\, t_0} - 1)\, k_z\, t_0}{2(e^{k_z t_0} - 1)^2} - 1\right\}, \qquad (3)$$

where t_0 is the zero-offset reflection time. Therefore, flattening a single image gather requires that $v_{\text{nmo},T}(t_0) = v_{\text{nmo},M}(t_0)$ and $\hat{\eta}_T(t_0) = \hat{\eta}_M(t_0)$.

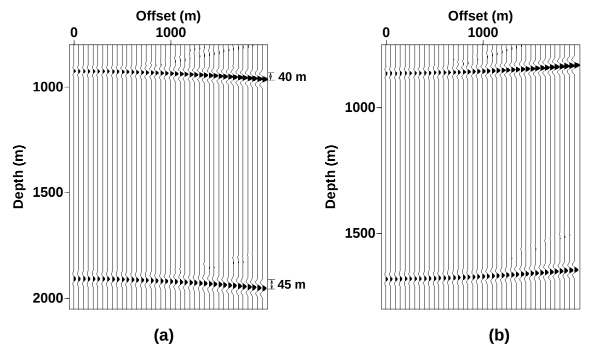

Fig. 3: Image gathers for two horizontal reflectors computed for inaccurate values of the vertical velocity gradient. The true parameters are $V_{P0,T} = 2000$ m/s, $k_{z,T} = 0.6$ 1/s, $\epsilon_T = 0.1$ and $\delta_T = -0.1$. Migration was done with the correct $V_{\text{nmo},M} = V_{\text{nmo},T}$ and $\eta_M = \eta_T$, but distorted values of $k_{z,M}$. (a) $k_{z,M} - k_{z,T} = 0.15$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.75$ 1/s, $\epsilon_M = 0.25$ and $\delta_M = 0$). (b) $k_{z,M} - k_{z,T} = -0.15$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.45$ 1/s, $\epsilon_M = 0.25$ and $\delta_M = 0$).

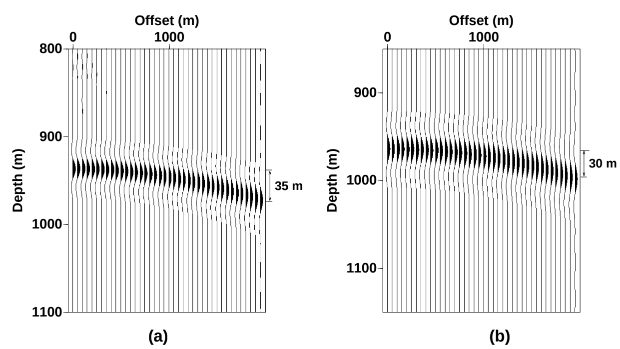

Fig. 4: Image gathers for reflectors dipping at (a) 30° and (b) 45° computed for a value of k_z overstated by 0.15. The true parameters are $V_{P0,T} = 2000$ m/s, $k_{z,T} = 0.6$ 1/s, $\epsilon_T = 0.1$ and $\delta_T = -0.1$. Migration was done with the parameters $V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.75$ 1/s, $\epsilon_M = 0.25$ and $\delta_M = 0$.

To flatten image gathers for the whole depth range occupied by the reflectors, the effective NMO velocity and the parameter $\hat{\eta}$ for the migration and true models should be equal at all zero-offset times t_0. This implies that both the exponential term in equation (2) and the coefficient in front of it should be preserved in the migration model. Hence, migration should be done with the correct values of the vertical velocity gradient and the NMO velocity at the surface: $k_{z,M} = k_{z,T}$ and $V_{\mathrm{nmo},M} = V_{P0,M}\sqrt{1+2\delta_M} = V_{P0,T}\sqrt{1+2\delta_T} = V_{\mathrm{nmo},T}$. Taking into account that $k_{z,M}$ has to be equal to $k_{z,T}$, the condition $\hat{\eta}_M = \hat{\eta}_T$ can be satisfied at all t_0 only if $\eta_M = \eta_T$ [see equation (3)].

We conclude that to flatten gathers of all horizontal events in a factorized $v(z)$ medium, three conditions need to be satisfied: (1) $V_{\mathrm{nmo},M} = V_{\mathrm{nmo},T}$, (2) $k_{z,M} = k_{z,T}$, and (3) $\eta_M = \eta_T$ (Figure 2a). Figure 2b confirms that these conditions are valid for dipping reflectors as well. The reflector depths, however, are scaled by the factor (≈ 0.9) equal to the ratio of the migrated and the true average vertical velocities. Note that the residual moveout caused by an error in k_z decreases with dip, but increases with depth (Figures 3 and 4).

Factorized $v(x,z)$ VTI medium

Factorized $v(x,z)$ VTI media with linear velocity variation can be described by five independent parameters: the vertical velocity $V_{P0} = V_{P0}(0,0)$ defined at zero depth and lateral location $x = 0$, the velocity gradients k_x and k_z responsible for the linear variation of V_{P0} in the x– and z–directions, respectively, and the parameters ϵ and δ (for P-waves). In principle, we can treat a factorized $v(x,z)$ model as being composed of narrow vertical strips of $v(z)$ factorized media discussed above. Therefore, it is natural to assume that image gathers in $v(x,z)$ media will be flat if $v_{\mathrm{nmo},M}[x,t_0(x)] = v_{\mathrm{nmo},T}[x,t_0(x)]$ and $\hat{\eta}_M[x,t_0(x)] = \hat{\eta}_T[x,t_0(x)]$ not only for all vertical times t_0, but also for all coordinates x.

The influence of weak lateral velocity variation on the NMO velocity in horizontally layered anisotropic media was discussed by Grechka and Tsvankin (1999). They showed that the NMO ellipse has to be corrected for lateral velocity variation by including a term dependent on the *second* derivatives of the vertical velocity with respect to the horizontal coordinates. For P-waves in the 2-D model considered here, the equation of Grechka and Tsvankin (1999) takes the form

$$v_{\mathrm{nmo}}^{-2}(x,z) = v_{\mathrm{nmo,hom}}^{-2}(x,z) + \frac{\tau_0(x,z)}{3}\,\frac{\partial^2 \tau_0(x,z)}{\partial x^2}, \qquad (4)$$

where $v_{\mathrm{nmo,hom}}$ is the NMO velocity in the background laterally homogeneous medium at the coordinate x, z is the reflector depth and $\tau_0(x,z)$ is the one-way zero-offset reflection traveltime. Since in our model $V_{P0}(x,z)$ [and, for weak lateral velocity variation, $\tau_0(x,z)$] is a linear function of x, $v_{\mathrm{nmo}}(x,z)$ from equation (4) is simply equal to the NMO velocity in the background factorized $v(z)$ medium at the lateral location x. The background

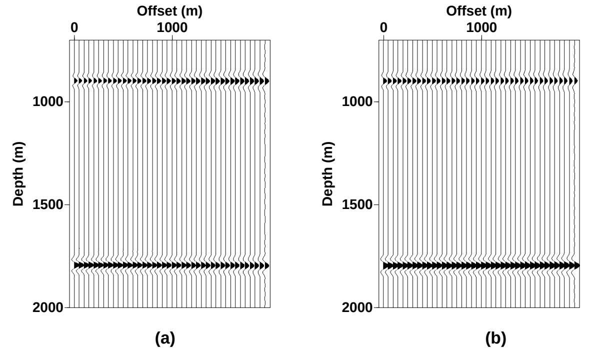

Fig. 5: Image gathers for (a) two horizontal reflectors and (b) two reflectors dipping at 30° embedded in a factorized $v(x,z)$ VTI medium. The true model parameters are $V_{P0,T} = 2000$ m/s, $k_{z,T} = 0.6$ 1/s, $k_{x,T} = 0.2$ 1/s, $\epsilon_T = 0.1$, and $\delta_T = -0.1$. Prestack depth migration was performed for a different model that has with the correct $V_{\mathrm{nmo},M} = V_{\mathrm{nmo},T} = 1789$ m/s, $k_{z,M} = k_{z,T} = 0.6$ 1/s, $k_{x,M}\sqrt{1+2\delta_M} = k_{x,T}\sqrt{1+2\delta_T} = 0.18$ 1/s, and $\eta_M = \eta_T = 0.25$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.6$ 1/s, $k_{x,M} = 0.18$ 1/s, $\epsilon_M = 0.25$ and $\delta_M = 0$). The gathers in Figures 5-6 are located 6000 m away from $x = 0$.

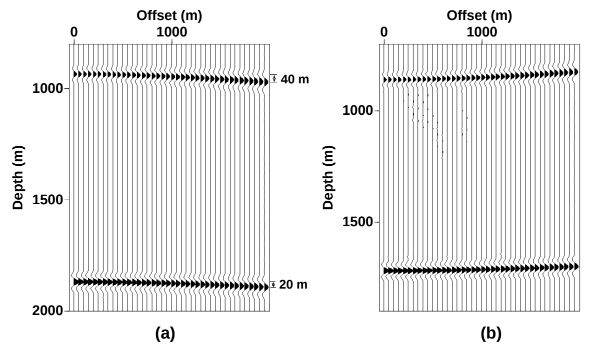

Fig. 6: Image gathers for two horizontal reflectors computed for inaccurate values of $k_x\sqrt{1+2\delta}$, but correct $V_{\mathrm{nmo},M} = V_{\mathrm{nmo},T} = 1789$ m/s, $k_{z,M} = k_{z,T}$, and $\eta_M = \eta_T = 0.25$. (a) $k_{x,M}\sqrt{1+2\delta_M} - k_{x,T}\sqrt{1+2\delta_T} = 0.02$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.6$ 1/s, $k_{x,M} = 0.20$ 1/s, $\epsilon_M = 0.25$, and $\delta_M = 0$). (b) $k_{x,M}\sqrt{1+2\delta_M} - k_{x,T}\sqrt{1+2\delta_T} = -0.02$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.6$ 1/s, $k_{x,M} = 0.16$ 1/s, $\epsilon_M = 0.25$, and $\delta_M = 0$).

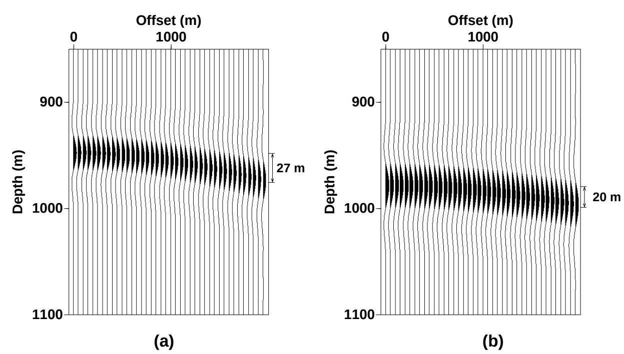

Fig. 7: Image gathers for reflectors dipping at (a) 30° and (b) 45° computed for a value of $k_x\sqrt{1+2\delta}$ overstated by 0.02, but correct $V_{\mathrm{nmo},M} = V_{\mathrm{nmo},T} = 1789$ m/s, $k_{z,M} = k_{z,T}$, and $\eta_M = \eta_T = 0.25$ ($V_{P0,M} = 1789$ m/s, $k_{z,M} = 0.6$ 1/s, $k_{x,M} = 0.2$ 1/s, $\epsilon_M = 0.25$, and $\delta_M = 0$).

NMO velocity can be represented as [equation (2)]

$$v^2_{\mathrm{nmo,hom}}(x) \;=\; \frac{V^2_{P0}(x)(1+2\delta)}{2\,t_{\mathrm{hom}}(x)\,k_z}\,[e^{2k_z t_{\mathrm{hom}}(x)}-1]\,, \quad (5)$$

where $V_{P0}(x) = V_{P0} + k_x x$, $t_{\mathrm{hom}}(x) = z_T/\hat{V}_{P0}(x)$, and $\hat{V}_{P0}(x,z)$ is the average vertical velocity above the reflector.

According to our results for the $v(z)$ model, v_{nmo} of horizontal events are equal to the true NMO velocities for all vertical times t_0 if the migration is based on the correct values of the vertical velocity gradient k_z and the NMO velocity at the surface $[V_{P0}(x)\sqrt{1+2\delta}]$. Hence, $k_{z,M}$ should be equal to $k_{z,T}$ and, as follows from equation (5),

$$(V_{P0,M} + k_{x,M}\,x)\sqrt{1+2\delta_M} = (V_{P0,T} + k_{x,T}\,x)\sqrt{1+2\delta_T}\,. \quad (6)$$

Therefore, $k_{x,M}\sqrt{1+2\delta_M} = k_{x,T}\sqrt{1+2\delta_T}$ and $V_{\mathrm{nmo},M} = V_{P0,M}\sqrt{1+2\delta_M} = V_{P0,T}\sqrt{1+2\delta_T} = V_{\mathrm{nmo},T}$. Also, because $k_{z,M} = k_{z,T}$, setting $\hat{\eta}_M[x,t_0(x)] = \hat{\eta}_T[x,t_0(x)]$ for all zero-offset times implies that $\eta_M = \eta_T$.

Figure 5 confirms that setting $V_{\mathrm{nmo},M} = V_{\mathrm{nmo},T}$, $k_{z,M} = k_{z,T}$, $\eta_M = \eta_T$, and $k_{x,T}\sqrt{1+2\delta_T} = k_{x,M}\sqrt{1+2\delta_M}$ flattens image gathers associated with not only horizontal, but also dipping reflectors. Also, as was the case with k_z and V_{nmo}, an error in k_x causes residual moveout that decreases with dip (Figures 6 and 7).

Numerical tests also show that the magnitude of residual moveout for a fixed error in V_{nmo} is smaller in factorized media than in homogeneous media for the same $V_{P0,M}$, $V_{P0,T}$, ϵ_M, ϵ_T, δ_M, and δ_T. Likewise, the residual moveout for a fixed error in k_z is smaller in a $v(x,z)$ medium than in the corresponding laterally homogeneous $v(z)$ model. In contrast, lateral velocity gradients tend to increase the residual moveout caused by an error in η, and distorted values of η for $v(x,z)$ media may lead to measurable residual moveout of horizontal events for offset-to-depth ratios close to unity.

Discussion and conclusions

In conventional seismic processing for isotropic media, image gathers proved to be a convenient tool for refining velocity models as well as for a quick qualitative assessment of the accuracy of velocity analysis. If the medium is anisotropic, reflection moveout is governed by several anisotropic parameters, and the interpretation of image gathers becomes much more complicated. Here, we presented an analytic and numerical study of P-wave image gathers in homogeneous and factorized [$v(z)$ and $v(x,z)$] VTI media.

Flattening image gathers of events with any dip in $v(z)$ media requires the correct values of the NMO velocity at the surface ($V_{P0}\sqrt{1+2\delta}$), the anisotropic coefficient η and the vertical velocity gradient k_z. The influence of errors in k_z on the residual moveout decreases with dip (as is the case for errors in V_{nmo}) but increases with reflector depth. Moveout on image gathers in $v(x,z)$ media is controlled by the three factors listed above and a combination of the horizontal velocity gradient k_x and Thomsen parameter δ ($k_x\sqrt{1+2\delta}$).

Even if all four parameter combinations for $v(x,z)$ media have been resolved, separation of the lateral velocity variation from the anisotropic coefficients cannot be accomplished in a unique fashion. It is important to emphasize, however, that the vertical velocity gradient k_z is constrained not only in the $v(z)$ model, but also in laterally heterogeneous media. As a result, although the inversion of P-wave data for the vertical velocity and Thomsen coefficients will suffer from inherent ambiguities, minimal *a priori* assumptions may be sufficient to remove the trade-offs between the VTI parameters. For example, if the vertical velocity V_{P0} is known at any single surface location (which is quite possible), then the inverted gradient k_z can be used to reconstruct the function $V_{P0}(z)$ and find the depth scale of the model. Also, in this case the anisotropic parameter δ can be determined from the NMO velocity at the surface ($V_{P0}\sqrt{1+2\delta}$), and, in turn, used to estimate the horizontal gradient k_x from the combination $k_x\sqrt{1+2\delta}$.

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