Parameter estimation in orthorhombic media using multicomponent reflection data

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Summary

Orthorhombic media with a horizontal symmetry plane adequately describe seismic signatures recorded over many naturally fractured reservoirs. Here we show that for a range of orthorhombic models wide-azimuth traveltimes of PP and SS (the fast S_1 and slow S_2) reflections can be inverted for Tsvankin's anisotropic parameters and the azimuths of the vertical symmetry planes. If shear waves are not excited, SS traveltimes can be found from PP and PS (converted-wave) data, which makes the method applicable to offshore surveys.

The feasibility of parameter estimation is strongly dependent on reflector dip and orientation. For a horizontal reflector beneath a single orthorhombic layer, the vertical velocities and reflector depth cannot be found from reflection traveltimes in a unique fashion. If the reflector is dipping, the inversion becomes possible provided the dip plane deviates from the vertical symmetry planes of the orthorhombic layer above it.

To perform velocity analysis for orthorhombic models composed of homogeneous layers separated by plane or curved interfaces, we apply multicomponent stacking-velocity tomography. The tomographic algorithm, which operates with the NMO ellipses, zero-offset traveltimes and reflection time slopes, is designed to estimate the interval anisotropic parameters and the shapes of the reflecting interfaces. Numerical tests show that for layered orthorhombic media it is preferable to have *a priori* knowledge of the vertical velocities to avoid instability in the inversion of noise-contaminated reflection data.

Introduction

Estimation of anisotropic velocity fields from reflection traveltimes of seismic waves is a nonlinear inverse problem that becomes increasingly more complicated for lower medium symmetries. Even for the simplest anisotropic model – transverse isotropy with a vertical symmetry axis (VTI) – PP-wave moveout alone is generally insufficient for estimating Thomsen anisotropic coefficients ϵ and δ. Therefore, it is imperative to combine PP data with shear information typically provided by PS-waves.

To avoid difficulties in converted-wave processing caused by polarity reversals, conversion-point dispersal, and moveout asymmetry, PP and PS data can be used to obtain the traveltimes of the pure (nonconverted) SS-waves for the same reflector. The method of Grechka and Tsvankin (2001) makes it possible to compute SS traveltimes from those of the PP- and PS-waves prior to velocity analysis (although correlation of the PP and PS reflections is required). Then one can estimate SS-wave normal-moveout (NMO) velocities (in 2-D) or NMO ellipses (in 3-D) and employ them, along with the PP data, in anisotropic parameter estimation. A practical velocity-analysis algorithm (we call it "stacking-velocity tomography") based on these ideas was suggested for TI media by Grechka et al. (2001) and successfully applied to field data by Grechka et al. (2002). Here, we extend stacking-velocity tomography to more complicated orthorhombic media believed to be typical for naturally fractured oil and gas reservoirs (Bakulin et al., 2000).

Tomographic parameter estimation using multicomponent data

The first step of our processing sequence is picking of the traveltimes of PP- and PS-waves on prestack data, which typically requires application of Alford rotation to separate the split converted modes PS_1 and PS_2. Then the method of Grechka and Tsvankin (2001) is employed to compute the traveltimes t_{S1} and t_{S2} of the pure shear-wave reflections S_1 and S_2 from the same interface. The times t_{S1} and t_{S2} are symmetric with respect to the common midpoint, so they can be processed by means of hyperbolic moveout analysis to obtain the shear-wave NMO ellipses (or, in 2-D, NMO velocities).

The PP- and SS-wave NMO ellipses, along with the corresponding horizontal projections of the slowness vectors $\boldsymbol{p}_Q$ (i.e., the reflection slopes on zero-offset time sections; $Q = P$, S_1, or S_2) and zero-offset traveltimes τ_Q are used as the input data for anisotropic stacking-velocity tomography. The data vector for an N-layered orthorhombic medium is

$$\boldsymbol{d}(Q,\, \boldsymbol{Y},\, n) \equiv \{\tau_Q(\boldsymbol{Y},\, n),\, \boldsymbol{p}_Q(\boldsymbol{Y},\, n),\, \boldsymbol{W}_Q(\boldsymbol{Y},\, n)\}\,, \quad (1)$$

where $\boldsymbol{Y} = [Y_1, Y_2]$ is the CMP location, $n = 1, 2, ..., N$ is the reflector number and $\boldsymbol{W}$ is the 2×2 matrix describing the NMO ellipse.

The main advantages of restricting the inversion to the hyperbolic portion of reflection moveout is high computational efficiency (only one zero-offset ray for each reflection event needs to be traced) and the analytic insight into constrained parameter combinations provided by the NMO ellipses. The tomographic algorithm is based on a two-step procedure for estimating the model vector $\boldsymbol{m}$ that contains the interval parameters of orthorhombic media and the coefficients of the polynomials specifying the model interfaces. For a given set of trial interval anisotropic parameters, the zero-offset traveltimes and reflection slopes are used to compute the one-way zero-offset rays and reconstruct the medium interfaces (i.e., build the trial model in depth). Then the interval parameters are obtained by minimizing the differences between the computed and measured NMO ellipses $\boldsymbol{W}$ for all available reflection events.

Parameter estimation in a single orthorhombic layer

Horizontal reflector

The semi-axes of NMO ellipses for all reflection modes in a horizontal orthorhombic layer are co-oriented with the vertical symmetry planes and have the following simple form (Grechka et al., 1999):

$$\frac{1}{V^2_{Q,\mathrm{nmo}}(\alpha)} = \frac{\cos^2\alpha}{\left[V^{(2)}_{Q,\mathrm{nmo}}\right]^2} + \frac{\sin^2\alpha}{\left[V^{(1)}_{Q,\mathrm{nmo}}\right]^2}\,; \qquad (2)$$

α is the azimuth with respect to the $[x_1,\, x_3]$-plane. Because of the kinematic equivalence between the symmetry planes of orthorhombic and VTI media, the semi-axes of the NMO ellipses ($V^{(i)}_{Q,\mathrm{nmo}}$) can be obtained by adapting the known VTI equations (Tsvankin, 2001).

Seismic signatures in orthorhombic media can be conveniently described in terms of the two vertical velocities, V_{P0} and V_{S0}, and Thomsen-style anisotropic coefficients $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$, and $\gamma^{(1,2)}$ introduced by Tsvankin (1997, 2001). As shown by Grechka et al. (1999), the coefficients $\epsilon^{(1,2)}$, $\delta^{(1,2)}$ and $\gamma^{(1,2)}$ can be estimated in a unique fashion only if either one of the vertical velocities or the reflector depth is known. The parameter $\delta^{(3)}$ has no influence on the NMO ellipses in a horizontal layer and, therefore, cannot be determined from conventional-spread reflection moveout.

Plane dipping reflector

First, we assume that the dip plane of the reflector coincides with the $[x_1,\, x_3]$ symmetry plane of the orthorhombic medium. Since the slowness vector of the zero-offset ray for any pure mode is orthogonal to the reflector, the horizontal slowness projection at zero offset is confined to the dip plane. Therefore, reflection time slopes can be used to find the azimuthal direction of both the reflector and the $[x_1,\, x_3]$ symmetry plane.

Then, the elements of the vector $\boldsymbol{m}$ to be estimated from moveout data include nine medium parameters (V_{P0}, V_{S0}, $\epsilon^{(1)}$, $\epsilon^{(2)}$, $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\gamma^{(1)}$, and $\gamma^{(2)}$), the reflector dip ϕ, and the distance D between the CMP and the reflector. The vector $\boldsymbol{m}$ has to be found from the eighteen-component data vector (1) that has six zero elements,

$$p_{2,\,Q} = 0 \quad \text{and} \quad W_{12,\,Q} = 0 \quad (Q = P,\ S_1,\ \text{or}\ S_2)\,, \qquad (3)$$

because $[x_1,\, x_3]$ is the symmetry plane for the whole model. The remaining twelve elements include three zero-offset traveltimes τ_Q, three horizontal slownesses $p_{1,\,Q}$ and six diagonal elements of the matrices $\boldsymbol{W}_Q$ (which correspond to the dip- and strike-components of the NMO velocities). However, the expressions for the traveltimes τ_Q are not independent because it can be shown that $\tau_Q = (D\, p_{1,\,Q})/\sin\phi$, and the zero-offset traveltimes τ_P, τ_{S1}, and τ_{S2} constrain only *one* combination of the model parameters – $D/\sin\phi$. Therefore, the number of independent equations to be solved for the 11 unknown parameters reduces from 12 to 10. Clearly, this inverse problem does not have a unique solution.

Next, consider a model in which the dip plane of the reflector *does not* coincide with either vertical symmetry plane, and each NMO ellipse generally has a different orientation. That increases the number of independent components of the data vector (1) and can make the inversion unambiguous. Still, the inverse problem is nonlinear, and the feasibility of parameter estimation has to be assessed by numerical testing on noise-contaminated data. Our analysis shows that the inversion for a range of typical orthorhombic models is not only unique but also quite stable. The results in Figure 1 are obtained from the data $\boldsymbol{d} = \left\{\tau_Q,\, \boldsymbol{p}_Q,\, \boldsymbol{W}_Q\right\}$ generated for all three modes (P, S_1 and S_2) at a single CMP location. The traveltimes τ_Q, the horizontal slowness components $\boldsymbol{p}_Q$, and the NMO ellipses $\boldsymbol{W}_Q$ were contaminated by Gaussian noise with standard deviations of 1% for τ_Q and $\boldsymbol{p}_Q$ and 2% for $\boldsymbol{W}_Q$. The parameter estimation was repeated 100 times for different realizations of the noise using a nonlinear least-squares algorithm. The maximum standard deviation in the recovered anisotropic coefficients does not exceed 0.025 (the value for $\delta^{(1)}$ in Figure 1), which means that the inversion is sufficiently stable. The standard deviations for the parameters not displayed in Figure 1 are small as well (1.2% and 0.6% for the vertical velocities V_{P0} and V_{S0}, respectively, and less than 1° for the angles β, ϕ and ψ).

For the model from Figure 1, the dip plane of the reflector deviates by 30° from the nearest vertical symmetry plane, which ensures the high stability of the inversion result. A reduction in the difference $|\psi - \beta|$ leads to a rapid increase in the errors (i.e., increasing standard deviations). Likewise, the inversion becomes unstable if the reflector dip $\phi < 15°$; for subhorizontal reflectors the anisotropic coefficients cannot be estimated without *a priori* knowledge of the vertical velocities or reflector depth (see above). A similar dependence of the quality of moveout-inversion results on reflector dip was found for VTI media by Tsvankin and Grechka (2000).

Curved reflector

If the reflector beneath an orthorhombic layer has arbitrary spatially varying curvature, the uniqueness of the inversion procedure is much more difficult to evaluate analytically. The feasibility of estimating the medium parameters in this case strongly depends on the shape and parameterization of the reflector and the spatial distribution of the common midpoints. Extensive testing on synthetic traveltimes shows that reflector curvature does not pose any serious problems for the parameter estimation in a single orthorhombic layer, provided the

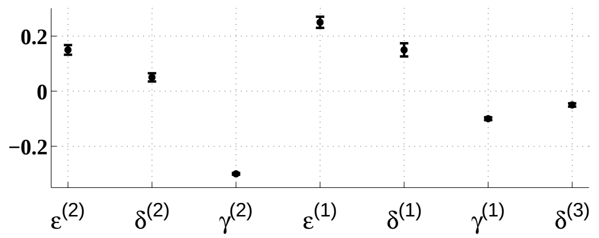

Fig. 1: Inversion results for a single orthorhombic layer above a dipping reflector. The exact values of the anisotropic coefficients are marked by the dots; the bars correspond to the ± standard deviation in each parameter. The reflector dip ϕ is 30°, the azimuth of the dip plane $\psi = 0°$, and the azimuth of the $[x_1, x_3]$ symmetry plane of the layer $\beta = 60°$.

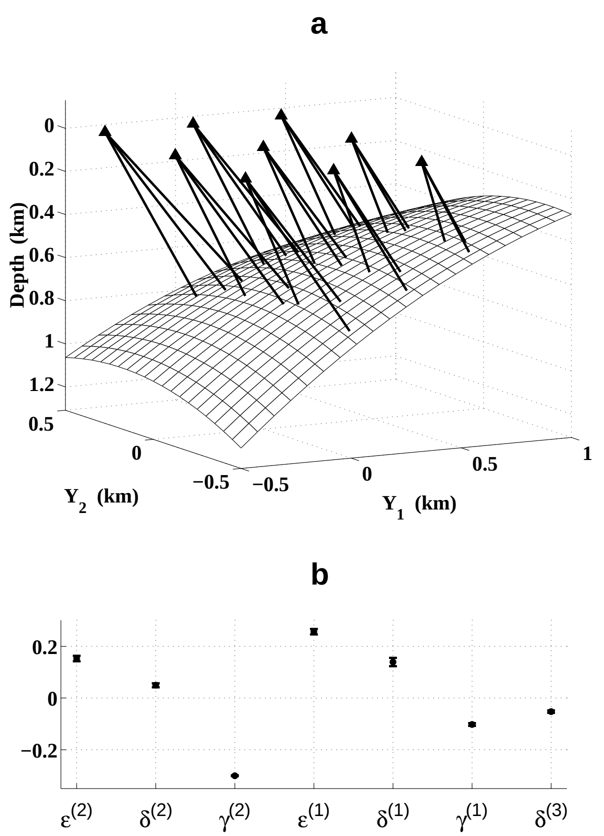

Fig. 2: (a) Zero-offset rays of the P- S_1- and S_2-waves reflected from a curved interface below an orthorhombic layer and (b) the estimated anisotropic parameters. The $[x_1, x_3]$ symmetry plane makes an angle of 60° with the Y_1-axis. The data were contaminated with Gaussian noise that had the same standard deviations as in Figure 1.

reflecting interface contains dips over 15° and the dip directions deviate from the vertical symmetry planes. Typical inversion results for noise-contaminated data τ_Q, $\boldsymbol{p}_Q$ and $\boldsymbol{W}_Q$ from nine CMP locations are displayed in Figure 2. As in the previous test, the inversion was repeated 100 times to study the distribution of the estimated parameters. The standard deviations in Figure 2b are even smaller than those in Figure 1. The increase in stability for the curved reflector in Figure 2 is explained by the relatively wide range of reflector dips and azimuths sampled by the zero-offset rays. This advantage of curved interfaces, of course, can be fully exploited only if the medium above the reflector can be treated as homogeneous.

Stacking-velocity tomography in layered media

The methodology of stacking-velocity tomography helps to extend parameter estimation to more realistic models composed of homogeneous orthorhombic layers separated by plane or curved interfaces. If *noise-free* data $\left\{\tau_Q, \boldsymbol{p}_Q, \boldsymbol{W}_Q\right\}$ ($Q = P, S_1, S_2$) for all interfaces are available, and the reflectors satisfy the conditions established for a single layer (i.e., the dips exceed 15-20° and the reflector azimuths sufficiently deviate from those of the vertical symmetry planes), the tomographic algorithm can recover the interval orthorhombic parameters along with the shapes of the interfaces. However, due to error accumulation with depth and a lower sensitivity of surface data to the parameters of deeper layers, even moderate noise causes substantial errors in the interval parameters. To make the inversion sufficiently stable for field-data applications, we assume that the interval vertical velocities ($V_{P0,\,n}$ or $V_{S0,\,n}$) are known in all layers $n = 1, \ldots, N$; alternatively, it is possible to specify the thickness of each layer.

Application of our tomographic algorithm to a layered model with curved interfaces is illustrated by Figure 3. The input data were computed for nine common midpoints (Figure 3a) and contaminated by Gaussian noise with the same standard deviations as in the previous examples. Although, as expected, parameter estimation becomes less stable with depth, knowledge of the vertical velocities ensures that all anisotropic coefficients are estimated with acceptable accuracy (Figures 3b,c,d). The standard deviations even in the deepest layer do not exceed 0.1 – the value for $\delta_3^{(1)}$ in Figure 3d. The azimuths of the symmetry planes and the shapes of the interfaces are also tightly constrained; the standard deviations in β_1, β_2 and β_3 are 1.0°, 1.6° and 4.6°, respectively.

Discussion and conclusions

We demonstrated that 3-D multi-azimuth reflection moveout of PP-waves and two split SS-waves (possibly computed from PS-waves) on conventional-length spreads can be used to reconstruct orthorhombic velocity fields in the depth domain. For a single layer, the theoretical conditions needed to make this inversion unique require that the reflector has at least a mild

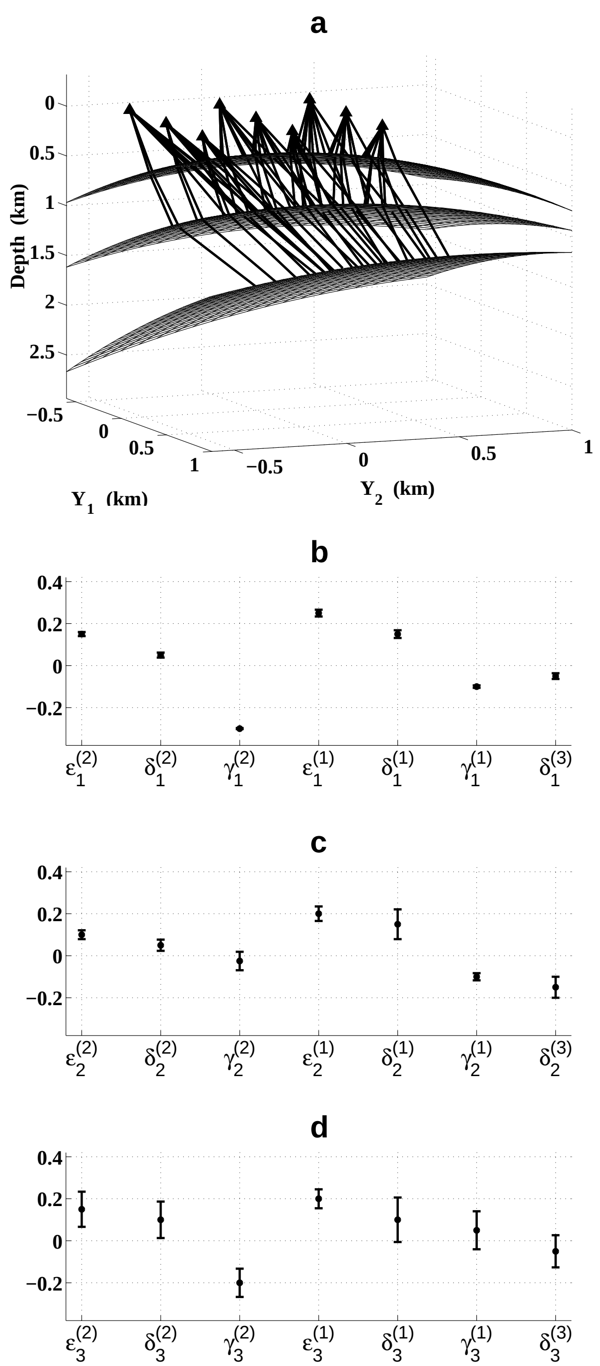

Fig. 3: (a) Reflected zero-offset rays of the P-, S_1- and S_2-waves in a three-layer orthorhombic medium, and (b,c,d) the results of stacking-velocity tomography. The vertical velocities $V_{P0,1} = 1.0$ km/s, $V_{S0,1} = 0.6$ km/s, $V_{P0,2} = 1.5$ km/s, $V_{S0,2} = 0.8$ km/s, and $V_{P0,3} = 1.8$ km/s, $V_{S0,3} = 1.0$ km/s were assumed to be known. The azimuths of the symmetry planes are $\beta_1 = 40°$, $\beta_2 = 50°$ and $\beta_3 = 10°$.

dip ($\phi > 15 - 20°$) and the azimuth of the dip plane be different by 10° or more from those of the vertical symmetry planes of the overburden. If the reflector is horizontal, the vertical velocities and the anisotropic parameters $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$ and $\gamma^{(1,2)}$ cannot be obtained from the reflection traveltimes of PP- and SS-waves (or PS-waves) alone (Grechka et al, 1999).

Synthetic tests on noise-contaminated data for typical orthorhombic models show that for a single layer noise does not get amplified by the inversion algorithm. In contrast, the estimated anisotropic parameters in layered orthorhombic media often are significantly distorted as a result of error accumulation with depth and a reduced sensitivity of surface data to the parameters of deeper layers. We suggested to avoid this instability in stacking-velocity tomography by supplementing the surface data with the vertical velocities of the P- and S-waves measured from well logs or check shots. Such an assumption may not be too restrictive in reservoir characterization because seismic inversion for orthorhombic anisotropy would typically be performed at later stages of the reservoir development. Constraining vertical velocities makes the inversion sufficiently stable for application in quantitative fracture characterization for orthorhombic reservoir models (Bakulin et al., 2000).

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