Quartic moveout coefficient: 3-D description and application to tilted TI media

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Summary

Nonhyperbolic (long-spread) moveout provides essential information for a number of seismic inversion/processing applications, particularly for parameter estimation in anisotropic media. Here, we present a 3-D analytic expression for the quartic moveout coefficient A_4 which is responsible for the magnitude of nonhyperbolic moveout of pure (non-converted) modes. Our result takes into account reflection-point dispersal on irregular interfaces and is valid for arbitrarily anisotropic, heterogeneous media.

This general equation is used to study azimuthally varying nonhyperbolic moveout of P-waves in a dipping transversely isotropic (TI) layer with an arbitrary tilt ν of the symmetry axis. The weak-anisotropy approximation for A_4 is proportional to the anellipticity coefficient $\eta \approx \epsilon - \delta$ and does not depend on the individual values of the Thomsen parameters ϵ and δ. The azimuthal variation of the quartic coefficient is governed by the tilt ν and reflector dip ϕ and has a much more complicated character than the NMO ellipse. If the symmetry axis is orthogonal to the reflector, which is typical for thrust-and-fold belts, the dip-line quartic coefficient rapidly decreases with ϕ, while the strike-line A_4 for any dip is defined by the well-known expression for a horizontal VTI (TI with a vertical symmetry axis) layer. Therefore, the magnitude of nonhyperbolic moveout for this model (and, typically, for other TI media as well) is largest near the reflector strike. The high sensitivity of the quartic moveout coefficient to the parameter η and tilt ν can be exploited in the anisotropic inversion of wide-azimuth, long-spread P-wave data.

Introduction

The influence of heterogeneity or anisotropy causes deviations from hyperbolic reflection moveout which sometimes cannot be ignored even for offsets-to-depth ratios close to unity. A detailed overview of nonhyperbolic moveout analysis in anisotropic media can be found in Tsvankin (2001). Most existing algorithms operating with nonhyperbolic moveout are based on the equation of Tsvankin and Thomsen (1994) that can be applied to media with any symmetry if the quartic moveout coefficient A_4 and the normal-moveout (NMO) velocity $V_{\rm nmo}$ are known. For example, Al-Dajani and Tsvankin (1998) derived the coefficient A_4 for a TI medium with a horizontal symmetry axis (HTI) and extended the Tsvankin-Thomsen equation to horizontally layered HTI models. A more general approach to the analytic description of A_4 that accounts for reflection-point dispersal at dipping or curved interfaces was developed by Fomel and Grechka (2001).

Here, we introduce a 3-D expression for the quartic moveout coefficient and use it to describe nonhyperbolic moveout of P-waves for TI media with an arbitrary orientation of the symmetry axis. Models with the symmetry axis tilted away from the vertical (TTI, or tilted TI media) are typical for fold-and-thrust belts such as the Canadian Foothills and for sediments near the flanks of salt domes (e.g., Isaac and Lawton, 1999; Tsvankin, 2001).

Nonhyperbolic moveout equation

To describe reflection traveltimes of pure (non-converted) modes for the whole offset range used in seismic exploration, Tsvankin and Thomsen (1994) suggested the following equation:

$$t^2(X) = t_0^2 + \frac{X^2}{V_{\rm nmo}^2} + \frac{A_4 X^4}{1 + A X^2}\,, \qquad (1)$$

where t_0 is the zero-offset traveltime, X is the source-receiver offset,

$$A_4 = \frac{1}{2}\,\frac{d}{d(X^2)}\left[\frac{d(t^2)}{d(X^2)}\right]\Bigg|_{X=0}\,, \text{ and } A = \frac{A_4}{V_{\rm hor}^{-2} - V_{\rm nmo}^{-2}}\,;$$

$V_{\rm hor}$ is the horizontal group velocity. The first two terms in equation (1) describe the hyperbolic part of the moveout curve, while the quartic coefficient A_4 is primarily responsible for nonhyperbolic moveout. The coefficient A makes the equation convergent at infinitely large offsets.

Equation (1) was originally developed for VTI media, but its generic form makes it suitable for azimuthally anisotropic models as well (Al-Dajani and Tsvankin, 1998). The effective $V_{\rm nmo}$ in heterogeneous, anisotropic media can be found using the formalism for NMO ellipses developed by Grechka et al. (1999). Therefore, the key issue in applying equation (1) to a particular model is to obtain the corresponding quartic moveout coefficient A_4. The dependence of A_4 on the medium parameters also provides useful insight into the properties of nonhyperbolic moveout.

General expression for the quartic moveout coefficient

Using the so-called normal-incidence-point (NIP) theorem (e.g., Fomel and Grechka, 2001), we derived the quartic moveout coefficient A_4 for an arbitrarily anisotropic, heterogeneous medium overlaying an irregular reflector (Figure 1):

$$A_4(\mathbf{L}) = \frac{\tau_0}{48}\,\frac{\partial^4 \tau_0}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n$$

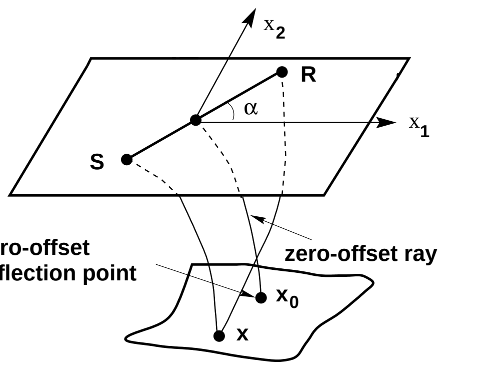

Fig. 1: Reflection traveltimes from an irregular interface are recorded in a multi-azimuth CMP gather over an arbitrarily anisotropic, heterogeneous medium. The derivation of the quartic moveout coefficient A_4 accounts for reflection-point dispersal.

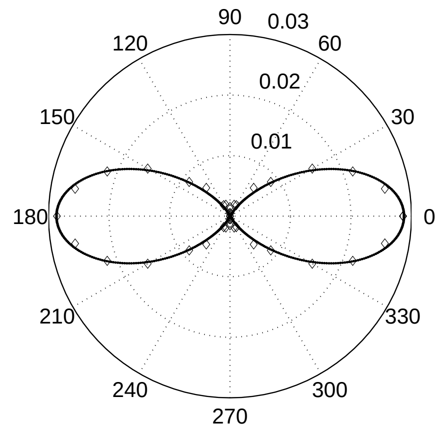

Fig. 2: Accuracy of the weak-anisotropy approximation for the coefficient A_4 in a tilted TI layer. The polar radius is equal to the magnitude of A_4 in the corresponding azimuthal direction. The diamonds mark the values of A_4 obtained for each azimuth (numbers on the perimeter) by fitting a quartic polynomial to the ray-traced $t^2(x^2)$–curve on the spreadlength $X_{\max} = 1.2z$, where $z = 1$ km is the reflector depth. The solid line is the linearized equation (3). The model parameters are $V_{P0} = 1$ km/s, $\epsilon = 0.1$, $\delta = 0.025$, $\nu = 80°$ and $\phi = 0°$.

$$- \frac{\tau_0}{16} \frac{\partial^3 \tau_0}{\partial x_i \partial y_k \partial y_m} \left(\frac{\partial^2 \tau_0}{\partial x_i \partial x_j} \right)^{-1} \frac{\partial^3 \tau_0}{\partial x_j \partial y_p \partial y_n} L_k L_m L_p L_n + \frac{1}{16\, \tau_0^2\, V_{\rm nmo}^4(\mathbf{L})} \, . \qquad (2)$$

Here $\mathbf{y}$ defines the CMP location, $\mathbf{x}$ corresponds to the zero-offset reflection point, τ_0 is the one-way zero-offset traveltime, $\mathbf{L}$ is a unit vector parallel to the CMP line and $V_{\rm nmo}(\mathbf{L})$ is the NMO velocity on the line $\mathbf{L}$.

For relatively simple models, the traveltime τ_0 can be expressed explicitly as a function of $\mathbf{y}$ and $\mathbf{x}$, and A_4 can be evaluated in closed form (see below). If the medium is laterally heterogeneous and/or has a low anisotropic symmetry, all derivatives in equation (2) can be computed during the tracing of the zero-offset ray.

Coefficient A_4 in a homogeneous TTI layer

Here, we apply equation (2) to analysis of P-wave nonhyperbolic moveout in a homogeneous TTI layer above a plane dipping reflector. The symmetry axis is assumed to be confined to the dip plane of the reflector, which is the quite common in practice. Hyperbolic reflection moveout and the dependence of NMO velocity on the anisotropic parameters for this model are discussed in Tsvankin (1997, 2001). Following Tsvankin (1997), we parameterize the medium by the symmetry-direction velocities of P-waves (V_{P0}) and S-waves (V_{S0}) and Thomsen's anisotropic coefficients ϵ, δ and γ specified with respect to the symmetry axis. The tilt ν of the symmetry axis is considered positive if the axis points towards the reflector (i.e., if the symmetry axis and the reflector normal deviate from the vertical in the same direction).

Weak-anisotropy approximation for A_4

To gain analytic insight into the dependence of A_4 on the model parameters, it is convenient to employ the weak-anisotropy approximation and linearize equation (2) in ϵ and δ:

$$A_4^{\rm TTI} = - \frac{2\,\eta}{t_{P0}^2\, V_{P0}^4} \left[\, F(\alpha,\, \phi,\, \nu) \,+\, C \,\right], \qquad (3)$$

where $\eta \equiv (\epsilon - \delta)/(1 + 2\delta) \approx \epsilon - \delta$ is the anellipticity parameter responsible for P-wave time-domain signatures in VTI media (Alkhalifah and Tsvankin, 1995), $C = 9/64$ is a constant, t_{P0} is the two-way zero-offset P-wave traveltime, α is the azimuth of the CMP line measured from the dip plane, ϕ is the reflector dip and F is a rather complicated function of the angles α, ϕ and ν, which is analyzed for special cases below. Clearly, regardless of the tilt of the symmetry axis and reflector dip, the P-wave coefficient A_4 for weak transverse isotropy is controlled by a single anisotropic parameter η. If the medium is elliptical ($\eta = 0$), A_4 vanishes and reflecton moveout becomes purely hyperbolic; this result remains valid for any strength of the anisotropy.

Figure 2 shows that the linearized equation (3) is sufficiently close to the exact quartic coefficient for relatively small values of the anisotropic parameters. As demonstrated by Tsvankin and Thomsen (1994), the weak-anisotropy approximation may rapidly lose its accuracy with increasing parameters ϵ and δ. However, equation (3) can still be used for qualitative analysis of nonhyperbolic moveout in tilted TI media. Note that equation (3) and all weak-anisotropy results below can be adapted for SV-waves by making the following substitutions (Tsvankin, 2001): $V_{P0} \to V_{S0}$, $\delta \to \sigma$, and $\epsilon \to 0$, where $\sigma \equiv (V_{P0}/V_{S0})^2(\epsilon - \delta)$.

Dip and strike components of A_4

Since the dip plane of the reflector contains the symmetry axis of the overburden, it represents a vertical symmetry plane for the whole model. Therefore, the dip and strike directions of the reflector determine "the principal axes" of the azimuthally-varying coefficient A_4. For the dip line of the reflector ($\alpha = 0^\circ$), equation (3) yields

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\cos^3\phi\,\cos(4\nu - 3\phi)\,. \tag{4}$$

Since the dip-line quartic coefficient is proportional to $\cos^3\phi$, $A_{4,\mathrm{dip}}^{\mathrm{TTI}}$ has a decreasing trend with dip. According to equation (4), nonhyperbolic moveout on the dip line vanishes if $(3\phi - 4\nu) = n\pi/2$ $(n = \pm1, \pm3, \pm5, ...)$. For a fixed reflector dip, $\cos(3\phi - 4\nu)$ goes to zero for two different values of the tilt ν between 0° and 90°.

For purposes of anisotropic parameter estimation, it is more convenient to rewrite equation (4) as a function of the horizontal component p of the slowness vector associated with the zero-offset ray (i.e., through the ray parameter responsible for reflection time slopes). It can be shown that both $A_{4,\mathrm{dip}}^{\mathrm{TTI}}(p)$ and the dip-line NMO velocity $V_{\mathrm{nmo,dip}}^{\mathrm{TTI}}(p)$ are controlled by the NMO velocity from a horizontal reflector $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$ and the combinations $[\eta\,\cos 4\nu]$ and $[\eta\,\sin 4\nu]$. In principle, it may be possible to estimate those three parameter combinations, if the dip-line NMO velocity and the coefficient A_4 are measured for two different dips.

The coefficient A_4 on the strike line ($\alpha = 90^\circ$) has the form

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\cos^4(\phi - \nu)\,. \tag{5}$$

While both the dip and strike components of A_4 are proportional to η, their dependencies on reflector dip ϕ and the symmetry-axis tilt ν are entirely different. $A_{4,\mathrm{strike}}^{\mathrm{TTI}}$ goes to zero only if the symmetry axis is perpendicular to the reflector normal (i.e., the symmetry axis is confined to the reflecting plane). For example, if the reflector is vertical ($\phi = 90^\circ$), the strike-line quartic coefficient vanishes for VTI media ($\nu = 0^\circ$). Indeed, for such a model reflected rays are confined to the horizontal (isotropy) plane where velocity is independent of angle.

Azimuthal dependence of A_4

Unlike NMO velocity that has a simple elliptical azimuthal dependence, the variation of the quartic moveout coefficient with azimuth has a much more complicated character. The function $A_4(\alpha)$ may have multiple zeros whose positions strongly depend on both dip ϕ and tilt ν. Figure 3 displays a polar plot with a typical azimuthal signature of A_4, with zeros at azimuths of 38° and 142°. (The quartic coefficient and moveout signature as a whole have an azimuthal period of 180°.) The sign of A_4 changes from negative near the dip direction (i.e., for the horizontally oriented lobe) to positive for the lobe corresponding to $38^\circ < \alpha < 142^\circ$. On the whole, the magnitude of nonhyperbolic moveout for dipping reflectors is usually highest in the strike direction.

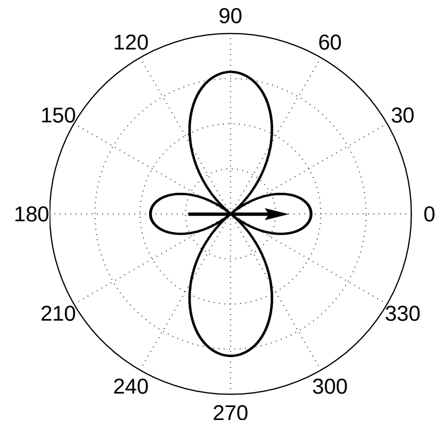

Fig. 3: Azimuthally-varying coefficient A_4 for a TTI layer computed from equation (3). The tilt ν is 40° and the reflector dip ϕ is 15°; the azimuth is measured from the dip plane marked by the arrow.

Evidently, the azimuthal signature of the quartic coefficient can provide useful information for anisotropic parameter estimation. In particular, the azimuthal directions of the CMP lines with vanishing A_4 depend on certain combinations of ν and ϕ and can be used to constrain the orientation of the symmetry axis.

Symmetry axis orthogonal to the reflector

Because of the complicated structure of equation (3), below we focus on two special cases of practical importance. Models with the symmetry axis orthogonal to the reflector ($\phi = \nu$) are typical, for example, for dipping TI shale layers in the Canadian Foothills (e.g., Isaac and Lawton, 1999). The azimuthal dependence of A_4 for $\phi = \nu$ has the form

$$A_4^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\left(1 - \sin^2\phi\cos^2\alpha\right)^2\,. \tag{6}$$

The quartic coefficient goes to zero when $|\cos\alpha| = 1/\sin\phi$, which is possible only on the dip line ($\alpha = 0^\circ$) of a vertical reflector ($\phi = 90^\circ$, which implies a horizontal symmetry axis). The dip and strike components of the quartic coefficient are given by

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\cos^4\phi\,, \tag{7}$$

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,. \tag{8}$$

Equation (8), which shows that the strike-line component of A_4 is independent of dip (or tilt), is well known for the special case of VTI media and a horizontal reflector, when $\phi = \nu = 0$ (Tsvankin and Thomsen, 1994). Whereas the strike-line component of A_4 does not change with dip, the dip-line component is proportional to $\cos^4\phi$ [equation (7)]. Therefore, nonhyperbolic moveout for dipping reflectors rapidly decays away from the strike direction.

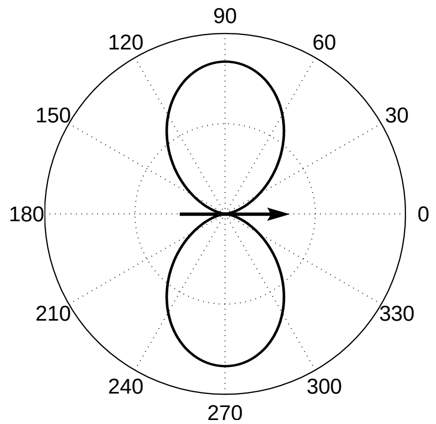

Fig. 4: Azimuthally-varying coefficient A_4 for a VTI layer computed from equation (9). Reflector dip is 30°; the dip direction is marked by the arrow.

Dipping reflector beneath a VTI layer

The linearized quartic coefficient in VTI media ($\nu = 0°$) is expressed as

$$A_4^{\rm VTI} = -\frac{2\,\eta\,\cos^4\phi}{t_{P0}^2\,V_{P0}^4}\,\left(1 - 4\,\sin^2\phi\,\cos^2\alpha\right)\,. \qquad (9)$$

For a horizontal reflector ($\phi = 0°$), the model as a whole is azimuthally isotropic, and A_4 is determined by equation (8). A discussion of the exact (i.e., not limited to weak anisotropy) quartic moveout coefficient of both P- and S-waves in horizontally layered VTI media can be found in Tsvankin (2001). If the reflector is vertical ($\phi = 90°$), A_4 vanishes regardless of the azimuth of the CMP line because reflected rays are confined to the horizontal isotropy plane where velocity is constant.

The coefficient A_4 for dipping reflectors goes to zero in azimuthal directions satisfying $|\cos\alpha| = 1/(2\sin\phi)$. If the dip is equal to 30°, A_4 vanishes only for a single azimuth $\alpha = 0°$ that corresponds to the dip plane (Figure 4); this analytic result is in good agreement with the numerical study of NMO velocity in Tsvankin (2001). For any dip between 30° and 90°, $A_4 = 0$ in two different azimuthal directions. Finally, if the dip is smaller than 30°, the quartic coefficient is negative for any azimuth.

Discussion and conclusions

We have introduced an exact expression for the quartic moveout coefficient A_4 valid for arbitrarily anisotropic, heterogeneous media. Substitution of the quartic coefficient into the general moveout equation of Tsvankin and Thomsen (1994) yields a good approximation for nonhyperbolic moveout of P-waves in anisotropic media with realistic structural complexity. All quantities needed to calculate A_4 for any orientation of the CMP line can be obtained by tracing a single (zero-offset) ray. Computing the zero-offset ray is also sufficient to construct the NMO ellipse (Grechka et al., 1999), so our results make it possible to model long-spread moveout without multi-offset, multi-azimuth ray tracing.

The developed equation for A_4 provides valuable insight into P-wave nonhyperbolic moveout in TI media. The azimuthal signature of the quartic coefficient (i.e., the azimuths of vanishing A_4 and the signs of A_4 in different azimuthal sectors) depends on the tilt ν of the symmetry axis and reflector dip ϕ. For example, if $\nu = 0°$ (VTI media) and reflector dip is mild ($\phi < 30°$), A_4 is negative for all azimuths, and its magnitude increases away from the dip direction. For a 30° dip, nonhyperbolic moveout in VTI media vanishes on the dip line; if the dip exceeds 30°, A_4 goes to zero in two different azimuths that do not coincide with either strike or dip directions. We also gave a detailed analysis of A_4 for the symmetry axis orthogonal to the reflector.

The dip components of both $V_{\rm nmo}$ and A_4 expressed through the ray parameter (i.e., through the reflection time slope) depend on the same three parameter combinations involving η, ν and the NMO velocity from a horizontal reflector. This result and the high sensitivity of the azimuthal signature of A_4 to the symmetry-axis orientation indicate that P-wave nonhyperbolic moveout may provide valuable information for velocity analysis in TTI media. Although the trade-off between $V_{\rm nmo}$ and A_4 makes quantitative estimates of the quartic coefficient relatively unstable (Tsvankin, 2001), the azimuthal variation of the sign of A_4 and the directions of vanishing or small nonhyperbolic moveout should be detectable from wide-azimuth reflection data.

References

Al-Dajani, A., and Tsvankin, I., 1998, Nonhyperbolic reflection moveout for horizontal transverse isotropy: Geophysics, **63**, 1738–1753.

Alkhalifah, T., and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Fomel, S., and Grechka V., 2001, Nonhyperbolic reflection moveout of P-waves: An overview and comparison of reasons: CWP Research Report (CWP-372).

Grechka, V., Tsvankin, I., and Cohen, J.K., 1999, Generalized Dix equation and analytic treatment of normal-moveout velocity for anisotropic media: Geophys. Prosp., **47**, 117–148.

Isaac, J.H., and Lawton, D.C., 1999, Image mispositioning due to dipping TI media: A physical seismic modeling study: Geophysics, **64**, 1230–1238.

Tsvankin, I., 1997, Moveout analysis for transversely isotropic media with a tilted symmetry axis: Geophys. Prosp., **45**, 479–512.

Tsvankin, I., 2001, Seismic signatures and analysis of reflection data in anisotropic media: Elsevier Science.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, **59**, 1290–1304.