

Total magnetization direction and dip from multiscale edges

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Summary

By adapting a multiscale edge analysis originally developed for gravity surveys, we show that information about both the total magnetization direction and dip can be extracted from total-field measurements. Such a separation becomes important in the presence of geologic units exhibiting remanent magnetization. The total magnetization in this case is not aligned with the Earth's background field. For processing steps, such as reduction to pole, and quantitative analysis of magnetic data, the direction of the total magnetization is critical. We first develop the method for isolating magnetization direction and dip using theoretical models and then illustrate the processing steps in a synthetic example.

Introduction

Continuous wavelets seemingly bear little resemblance to their compact, discrete, and much more widely known cousins used in the compression of data. However, these overlooked wavelets are a natural tool for the study of potential fields. Inspired by this observation, Hornby *et al.* (1999) recast several standard gravity and magnetic data processing techniques into the language of wavelets, specifically in the context of multiscale edge analysis. This analysis provides information on the singular nature of a generating signal by blurring it over wider and wider scales.

Within this framework, it is not a stretch to consider the issue of identifying total magnetization direction. The total magnetization of a source body results from the vector addition of the induced and remanent magnetization. In the absence of both demagnetization and remanence, the magnetization of the source is in the direction of the Earth's background field. The processing of magnetic data is greatly simplified under this assumption since any remanent magnetization causes the total magnetization direction to change. Several mechanisms exist that produce remanent magnetization in rocks (Telford, 1996), including shock remanent magnetization resulting from high velocity impact (Roest and Pilkington, 1993). Techniques put forward to solve for the inclination of the total magnetization vector depend on either a comparison of gravity and magnetic data or the assumption of a 2D subsurface (Roest and Pilkington, 1993; Thurston, 2002). Furthermore, geologic contacts are taken to be vertical (Roest and Pilkington, 1993).

Constraining the direction of the total magnetization de-

termines the accuracy of magnetic data processing steps, such as the reduction to pole, and properties of the paleomagnetic field. The presence of remanent magnetization adds skewness, or lateral shift, to magnetic anomalies. Compounding the problem of identifying remanence in practice, the dip of a geologic unit alters a magnetic profile much in the same way as remanent magnetization. Being able to separate the effects of total magnetization direction and dip is important for the mapping of faults with total-field aeromagnetic data and has been the focus of current research in exploration magnetics (Thurston, 2002).

With a needed modification, the recipe for continuous wavelets presented in Hornby *et al.* (1999) can be used to determine the total magnetization direction. A new continuous wavelet, tuned specifically for the magnetic problem, simplifies the determination of the total magnetization inclination. In the presence of dip, our method effectively separates the similar patterns introduced by both the dip and remanent magnetization. Previous studies applying continuous wavelets to the magnetic problem (Sailhac *et al.*, 2000) have been done for sources with induced magnetization only. By relaxing this assumption, we aim to highlight some of the complications and resolve some of the issues associated with identifying remanent magnetization.

Multiscale Edge Analysis

Using processing techniques well-known within the study of potential fields, Hornby *et al.* (1999) have shown how to obtain a continuous wavelet transform (CWT) of gravity data. The magnitude and location of the extrema of the CWT comprise the multiscale edges. Due to the localizing action of the CWT, most of the information carried by a signal is concentrated in its multiscale edges. Hornby *et al.* (1999) have demonstrated that, with only multiscale edges, a complete CWT can be reconstructed by defining an inverse CWT to act on the multiscale edges.

Given aeromagnetic data collected at a height z_0 in an ambient magnetic field with inclination I , a simple recipe consisting of three items yields a CWT of the data. The CWT given by Hornby *et al.* (1999) can be modified for the magnetic problem in order to identify the total magnetization direction:

1. upward continue to the new height $z = sz_0$
2. take directional derivative in the direction of $-I$
3. multiply by the dilation factor s .

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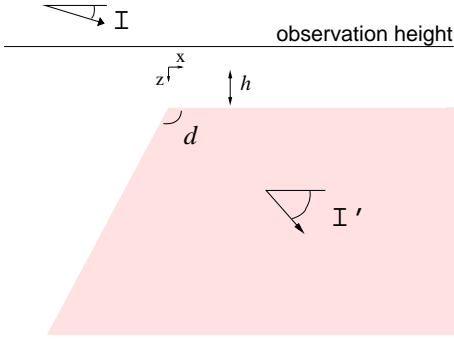


Fig. 1: The parameters describing the infinite dipping step

The need for taking a directional derivative in item 2., instead of a horizontal gradient as in Hornby *et al.* (1999), allows a CWT that only depends on the unknown total magnetization direction. Once the wavelet representation of the magnetic data has been calculated, the extrema, or multiscale edges, of the CWT can be found by taking its horizontal derivative and setting it equal to zero.

The two components of the multiscale edges, the *magnitudes* and *locations* of the extrema of the CWT, hold different information about the source bodies. The *magnitudes* of the extrema of the CWT, as a function of the dilation factor s , depend on the geometry of the source. This dependence is related to the Lipschitz exponent, or structural index, of the source distribution. Intrinsic properties of the sources, e.g. remanent magnetization, are contained in the *locations* of the extrema of the CWT. In the following theoretical examples, we explore how these locations depend on the total magnetization and develop a method for estimating its direction.

2D Infinite Dipping Step

The simplest example of a 2D magnetic anomaly is the infinite dipping step pictured in Fig. ???. The Earth's ambient field, with direction I , is in general oriented differently than the total magnetization of the source body, I' . From Nabighian (1972), the horizontal and vertical derivatives of the total-field anomaly for the infinite dipping step are given by:

$$\frac{\partial(\Delta M)}{\partial x} = 2k \sin d \frac{(h-z) \cos \psi + x \sin \psi}{(h-z)^2 + x^2} \quad (1)$$

$$\frac{\partial(\Delta M)}{\partial z} = 2k \sin d \frac{x \cos \psi - (h-z) \sin \psi}{(h-z)^2 + x^2} \quad (2)$$

where d is the dip of the interface, k is the susceptibility contrast, and h is the depth. For simplicity, the infinite dipping step in Fig. ??? strikes perpendicular to the observation line and the declination is in the (x, z) -plane. The angle ψ , assuming only induced magnetization, is given in

degrees by $2I - d - 90^\circ$. More generally, in the presence of remanence, ψ is determined by both the ambient field direction, I , and the total magnetization direction, I' :

$$\psi = I + I' - d - 90^\circ. \quad (3)$$

From this expression, it is evident that I' and d both enter into the phase of the total-field anomaly in exactly the same way. Similarly, for the vertical magnetic anomaly, the equations (??) and (??) have the same form, except that:

$$\psi = I' - d. \quad (4)$$

Thurston (2002) attempts to untangle remanent direction and dip by subtracting equations (??) and (??) via the local phase of the analytic signal. However, subtraction of these two equations cancels out both the effects of the remanent magnetization (I') and dip (d). What is left is a function that depends on a known quantity, the inclination of the ambient field I . Besides phase, the amplitude of the analytic signal has been used to constrain the total magnetization direction (Roest and Pilkington, 1993), but this method suffers from the assumption of vertical geologic contacts. Hence, at any single height above a magnetic layer, the amplitude and the phase of the analytic signal cannot differentiate between remanence and dip. Only by monitoring the location of the multiscale edges over many heights can total magnetization direction and dip be separated. In the following example, we will analyze the multiscale edges from an infinite dipping step with dip d at a depth of $h = 0$.

Since equations (??) and (??) describe the magnetic field for all heights, item 1. in the recipe for the CWT is not as important for the theoretical analysis as it will be for data examples. In accordance with Hornby *et al.* (1999), we define the observation height z_0 as -1. This means that, from the relationship in item 1., the dilation factor, s , is the negative of the z -coordinate. Note that the z -coordinate must be similarly scaled to this unit distance. For items 2. and 3., the derivative in the direction of $-I$ needs to be taken and multiplied by $s = -z$, yielding the CWT:

$$W[s, x] = s \left[\cos I \frac{\partial(\Delta M)}{\partial x} - \sin I \frac{\partial(\Delta M)}{\partial z} \right] \quad (5)$$

The use of this particular directional derivative effectively cancels all terms depending on I , the known ambient field, and leaves a wavelet transform that only depend on I' , the desired total magnetization direction.

Taking the horizontal derivative of the CWT and setting it equal to zero yields the equation for the location of the multiscale edges:

$$\frac{\partial W[s, x]}{\partial x} = 0. \quad (6)$$

Solving equation (??) for s as a function of x yields two solutions depending on the difference of the total magne-

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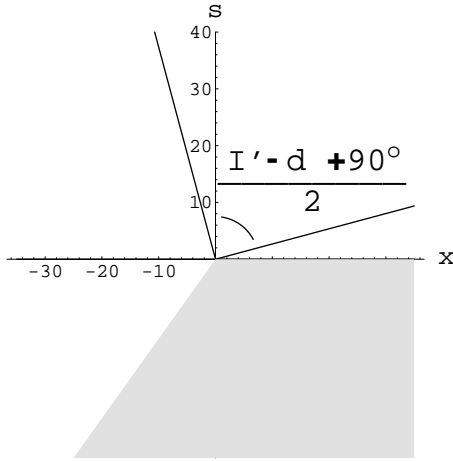


Fig. 2: The location of the extrema of the wavelet transform for the 2D infinite dipping step

tization and dip direction:

$$s = x \cot\left(\frac{I' - d - 90^\circ}{2}\right), x \cot\left(\frac{I' - d + 90^\circ}{2}\right). \quad (7)$$

As seen in Fig. ??, these solutions define two trajectories in the (x,s) -plane that are perpendicular to each other and point toward the vertex of the infinite dipping step. From equation (??), the angle between the right hand trajectory and the vertical axis can be simply related to the total magnetization direction, I' . This relationship is shown in Fig. ?. Since the two solutions depend on the difference between I' and d , the total magnetization and dip direction cannot be separated for the case of an infinite dipping step.

2D Infinitely Thin Horizontal Sheet

The total-field anomaly for a 2D thin horizontal sheet with dip can be determined by taking the derivative of the infinite dipping step in the direction of the dip (Nabighian, 1972). After taking this derivative, the dip dependence of the anomaly vanishes. As a result, the locations of the three multiscale edges for the 2D infinitely thin horizontal dyke only depend on I' :

$$s = x \cot\left(\frac{I'}{3}\right), x \cot\left(\frac{I' \pm 180^\circ}{3}\right). \quad (8)$$

Equation (??) shows that, in the presence of dip, the total magnetization direction from a *finite* dipping step can be extracted from the locations of the multiscale edges at large dilations. At large dilations, the magnetic anomaly of a finite dipping step approaches that of an infinitely thin horizontal sheet. Again, as seen from equation (??), this separation of total magnetization direction from dip cannot be done for the infinite step.

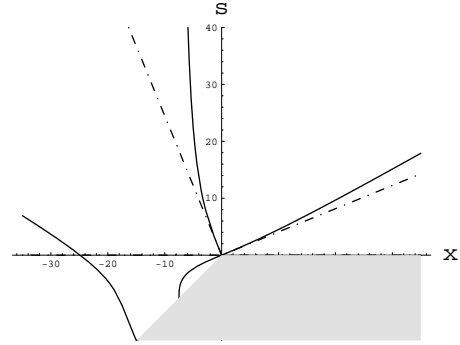


Fig. 3: The location of the extrema of the wavelet transform for the 2D finite dipping step are shown with the solid lines. Their small dilation trajectories are depicted as dash-dotted lines. The finite step shown here is 15 times thicker than the height of observation. The total magnetization direction is 0° and the dip is 135° for this example

2D Finite Dipping Step

The wavelet extrema for the 2D finite dipping step are not as simple as those for the previous two examples. However, the two previous cases represent asymptotes of these complex trajectories in the (x,s) -plane. Close to the finite dipping step, or at small dilations, the multiscale edges should behave as if they were over an infinite dipping step. For large dilations, the anomaly should gradually resemble that of an infinitely thin horizontal sheet. The quantitative understanding of the multiscale edges of these two end members, embodied in equations (??) and (??), allows insight into the behavior of the finite dipping step.

In Fig. ??, the true trajectories of the multiscale edges for the finite dipping step are shown with the trajectories from equation (??). The total magnetization direction for this example has been set to 0° and the dip is 135° . The true trajectories, as solutions of a quartic equation, are complicated and display curvature. However, at small dilations, the straight lines of equation (??) approximate the multiscale edges well.

The large dilation behavior of the finite dipping step can be surmised from Fig. ?. Three multiscale edges develop away from the top of the dipping face and begin to converge to their asymptotes. The asymptotes only depend on the total magnetization direction, and not the dip. The analytic forms of these asymptotes are similar to equation (??), except for the fact that, at low dilations, they do not intersect the origin. Instead, the three asymptotes collectively intersect the barycenter of the dipping face. Such behavior has been observed before in the application of the CWT to magnetic surveys (Sailhac *et al.*, 2000). The triangular intersection of the three asymptotes can also be used to constrain the depth to the finite dipping

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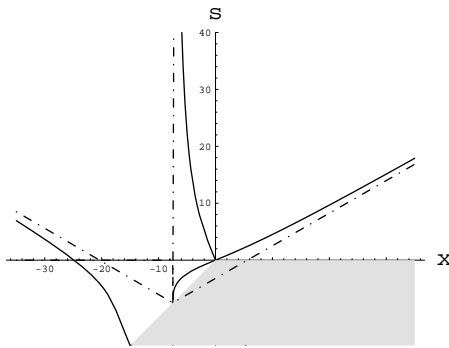


Fig. 4: The location of the extrema of the wavelet transform for the 2D finite dipping step are shown with the solid lines. Their large dilation asymptotes are depicted as dash-dotted lines. The finite step shown here is 15 times thicker than the height of observation. The total magnetization direction is 0° and the dip is 135° for this example

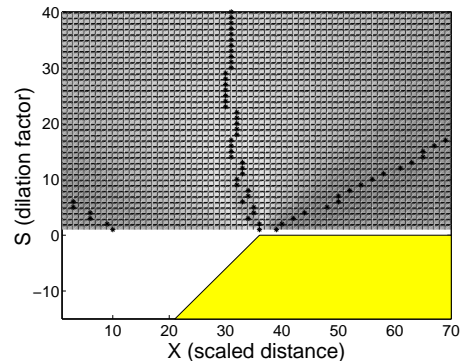


Fig. 5: The results of the synthetic experiment of a magnetic survey over a dipping layer. The values of the CWT are shown in contour shading at all values above the observation height (z_0). The locations of the extrema of the wavelets are shown as black dots.

step with error estimates. The rate of convergence of the true trajectories to their large dilation asymptotic values determines the layer's thickness.

Synthetic Example

To test the robustness of the multiscale edges, we performed a numerical experiment using magnetic data generated from SAKI, a modeling program designed by Michael Webring of the USGS. Shown in Fig. ?? is the numerically calculated CWT of the magnetic field over a layer dipping at 135° with a total magnetization direction $I' = 0^\circ$ (same values as in the theoretical examples). Fourier-domain upward continuation and space-domain derivatives were used to accomplish the items 1., 2., and 3. in the recipe for wavelets. Comparing the picked wavelet extrema from Fig. ?? to the solid lines of Fig. ??, the synthetics are seen to match up well with the theory. From a linear regression on the picked multiscale edges, the total magnetization is recovered from the slope at $\sim 2.5^\circ$ (true value of $I' = 0^\circ$).

Conclusions

We have extended the wavelet-based processing steps to the magnetic problem in the general case of an unknown total magnetization direction. Such a discrepancy between the induced and total directions results from the presence of remanent magnetization. From theory and a synthetic example, the coupled skewness patterns introduced into total-field data by dip and remanence effectively separate in the wavelet domain, allowing the measurement of the total magnetization direction in the presence of finite dipping contacts. Future work will focus on the extension of the CWT to detect the total magnetization direction in 3D.

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