

Feasibility of seismic characterization of multiple fracture sets

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Summary

Estimation of parameters of multiple fracture sets is often required for successful exploration and development of naturally fractured reservoirs. The goal of this paper is to determine the maximum number of fracture sets of a certain rheological type which, in principle, can be resolved from seismic data. The main underlying assumption is that an estimate of the complete effective stiffness tensor has been obtained, for example, from multi-azimuth, multi-component surface seismic and VSP data. Although under typical circumstances only a subset of the stiffness elements (or some of their combinations) may be available, this study helps to establish the limits of seismic fracture-detection algorithms.

The number of uniquely resolvable fracture systems depends on the anisotropy of the host rock and the rheology and orientation of the fractures. Somewhat surprisingly, it is possible to characterize fewer vertical fracture sets than dipping ones, even though in the latter case the fracture dip has to be found from the data. For the simplest, rotationally invariant fractures embedded in either a purely isotropic or VTI (transversely isotropic with a vertical symmetry axis) host rock, the stiffness tensor can be inverted for up to *two* vertical or *four* dipping fracture sets. In contrast, only one fracture set of the most general (micro-corrugated) type, regardless of its orientation, is constrained by the effective stiffnesses. These results can serve as a guide for seismic fracture-characterization studies that should take into account the limitations of a particular data set in estimating the effective stiffnesses.

Introduction

Seismic methods are critically important in characterizing fractured reservoirs which often contain multiple, differently oriented fracture networks (e.g., Lynn et al., 1999). Processing of reflected waves produces such signatures as the NMO (normal-moveout) ellipses of different modes, amplitude variation with offset and azimuth, and shear-wave polarizations and splitting coefficients. Additionally, walkaway VSP data can be used to estimate slowness surfaces and polarization vectors at receiver locations in boreholes. Available combinations of those signatures can be inverted for the effective stiffness matrix. Then the linear-slip theory (Schoenberg, 1980; Schoenberg and Sayers, 1995) can be employed to infer the fracture compliances and orientations for a certain fracture model from the effective stiffnesses. This approach is discussed in detail by Bakulin et al. (2000a,b,c, 2002), who developed a number of practical seismic fracture-characterization algorithms for typical models

with one or two vertical fracture sets.

Since effective anisotropic media are fully described by up to 21 stiffness coefficients, there exists only a limited subset of fractured models constrained by seismic data. The goal of this paper is to identify fracture sets whose compliances and orientations can be unambiguously found from the effective stiffness tensor. We examine three different rheological types of fractures (rotationally invariant, diagonal and completely general) embedded in either a purely isotropic or VTI (transversely isotropic with a vertical symmetry axis) host rock. By computing the Fréchet derivatives of the effective stiffness coefficients, we determine how many different fracture sets of a certain type (along with the unknown background parameters) are constrained by the stiffness matrix.

Effective stiffness of fractured rock

According to the linear-slip theory, the effective compliance matrix \mathbf{s} of a medium containing N sets of aligned fractures is given by

$$\mathbf{s} = \mathbf{s}_b + \sum_{i=1}^N \mathbf{s}_f^{(i)}, \quad (1)$$

where \mathbf{s}_b and $\mathbf{s}_f^{(i)}$ are the 6×6 symmetric compliance matrices of the unfractured background and the i th fracture set, respectively. The effective stiffness matrix \mathbf{c} is found by inverting the compliance matrix \mathbf{s} .

The excess compliance of the most general (micro-corrugated) fracture set with the normal in the x_1 -direction is described by the following matrix (Schoenberg, 1980; Bakulin et al., 2000c):

$$\mathbf{s}_f^{\text{GN}, x_1} = \begin{pmatrix} K_N & 0 & 0 & 0 & K_{NV} & K_{NH} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K_{NV} & 0 & 0 & 0 & K_V & K_{VH} \\ K_{NH} & 0 & 0 & 0 & K_{VH} & K_H \end{pmatrix}. \quad (2)$$

The nonzero off-diagonal elements of the matrix $\mathbf{s}_f^{\text{GN}, x_1}$ imply the existence of coupling between the slips (jumps in displacement across the fractures) and tractions in the directions normal and tangential to the fractures.

For fractures with a less complicated rheology, normal slips are decoupled from tangential tractions, and the

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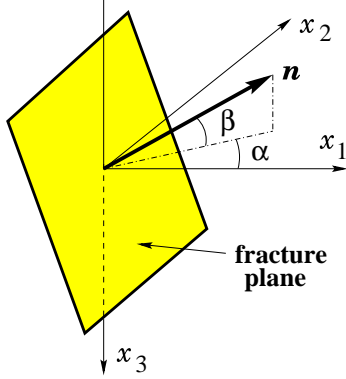


Fig. 1: Orientation of a fracture set is defined by the azimuth α and dip β of the unit vector \mathbf{n} orthogonal to the fracture plane.

compliance matrix simplifies to

$$\mathbf{s}_f^{\text{DI}, x_1} = \begin{pmatrix} K_N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_V & K_{VH} \\ 0 & 0 & 0 & 0 & K_{VH} & K_H \end{pmatrix}. \quad (3)$$

We call fractures of this type “diagonal” because after a special rotation around the x_1 -axis, the matrix (3) becomes diagonal. The simplest type of fractures, called “rotationally invariant” by Schoenberg and Sayers (1995), corresponds to diagonal fractures with equal tangential compliances:

$$\mathbf{s}_f^{\text{RI}, x_1} = \begin{pmatrix} K_N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_V & 0 \\ 0 & 0 & 0 & 0 & 0 & K_V \end{pmatrix}. \quad (4)$$

Applying the so-called Bond transformation yields the compliance matrix of a *rotated* fracture set of specific rheology (GN, DI or RI) whose orientation is described by the fracture normal \mathbf{n} (Figure 1):

$$\mathbf{n} = \{ \cos \alpha \cos \beta, \sin \alpha \cos \beta, -\sin \beta \}. \quad (5)$$

Then we use equation (1) to compute the effective compliance and stiffness matrices of the fractured rock.

Table 1 lists the number of independent physical model parameters for the two types of host rock (background) and three types of fracture systems considered here. Since fracture characterization can be unique only if there are no more than 21 physical parameters (the number of stiffness coefficients for the most general, triclinic symmetry), Table 1 helps to identify fractured models that cannot (even in principle) be constrained by seismic data. For instance, if three arbitrarily oriented diagonal fracture sets

Host rock		Fracture set		
ISO	VTI	RI	DI	GN
2	5	4	6	8

Table 1. Number of independent parameters needed to describe isotropic (ISO) and VTI background media, as well as rotationally invariant (RI), diagonal (DI) and general (GN) fractures. The numbers for the fracture sets include the orientation angles α and β .

are embedded in a VTI host rock, the total number of parameters is $3 \times 6 + 5 = 23 > 21$.

Uniqueness of fracture characterization

Frèchet derivatives of the stiffness matrix

Even if the number of effective stiffness coefficients is larger than the number of unknown physical parameters, the inversion for a particular parameter can still be ambiguous. To study the uniqueness of the inverse problem, we apply the singular value decomposition (SVD) to the matrix \mathcal{F} of Frèchet derivatives of the effective stiffnesses c_{ij} with respect to the unknown model parameters. Using equation (1), the matrix \mathcal{F} can be obtained in the following form:

$$\mathcal{F} = \mathbf{c} \left(\mathbf{s}_b \frac{\partial \mathbf{c}_b}{\partial \mathbf{m}} \mathbf{s}_b - \sum_{i=1}^N \frac{\partial \mathbf{s}_f^{(i)}}{\partial \mathbf{m}} \right) \mathbf{c}, \quad (6)$$

where \mathbf{m} is the model parameter vector that includes the background stiffnesses, fracture compliances [equations (2), (3) or (4)] and the orientation angles of each fracture set.

According to the standard SVD criterion, if the condition number (defined as the ratio of the greatest singular value to the smallest one) of \mathcal{F} is finite ($\kappa \equiv \text{cond } \mathcal{F} < \infty$), the inversion for \mathbf{m} is theoretically unique (for noise-free data). To identify the maximum possible number of fracture sets which can be resolved using the effective stiffness coefficients, we elected the following simple approach. After choosing the symmetry of the host rock (isotropic or VTI), we keep adding fracture sets of a certain type to the model until the condition number becomes “infinite” (i.e., exceeds the numerical limit set in the Matlab software – $1.8 \cdot 10^{308}$).

Arbitrarily oriented (dipping) fractures

The analysis of the condition number for fracture sets with arbitrary dip and azimuth is summarized in Table 2. The results show an intuitively obvious trend: the more complicated rheology the fracture systems have, the fewer such systems can be uniquely estimated from the effective stiffnesses. It is interesting that for rotationally invariant and diagonal fractures it should be possible to invert for as many fracture sets as allowed by the dimensionality constraint (i.e., the inversion is possible if the number of model parameters is smaller than 21). In contrast, only *one* general fracture set can be resolved,

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	RI	DI	GN
ISO	4	3	1
VTI	4 [°]	2	1

Table 2. Maximum number of dipping fracture sets that can be uniquely resolved from the stiffness matrix.

	RI	DI	GN
ISO	2 [*]	2	1 [†]
VTI	2 ^{**}	2 [†]	1

Table 3. Maximum number of vertical fracture sets that can be uniquely resolved from the stiffness matrix.

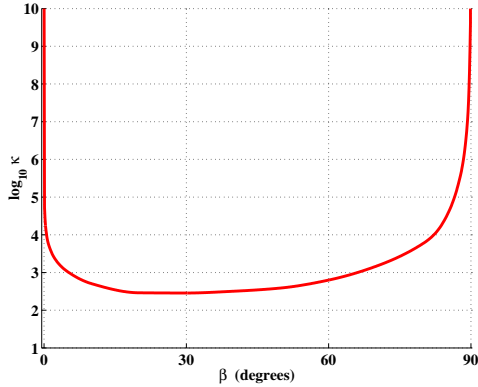


Fig. 2: The influence of fracture dip on the condition number κ of the Fréchet matrix for the model of three rotationally invariant fracture sets in a purely isotropic background with $V_P/V_S = 2$. The dip β (the x -axis of the plot) is the same for all three sets; the azimuths α of the fracture normals are 0° , 60° and 120° .

even for the simplest isotropic background rock.

Seismic signatures needed for characterization of dipping fractures have seldom been discussed in the literature. A relatively simple model with a single set of dipping rotationally invariant fractures in a VTI host rock (diamond in Table 2) was analyzed by Grechka and Tsvankin (2002). The effective model in this case has monoclinic symmetry with a vertical symmetry plane parallel to the dip plane of the fractures. Grechka and Tsvankin (2002) showed that all fracture and background parameters can be obtained from the vertical velocities and NMO ellipses of PP -waves and two split SS -waves (or mode-converted waves) reflected from horizontal interfaces. The analysis of the Fréchet matrix indicates that it is possible to resolve a total of *four* systems of rotationally invariant fractures in a VTI background.

Figure 2 shows a typical example of the relationship between the condition number κ and the fracture dip β for a model that includes three rotationally invariant fracture sets. For a wide range of dips the curve is almost flat with the condition number satisfying $10^2 < \kappa < 10^3$, which indicates that the inverse problem is well-posed (according to our criterion). Only for dips extremely close to 0° (near-vertical fractures) and 90° (near-horizontal fractures) does the condition number rapidly go to infinity.

Vertical fractures

Most existing papers on fracture characterization treat

vertical fractures, which are believed to be most common in the subsurface. Although it seems that making fractures vertical (i.e., setting the fracture tilt β to zero) should help in fracture detection because each set is described by one fewer parameter, Table 3 proves this expectation to be wrong. For the simplest, rotationally invariant fractures in both isotropic and VTI background, the maximum number of resolvable vertical fracture sets is just two compared to four dipping sets in Table 2. To explain this paradox, note that fixing the tilt of fractures ($\beta = 0$) removes one degree of freedom in the description of each fracture set. As a consequence, certain excess compliances of different fracture systems can map into the same element of the effective stiffness matrix, making estimation of these individual compliances impossible. An example of this type of ambiguity is discussed by Bakulin et al. (2002).

The simplest fractured model (star in Table 3) is that of vertical, rotationally invariant fractures in an otherwise isotropic host rock. For a single fracture set, the effective medium is transversely isotropic with a horizontal symmetry axis (HTI). Fracture-characterization algorithms for this model can be based entirely on surface reflection data (Bakulin et al., 2000a). Two vertical fracture systems making an arbitrary angle with each other lower the effective medium symmetry to monoclinic (the only symmetry plane of the model is horizontal). In the special case of orthogonal fracture sets, the effective model is orthorhombic, with the vertical symmetry planes parallel to the fracture strike directions. For both orthorhombic and monoclinic models of this type, the fracture and background parameters can be estimated using the vertical velocities and NMO ellipses of PP - and two split SS -waves (or converted PS -waves) from horizontal interfaces (Bakulin et al., 2000b,c).

Two vertical RI fracture sets in a VTI background (two stars in Table 3) also yield an effective monoclinic medium with a horizontal symmetry plane. Bakulin et al. (2002) showed that if the sets are orthogonal (then the effective medium is orthorhombic), the fracture and background parameters cannot be estimated in a unique fashion. However, our study reveals that the orthogonal orientation of the fracture systems is the *only* special case for this model when the Fréchet matrix \mathcal{F} degenerates. As illustrated by Figure 3, the condition number κ rapidly increases only when the angle $\Delta\alpha$ between the fractures is in a narrow vicinity of 0° or 90° .

A single system of vertical diagonal fractures [equation (3)] with $K_{VH} = 0$ in a VTI host rock (dagger in Table 3) creates an effective orthorhombic medium. Bakulin

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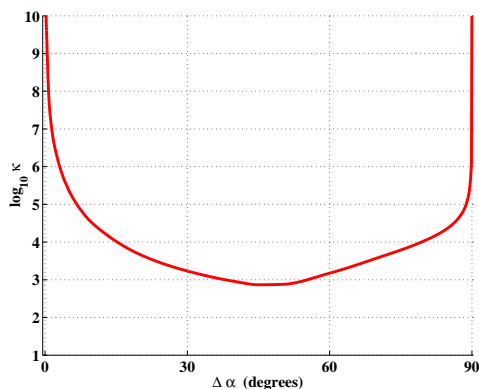


Fig. 3: The condition number κ for the model of two vertical, rotationally invariant fracture sets in a VTI background. $\Delta\alpha$ denotes the difference between the fracture azimuths.

et al. (2000b) showed that the fracture compliances and orientations for this model can be estimated from *PP* and *SS* (or *PS*) reflection traveltimes. Finally, characterization of vertical fractures of the most general type embedded in a purely isotropic host rock (double dagger in Table 3) was studied by Grechka et al. (2001), who proved that the fracture and background parameters of this (triclinic) model can be found from a combination of reflection and borehole data.

Discussion and conclusions

The key assumption in our study was that all elements of the effective stiffness matrix c can be recovered from seismic data. By expressing the stiffnesses through the fracture and background parameters and analyzing the condition number of the corresponding Fréchet matrix, we determined the maximum number of fracture sets with a given rheology which can be uniquely resolved under those “ideal” conditions. If the fracture sets are tilted away from the vertical (i.e., the fractures are dipping), it may be possible to characterize up to four rotationally invariant fracture sets in either isotropic or VTI background. Surprisingly, parameter estimation becomes more ambiguous for vertical fractures which create simpler (i.e., higher-symmetry) effective anisotropic models. Because of the trade-offs between different fracture parameters, no more than two vertical fracture sets can be resolved from the effective stiffnesses. Even a small fracture dip, however, is sufficient to sharply reduce the condition number and make the inverse problem much better posed.

Estimating the stiffness matrix is a highly challenging problem that requires acquisition and processing of multi-component, multi-azimuth seismic data (e.g., Horne and Leaney, 2000; Bakulin et al., 2000a,b,c; Grechka et al., 2001). In practice, fracture characterization has to be based on available (incomplete) data which usually cannot be inverted for all stiffness coefficients c_{ij} individually. Such incomplete stiffness matrix may constrain fewer fracture sets than indicated by our analysis (if any), so it may

become necessary to impose restrictions on the fracture rheology (e.g., assume rotationally invariant fractures) and orientation, or ignore the background anisotropy.

Another reason for our results to be overly optimistic is that we deem the inversion to be non-unique only when the condition number of the Fréchet matrix goes to infinity. In fact, there is a vast “transitional” set of models with relatively large condition numbers that requires a special analysis based on the accuracy of the input data. For instance, two fracture sets making a small angle with each other may not be resolvable even if the corresponding condition number is finite because the effective stiffness coefficients are estimated with an error (see Figure 3). This issue can be accounted for by using a more conservative condition to define nonuniqueness.

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