

# Inversion of multi-component, multi-azimuth, walkaway VSP data for the stiffness tensor

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## Summary

Vertical seismic profiling (VSP) is an established method for estimating in-situ anisotropy that might provide valuable information for characterization of reservoir lithology, fractures, and fluids. The  $P$ -wave slowness components, conventionally measured in multi-azimuth, walkaway VSP surveys, allow one to reconstruct some portion of the corresponding slowness surface. A major limitation of this technique is that the  $P$ -wave slowness surface alone does not constrain a number of stiffness coefficients that may be crucial for inferring certain rock properties. Those stiffnesses can be obtained only by combining the measurements of  $P$ -waves with those of  $S$  (or  $PS$ ) modes.

We show that, when polar and azimuthal coverage of the data is good, the polarizations and slownesses of  $P$  and two split shear ( $S_1$  and  $S_2$ ) waves are sufficient for estimating all 21 local elastic stiffness coefficients  $c_{ij}$  that characterize the most general triclinic anisotropy. The inverted stiffnesses themselves indicate whether or not the data can be described by a higher-symmetry model. We discuss three different scenarios for inverting noise-contaminated data and then apply our methodology to a multi-azimuth, multi-component VSP data set acquired in the Vacuum Field, New Mexico. Our inversion indicates that the medium at the receiver level can be approximated by an orthorhombic model.

## Introduction

Multi-azimuth walkaway vertical seismic profiling (VSP) can be used for measuring in-situ anisotropy. Anisotropic parameters are usually estimated from  $P$ -wave slowness surfaces  $\mathbf{p}^{(P)}$  obtained by differentiating the traveltimes  $t^{(P)}$  of the first arrivals with respect to the coordinates  $\mathbf{x}$  of the surface sources as well as for the downhole geophones:  $p_i^{(P)} = \partial t^{(P)} / \partial x_i$  ( $i = 1, 2, 3$ ). Because of the acquisition geometry, the vertical slowness components  $p_3^{(P)}$  are obtained only at the geophone levels, whereas the horizontal components  $p_1^{(P)}$  and  $p_2^{(P)}$  are computed only at the earth surface. To reconstruct the slowness surfaces at the geophone levels, one usually assumes lateral homogeneity of the overburden. Then, according to Snell's law, the horizontal slownesses  $p_1^{(P)}$  and  $p_2^{(P)}$  at the downhole receivers coincide with those measured at the surface.

The  $P$ -wave slowness surfaces  $\mathbf{p}^{(P)}$ , however, are inherently insufficient for constraining a number of stiffness

coefficients needed for describing certain rock properties. Combining  $P$ - and  $S$ -wave VSP data is necessary for a more comprehensive reservoir characterization. Horne and Leaney (2000) recognized this possibility and developed a procedure for joint inversion of  $P$ - and  $SV$ -wave VSP data for parameters of transversely isotropic media with a vertical symmetry axis (VTI). The VTI model, however, might be too simplistic for characterization of realistic reservoirs. Thus, it is important to examine whether or not multi-component, multi-azimuth, walkaway VSP data can be used to obtain parameters of lower-symmetry anisotropic media.

Here we show that all 21 stiffness coefficients can be found given sufficient polar and azimuthal coverage of the data. Errors in their estimated values turn out to depend on the complexity of the overburden, which determines our ability to use the horizontal slowness components (measured at the surface) in the inversion.

## Analytic background

The forward model for the inverse problem at hand is the Christoffel equation, which determines the polarization and slowness vectors of plane waves propagating in anisotropic media:

$$F_i^{(Q)} \equiv c_{ijkl} p_j^{(Q)} p_k^{(Q)} A_l^{(Q)} - A_i^{(Q)} = 0, \quad (1)$$

$$(Q = P, S_1, S_2; i, j, k, l = 1, 2, 3).$$

We require the polarization vectors  $\mathbf{A}^{(Q)}$  to be normalized,  $|\mathbf{A}^{(Q)}| = 1$ , and assume summation over all repeated indexes from 1 to 3. The density-normalized stiffness tensor  $\mathbf{c}$ , as well as the vectors  $\mathbf{p}$  and  $\mathbf{A}$ , are defined in a cartesian coordinate frame with the  $x_3$ -axis pointing downward. Our goal is to estimate the elastic stiffness tensor  $\mathbf{c}$  by inverting equations (1). We consider three possible scenarios for the inversion of VSP data.

**Scenario 1.** When the overburden is close to homogeneous horizontally layered (Figure 1a), the model and data vectors are given by

$$\mathbf{m} = \mathbf{c}, \quad \mathbf{d} = \{\mathbf{p}_n^{(Q)}, \mathbf{A}_n^{(Q)}\}, \quad (2)$$

$$(Q = P, S_1, S_2; n = 1, \dots, N).$$

Clearly, obtaining  $\mathbf{c}$  from equations (1) is a linear inverse problem.

**Scenario 2.** It may happen that only conventional  $P$ -wave surface-source data were acquired, and the  $S$ -wave

## Inversion of VSP data

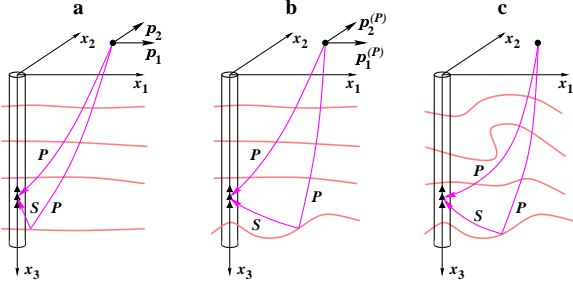


Fig. 1: Three different scenarios of the inversion of multi-component VSP data corresponding to different levels of the complexity of the subsurface: (a) scenario 1, (b) scenario 2, and (c) scenario 3.

velocity distribution is unknown (Figure 1b). In this case, the horizontal slowness components  $p_1^{(S_1)}$ ,  $p_2^{(S_1)}$ , and  $p_1^{(S_2)}$ ,  $p_2^{(S_2)}$  of  $S$ -waves are unknown at receiver locations. Then model and data vectors are

$$\begin{aligned} \mathbf{m} &= \left\{ \mathbf{c}, p_{1,n}^{(S_1)}, p_{2,n}^{(S_1)}, p_{1,n}^{(S_2)}, p_{2,n}^{(S_2)} \right\}, \\ \mathbf{d} &= \left\{ \mathbf{p}_n^{(P)}, p_{3,n}^{(S_1)}, p_{3,n}^{(S_2)}, \mathbf{A}_n^{(Q)} \right\}, \\ (Q &= P, S_1, S_2; n = 1, \dots, N). \end{aligned} \quad (3)$$

Since the slowness components  $p_{i,n}^{(S_1)}$  and  $p_{i,n}^{(S_2)}$  ( $i = 1, 2$ ) have to be estimated along with the components of the tensor  $\mathbf{c}$ , the inversion becomes nonlinear.

**Scenario 3.** If the subsurface is so complicated that it is impossible to get acceptable estimates of the horizontal slownesses of either  $P$ - and  $S$ -waves at the receiver locations (Figure 1c), we operate with the model and data vectors

$$\begin{aligned} \mathbf{m} &= \left\{ \mathbf{c}, p_{1,n}^{(Q)}, p_{2,n}^{(Q)} \right\}, \quad \mathbf{d} = \left\{ p_{3,n}^{(Q)}, \mathbf{A}_n^{(Q)} \right\}, \\ (Q &= P, S_1, S_2; n = 1, \dots, N). \end{aligned} \quad (4)$$

The inversion procedure is again nonlinear.

### Inversion scheme and numerical results

The Christoffel equation (1) written in the form

$$\mathbf{F}(\mathbf{m}, \mathbf{d}(\mathbf{m})) = 0, \quad (5)$$

where  $\mathbf{m}$  and  $\mathbf{d}$  are the model and data vectors given by equations (2)–(4), allows us to relate the data and model perturbations as

$$\Delta \mathbf{d} = \mathcal{F} \Delta \mathbf{m}. \quad (6)$$

Here the matrix

$$\mathcal{F} = - \left( \frac{\partial \mathbf{F}}{\partial \mathbf{d}} \right)^\dagger \frac{\partial \mathbf{F}}{\partial \mathbf{m}} \quad (7)$$

has the meaning of the Frechét derivative matrix, and  $\dagger$  denotes the pseudo-inverse.

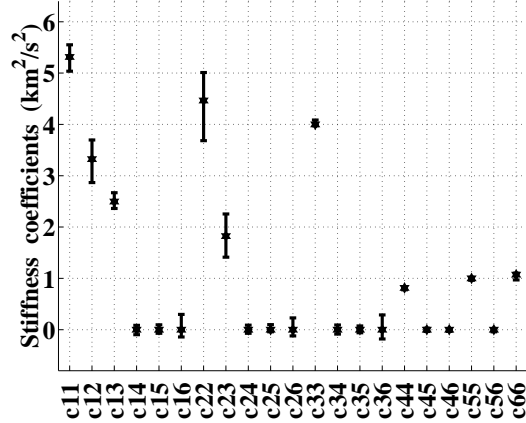


Fig. 2: Results of linear inversion (scenario 1) of polarization and slowness vectors. Stars denote the exact values of the stiffness coefficients (corresponding to an orthorhombic model), bars represent  $\pm$  one standard deviation in the estimated quantities. The data were computed at 24 directions specified by four polar angles  $\alpha^{(1)} = [13^\circ, 26^\circ, 39^\circ, 52^\circ]$  and six azimuths  $\alpha^{(2)} = [0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ]$ .

In scenario 1, the problem of obtaining the stiffnesses  $c_{ij}$  from equations (1) is linear. Its solution is

$$\mathbf{m} = \mathcal{F}^\dagger \mathbf{d}. \quad (8)$$

For scenarios 2 and 3, when the inversion becomes nonlinear, we implemented the following procedure. First, given a trial stiffness tensor  $\mathbf{c}$ , we minimize  $\mathbf{F}^2(\mathbf{c}, \mathbf{p}^{(Q)})$  [equation (1)] with respect to the horizontal slownesses  $p_1^{(Q)}$  and  $p_2^{(Q)}$  at each data point. Once estimates of the horizontal slownesses are obtained, we seek a minimum of the objective function

$$\Phi(\mathbf{c}) = \sum \mathbf{F}^2(\mathbf{c}, \mathbf{p}^{(Q)}) \quad (9)$$

with respect to  $\mathbf{c}$ . The summation here is performed over all data points, and the conjugate gradient method is used in both optimization steps.

To test the inversion algorithm, we computed the polarization and slowness vectors in an orthorhombic model. Then, the data were contaminated with Gaussian noise that had standard deviations 2% for the slownesses and  $10^\circ$  for the polarization vectors. The stiffnesses were estimated for different realizations of the noise. We did not use information about the symmetry of the model, so the data were inverted for triclinic media.

The result presented in Figure 2 (scenario 1) clearly indicates that all elements of the stiffness tensor that are strictly zero in the original orthorhombic medium are, indeed, small. Thus, knowing the symmetry of the model *a priori* is not needed for the inversion; the symmetry can be inferred from the results. The larger error bars for the computed stiffness coefficients  $c_{11}$ ,  $c_{12}$ , and  $c_{22}$

## Inversion of VSP data

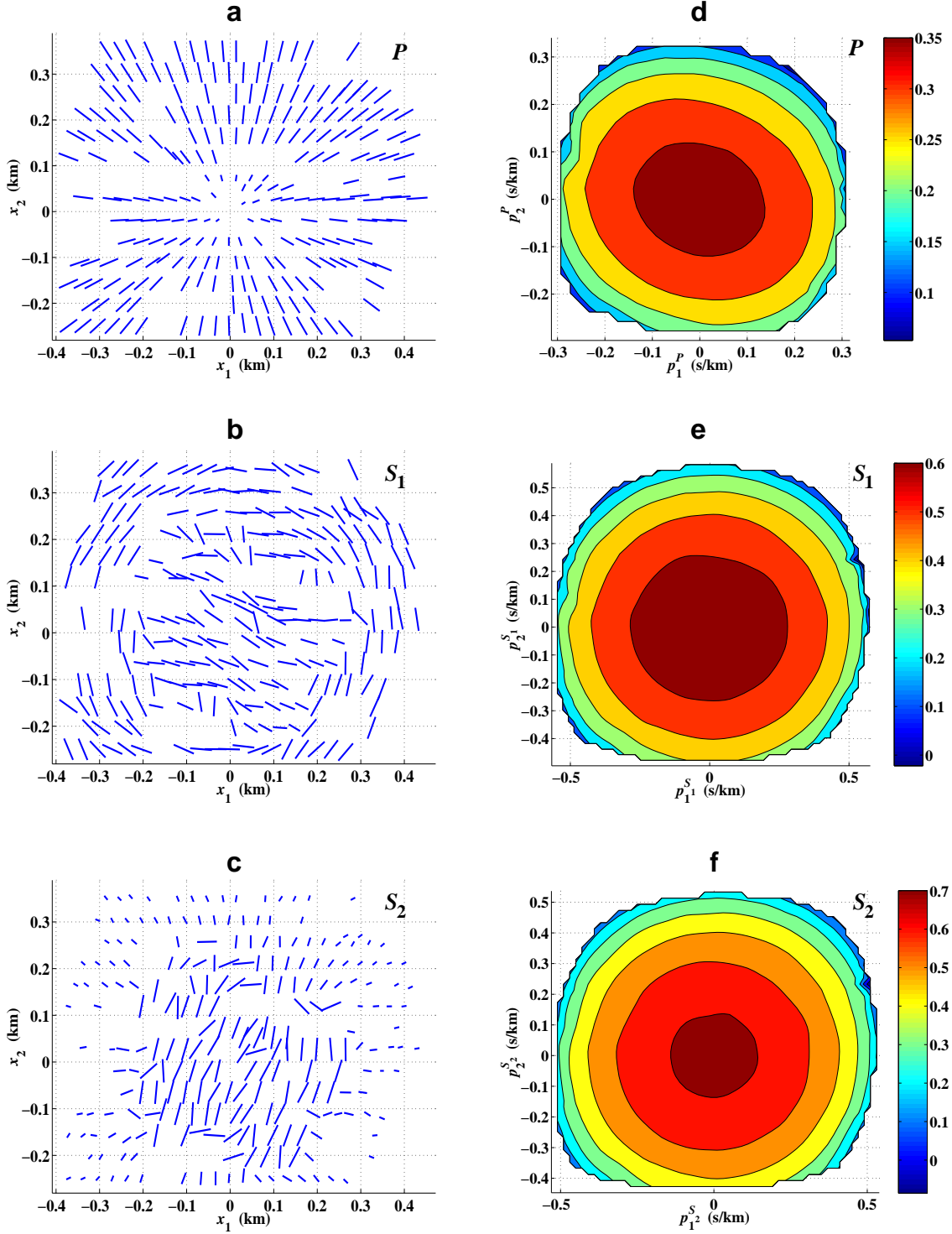


Fig. 3: (a, b, c): Horizontal components of the polarization vectors  $A_i^{(Q)}$  ( $i = 1, 2$ ); (d, e, f): vertical slowness components  $p_3^{(Q)}$  ( $p_1^{(Q)}, p_2^{(Q)}$ ) of  $P$ -,  $S_1$ -, and  $S_2$ -waves for a receiver at depth 304.8 m in well VGWU 127. The well is located at  $x_1 = x_2 = 0$ .

## Inversion of VSP data

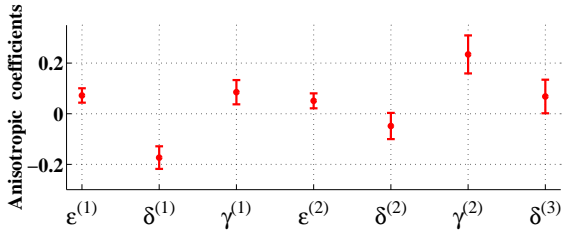


Fig. 4: Tsvankin’s (1997) anisotropic coefficients for the estimated orthorhombic medium. Dots indicate the mean values, bars correspond to the 95% confidence intervals. The vertical velocities for the obtained models are  $V_{P0} = 2.66 \pm 0.05$  km/s and  $V_{S0} = 1.44 \pm 0.04$  km/s (not shown).

as compared to those for  $c_{33}$ ,  $c_{44}$ , and  $c_{55}$  can be explained by the limited polar coverage of the data (less than  $52^\circ$  with the vertical). When we follow scenario 2 to invert the stiffnesses from the same data [except for the slowness components  $p_{1,n}^{(S_1)}$ ,  $p_{2,n}^{(S_1)}$ ,  $p_{1,n}^{(S_2)}$ , and  $p_{2,n}^{(S_2)}$  that now become a part of the model; compare equations (2) and (3)], we obtain less accurate but still reasonable results (not shown). The inversion for scenario 3 reveals a serious deterioration in the obtained estimates. Not only do the error bars generally increase compared to those in Figure 2, but the method also fails to constrain a number of stiffness coefficients.

### Field-data example

Next, we apply our inversion scheme to a 3-D walkaway VSP data set acquired in the Vacuum field, Lea County, New Mexico. The data were recorded by a string of 10 three-component geophones placed in the vertical well VGWU 127 with a 15 m depth increment over the depth range 304.8 m to 439.8 m. Vertical and horizontal vibrators were used to excite  $P$ - and  $S$ -waves. The details of data acquisition and initial processing can be found in Michaud (2001).

Figure 3 shows the polarizations and slownesses used to invert the stiffness coefficients. Since the subsurface structure in the vicinity of well VGWU 127 is close to horizontally layered with almost no lateral velocity variation (e.g., Michaud, 2001), we apply the linear inversion strategy of scenario 1 and make no assumption about the medium symmetry. To obtain a suite of inversion results rather than a single model, we contaminate the smoothed polarizations and slownesses (such as those shown in Figure 3) with Gaussian noise. According to the estimates of Michaud (2001), the errors in the polarization and slowness vectors are  $12^\circ$  and 5%, respectively. We use these values as the corresponding standard deviations. It turns out that the inverted stiffness coefficients are accurately described by an *azimuthally rotated* orthorhombic medium with horizontal symmetry plane and the local  $x'_1$ -axis pointing at  $28^\circ$  SE. The velocities (along the  $x'_3$ -axis) of the obtained orthorhombic model and its anisotropic coefficients (Tsvankin, 1997) are shown in Figure 4.

Our results agree with existing studies of the Vacuum

Field performed by students of the Reservoir Characterization Project at Colorado School of Mines. For instance, the analysis of the polarization of  $S$ -waves and the focusing of shear-wave energy led to the conclusion that the overburden (shallow 300 m) of the Vacuum Field has orthorhombic symmetry. In addition, the study of borehole breakouts resulted in an estimate of the azimuth of the maximum horizontal stress of about  $32^\circ$  SE.

### Discussion

The feasibility of estimating the full stiffness tensor  $\mathbf{c}$  from multi-component, multi-azimuth, walkaway VSP data depends on several factors. First, the overburden complexity limits our ability to constrain the horizontal slowness components  $p_1$  and  $p_2$  at the geophone levels. If estimates of  $p_1$  and  $p_2$  cannot be obtained (scenario 3), the problem is under-determined, rendering the inverted stiffnesses almost useless. We showed that knowledge of the horizontal slowness components of only  $P$ -waves (scenario 2) makes the inversion for all  $c_{ij}$  feasible. When the components of  $p_1$  and  $p_2$  of the shear-waves can be measured as well (scenario 1), the stability of the inverted stiffness coefficients increases.

Another factor that governs the accuracy of the inverted  $c_{ij}$  is the data coverage. Since different stiffness coefficients influence wave propagation for different ranges of polar and azimuthal angles, full coverage may be needed to obtain all  $c_{ij}$  with a comparable accuracy. While the inversion becomes more stable when the medium has a known symmetry, we showed that assuming a particular anisotropic model is unnecessary.

The presented case study corroborates this last point. We estimated the stiffness tensor of a triclinic model by fitting both the polarization and slowness vectors of the  $P$ - and two split  $S$ -waves. We saw that the obtained stiffnesses are close to those describing an azimuthally rotated orthorhombic model. The orientation of its symmetry planes fits a number of independent observations and seems to relate to the subsurface stresses.

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