PARAMETER ESTIMATION FOR VTI MEDIA USING *PP* AND *PS* REFLECTION DATA

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Summary

Combining reflection traveltimes of PP- and PS-waves provides valuable information for anisotropic velocity analysis that cannot be obtained from PP data alone. Here we show that for horizontally layered transversely isotropic media with a vertical symmetry axis (VTI), traveltimes of PSV-waves help to determine the ratio of the vertical P- and S-wave velocities, the NMO velocity of SV-waves, and give a more accurate estimate of the parameter η. However, mode conversions cannot resolve the nonuniqueness in reconstructing the depth scale of the model and the anisotropic parameters ϵ and δ, even for a single layer.

The depth-domain velocity analysis of PP and PS data becomes feasible in the presence of reflector dip. For wide-azimuth multicomponent 3-D surveys it is possible to constrain all VTI parameters (including the SH-wave coefficient γ) by combining the moveouts of PP- and split PS-waves from a *single* mildly dipping reflector. To avoid problems caused by such features of PS-waves as moveout asymmetry, polarity reversal and reflection-point dispersal, PP and PS traveltimes can be recomputed into the traveltimes of the corresponding pure SS reflections. Then the anisotropic velocity model is obtained by applying stacking-velocity tomography to the NMO ellipses of PP- and SS-waves. We implemented this inversion/processing methodology for VTI media composed of homogeneous layers separated by plane or curved interfaces.

Introduction

A major complication caused by anisotropy in velocity analysis is the uncertainty in estimating the vertical velocity and depth scale of the model from surface data. For VTI media, P-wave kinematic signatures are governed by the vertical velocity V_{P0} and Thomsen anisotropic parameters ϵ and δ. However, only two combinations of these parameters – the NMO velocity from a horizontal reflector and the “anellipticity” coefficient η – can be determined from P-wave reflection traveltimes, if the medium above the reflector is laterally homogeneous.

A practical option for implementing depth-domain velocity analysis in VTI media is to combine PP-waves with converted PS-wave data. We discuss the nonuniqueness of the joint inversion of PP and PS reflection traveltimes for horizontally layered VTI media and then introduce a tomographic inversion procedure operating with dipping events recorded in multicomponent wide-azimuth surveys.

Inversion of nonhyperbolic moveout of horizontal *PP* and *PS* events

As shown by Tsvankin and Thomsen (1995), the interval vertical velocities of P- and S-waves (V_{P0} and V_{S0}) and the coefficients ϵ and δ in horizontally layered VTI media can be estimated from long-spread (nonhyperbolic) moveout of PP and SS ($SVSV$) reflections. The question addressed here is whether or not it is possible to replace pure SS-waves by large-offset mode-converted PS (PSV) data acquired over a single horizontal VTI layer. Long-spread PP-wave moveout can be used to estimate the NMO velocity $V_{\mathrm{nmo},P}$ and the coefficient η:

$$V_{\mathrm{nmo},P} = V_{P0}\sqrt{1+2\delta}\,; \qquad \eta \equiv \frac{\epsilon-\delta}{1+2\delta}\,. \tag{1}$$

The trade-off between these parameters, however, may lead to sizable errors in η which can reach 0.1, even for this simple model (Grechka and Tsvankin, 1998). Hyperbolic velocity analysis of conventional-spread PS data yields the vertical PS traveltime $t_{PS0} = t_{P0} + t_{S0}$ (t_{P0} and t_{S0} are the one-way vertical times of the P- and S-waves) and the PS-wave NMO velocity. Using the relationship between the NMO velocities of pure and converted modes, we can obtain the

Data	t_{P0} (s)	t_{S0} (s)	$V_{\mathrm{nmo},P}$ (km/s)	$V_{\mathrm{nmo},S}$ (km/s)	η
Correct	0.800	2.000	2.622	1.696	0.136
Erroneous					
1	0.800	2.000	2.648	1.696	0.116
2	0.800	2.000	2.635	1.696	0.126
3	0.800	2.000	2.609	1.696	0.146
4	0.800	2.000	2.596	1.696	0.156

Table 1. Correct and erroneous sets of the moveout parameters. The four erroneous sets have slightly distorted values of $V_{\mathrm{nmo},P}$ and η.

Model parameters	V_{P0} (km/s)	V_{S0} (km/s)	ϵ	δ	σ	D (km)
Correct	2.500	1.000	0.200	0.050	0.938	1.000
Erroneous						
1	2.787	1.115	0.057	−0.049	0.657	1.115
2	2.646	1.058	0.121	−0.004	0.783	1.058
3	2.349	0.939	0.298	0.117	1.128	0.939
4	2.190	0.876	0.421	0.202	1.372	0.876

Table 2. Correct VTI parameters and those corresponding to the four erroneous sets of moveout parameters from Table 1. D is the reflector depth.

SV-wave NMO velocity $V_{\mathrm{nmo},S}$ given by

$$V_{\mathrm{nmo},S} = V_{S0}\,\sqrt{1+2\sigma}\,; \qquad \sigma \equiv \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\epsilon-\delta)\,. \tag{2}$$

In principle, the parameters $V_{\mathrm{nmo},P}$, η, $V_{\mathrm{nmo},S}$ and the vertical times t_{P0} and t_{S0} can be used to recover all four unknowns (V_{P0}, V_{S0}, ϵ and δ). For example, δ can be found as

$$1+2\,\delta = \left(\frac{t_{P0}}{t_{S0}}\right)^2 \frac{1}{\left(V_{\mathrm{nmo},S}/V_{\mathrm{nmo},P}\right)^2 - 2\,\eta}\,. \tag{3}$$

In vertically inhomogeneous media, the interval NMO velocities of PP- and SS-waves can be found from the conventional Dix equation and combined with the interval η values to perform parameter estimation. However, as indicated by the form of the denominator in equation (3), this inversion procedure is unstable, with realistic small errors in η and the NMO velocities propagating with considerable amplification into the inverted parameters (Tsvankin and Grechka, 2000a).

Li and Yuan (1999) suggested to overcome this instability by using PS moveout at large (compared with the reflector depth) offsets. Here, we demonstrate that the combination of long-spread moveouts of PP- and PS-waves is still insufficient for unambiguous parameter estimation. Let us perturb a typical VTI model from Table 1 (top row) by introducing small errors into the parameter $\eta = 0.136$. To keep the long-spread PP-wave moveout almost unchanged, the errors in η are compensated by small changes (up to 1%) in $V_{\mathrm{nmo},P}$ (Grechka and Tsvankin, 1998). Table 2 lists the VTI parameters of the correct model and of the four erroneous ones from Table 1. As expected, the small errors in $V_{\mathrm{nmo},P}$ and η lead to large (greater than 10%) errors in the vertical velocities and the reflector depth; also, the anisotropic parameters ϵ, δ and σ are severely distorted. However, all four vastly different erroneous models have practically the same *long-spread* moveout of both PP- and PS-waves as the correct model, with the difference limited by 1 ms (Figure 1). The maximum offset-to-depth ratio X/D in this test varies from 2.7 to 3.4 in accordance with the change in the reflector depth D (Table 2).

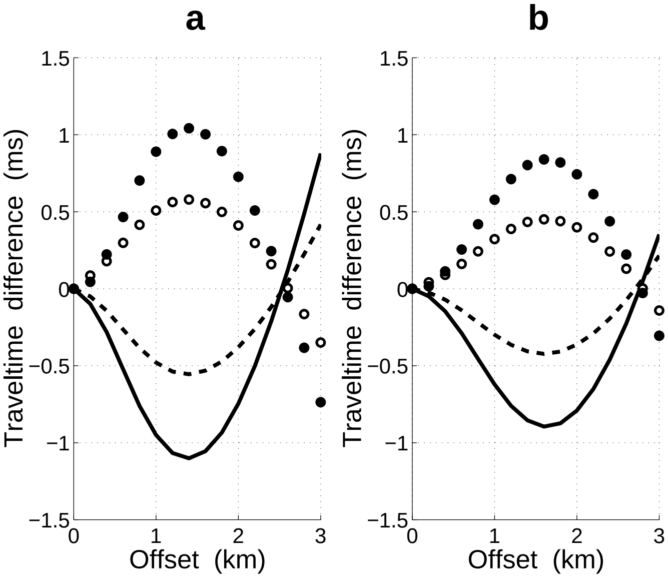

FIG. 1. Differences between the traveltimes of PP-waves (a) and PS-waves (b) computed for the erroneous models from Tables 1,2 and the correct model. The solid line corresponds to model 1 (see the left row) from Table 2, dashed line to model 2, circles to model 3, and dots to model 4.

Clearly, even for large offsets-to-depth ratios seldom attained in practice, the combination of PP and PS traveltimes cannot be inverted for the vertical velocities and the parameters ϵ and δ. Since the S-wave reflection angle for mode conversions is limited by the critical angle, the traveltimes of PS-waves at large offsets are primarily controlled by the S-wave NMO velocity and the parameter η responsible for PP-wave nonhyperbolic moveout. It should be mentioned, however, that the increased incidence angle of P-waves for large-offset PS arrivals helps to better constrain the parameter η.

Inversion of wide-azimuth PP and PS data from dipping reflectors

If the subsurface contains non-horizontal (dipping or irregular) interfaces, the nonuniqueness of the parameter estimation can be overcome by using *dipping* events. As demonstrated by Tsvankin and Grechka (2000a), the interval parameters ϵ and δ, along with the vertical velocities V_{P0} and V_{S0}, can be determined by combining the dip-line moveout of PP- and $PS(PSV)$-waves from two (horizontal and dipping) reflectors. Alternatively, it is possible to obtain all VTI parameters using a *single* dipping reflector, if PP and PS data are recorded for a wide range of source-receiver azimuths (Tsvankin and Grechka, 2000b). However, the dipping PS events possess such features as moveout asymmetry, reflection-point dispersal and polarity reversal, which substantially complicate anisotropic parameter estimation. Grechka and Tsvankin (2001) devised a technique for transforming PP and PS traveltimes into the moveout of the corresponding pure SS reflections that can be processed by pure-mode velocity-analysis algorithms.

Here, extending our results for PP-wave data (Grechka et al., 2001), we develop the so-called "stacking-velocity tomography" that operates with stacking (moveout) velocities of PP- and SS-waves measured on moderate-length CMP spreads (i.e., spreads close to the reflector depth). Although this approach excludes the far-offset information from analysis, it has two important advantages over conventional tomography. First, azimuthally-varying moveout velocity, or the NMO ellipse, can be computed by tracing only one zero-offset ray per common midpoint and per reflector, which makes anisotropic traveltime tomography computationally feasible (Grechka et al., 2001). Second, even for lower-symmetry systems NMO ellipses can be described by semi-analytic expressions providing valuable insight into the parameter combinations constrained by a certain set of input data. We implemented the multicomponent tomographic procedure for TI media composed of homogeneous layers separated by plane or smooth curved interfaces. The algorithm includes the following main steps:

- Picking PP and PS traveltimes from pre-stack 3-D data volumes and identifying the events reflected from the same interface.
- Computation of the traveltimes of the pure SS reflections from PP and PS data using the method of Grechka and Tsvankin (2001).
- Azimuthal velocity analysis to reconstruct the NMO ellipses of the PP- and SS-waves.
- Inversion of the NMO ellipses, reflection slopes and zero-offset traveltimes for the interval anisotropic parameters by extending the approach suggested by Grechka et al. (2001) for PP-

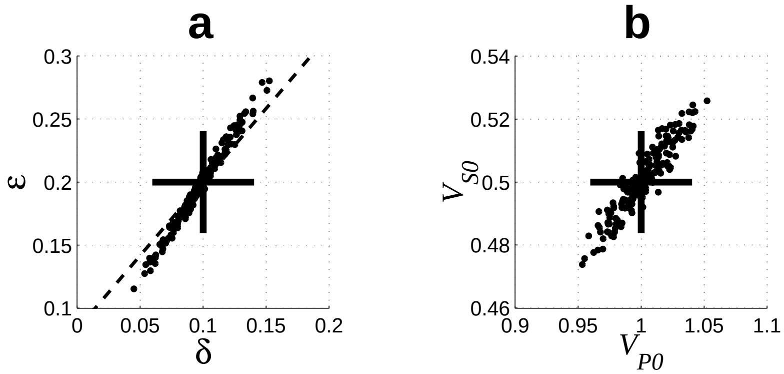

FIG. 2. Results (dots) of the joint inversion of PP and SS ($SVSV$) data for a single VTI layer above a dipping reflector. The correct layer parameters are marked by the crosses, the reflector dip is 15°, the velocities are given in km/s. The dashed line on plot (a) corresponds to the correct value of η. The data were contaminated by Gaussian noise with the standard deviations equal to 2% for the NMO velocities and 1% for the zero-offset traveltimes and the reflection slopes.

waves.

Figure 2 illustrates application of this methodology to wide-azimuth PP and PS (PSV) traveltimes (the latter were recomputed into SS traveltimes) generated for the simplest model of a homogeneous VTI layer with a *dipping* lower boundary. The dots in Figure 2 mark the estimated VTI parameters for different realizations of the noise added to the input data. The standard deviations in the inverted parameters (1% for V_{P0} and V_{S0}, 0.03 for ϵ and 0.02 for δ) indicate that the noise does not get amplified by the parameter-estimation procedure. This test corroborates the stability of the joint inversion of wide-azimuth PP and PS data from a mildly dipping reflector (the dip is just 15°).

The remaining anisotropic coefficient γ can be estimated using PSH conversions which exist for all azimuthal directions outside the dip plane of the reflector. Similar results were obtained for more complicated multilayered TI models with dipping and irregular interfaces.

Conclusions

The combination of long-spread PP and PS (PSV) data in horizontally layered VTI media is insufficient for constraining the vertical velocities V_{P0} and V_{S0} and Thomsen parameters ϵ and δ. There is an infinite number of models with vastly different VTI parameters which have practically the same reflection moveout of PP- and PS-waves for offsets as large as three times the reflector depth.

This ambiguity can be overcome by including dipping PP and PS events in the inversion, either in 2-D or 3-D geometry. 2-D parameter estimation on the dip line of the structure requires both horizontal and dipping events in each depth interval, whereas 3-D inversion can be performed for wide-azimuth multicomponent data from a single dipping interface (the dip can be as small as 15°). Our approach to interval parameter estimation in layered VTI media is based on stacking-velocity tomography that is several orders of magnitude more efficient than conventional tomographic algorithms operating directly with reflection traveltimes.

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