Processing-induced anisotropy

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Summary

Processing of seismic data is often performed under the assumption that the velocity distribution in the subsurface can be approximated by a macro-model composed of isotropic homogeneous layers or blocks. Despite being physically unrealistic, such models are believed to be sufficient for describing the kinematics of the reflection arrivals.

Here, we examine the distortions in normal-moveout (NMO) velocities caused by the vertical heterogeneity unaccounted for in velocity analysis. To match P-wave moveout measurements from a horizontal or a dipping reflector overlaid by a vertically heterogeneous *isotropic* medium, the effective homogeneous model has to be *anisotropic*. Assuming that the effective medium is transversely isotropic with a vertical symmetry axis (VTI), we express the VTI parameters through the depth-dependent isotropic velocity function. If the reflector is horizontal, combining the NMO and vertical velocities always produces non-negative values of the Thomsen's coefficient δ. The effective anellipticity coefficient η, obtained from the P-wave NMO ellipse for a dipping reflector, is non-negative as well. These results also indicate a potential bias toward positive δ and η values in the velocity analysis of reflection data for VTI media.

Introduction

Rapid progress in the development of anisotropic velocity-analysis methods made transverse isotropy with a vertical symmetry axis (VTI media) a common model in depth and time processing of P-wave data. Depth imaging in VTI media requires estimates of the P-wave vertical velocity V_0 and Thomsen's (1986) anisotropic coefficients ϵ and δ, while time imaging for models with a laterally homogeneous overburden is controlled by the NMO velocity for horizontal reflectors $V_{\rm nmo}$ and Alkhalifah-Tsvankin (1995) coefficient η defined as

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\,\delta}\,. \quad (1)$$

Both time-imaging parameters can be obtained from P-wave reflection traveltimes using either dip-dependent P-wave NMO velocity or nonhyperbolic moveout from horizontal interfaces (e.g., Alkhalifah and Tsvankin, 1995). Values of δ are often determined at borehole locations by combining the NMO (stacking) velocity $V_{\rm nmo}$ measured from surface seismic and the vertical velocity V_0 derived from check shots or well logs. For a single homogeneous VTI layer, the two velocities are related by

$$V_{\rm nmo} = V_0\,\sqrt{1 + 2\,\delta}\,. \quad (2)$$

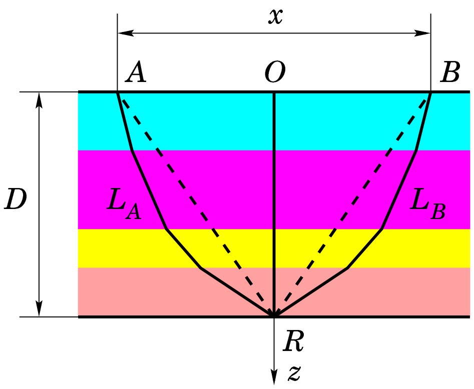

Figure 1. Zero-offset ray ORO and ray AL_ARL_BB at offset AB for the reflection from an interface beneath a vertically heterogeneous isotropic medium.

Here, we discuss *apparent* (or effective) transverse isotropy caused by simplified model assumptions commonly used in seismic processing. High-quality reflection events are recorded from only a limited number of interfaces, and it is often assumed that the medium between the interpreted reflectors is homogeneous. We show that ignoring vertical heterogeneity makes the model transversely isotropic with *non-negative* values of δ and η.

Effective *P*-wave NMO velocity from a horizontal reflector

Consider the vertical (zero-offset) and nonzero-offset traveltimes ($T(0)$ and $T(x)$, respectively) for the simple model of a horizontal reflector beneath a stack of homogeneous isotropic layers (Figure 1). If the reflector depth D is known, the traveltime $T(0)$ can be used to compute the vertical velocity V_0:

$$V_0 = \frac{2\,D}{T(0)}\,. \quad (3)$$

Since the model is vertically heterogeneous, V_0 becomes an effective quantity that averages the interval (or local) velocities $v(z)$. If the interfaces in the overburden are not strong enough to generate detectable reflection events, it is natural to treat the whole section above the reflector as homogeneous. Note that any permutation

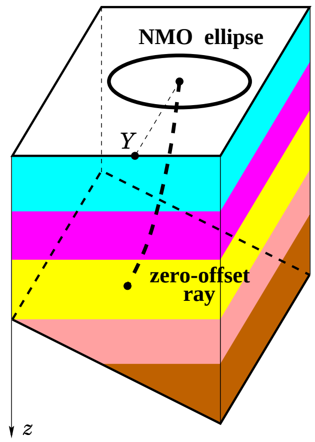

Figure 2. P-wave NMO ellipse from a dipping reflector overlaid by a vertically heterogeneous isotropic medium.

of layers in Figure 1 produces exactly the same reflection traveltime $T(x)$ at any offset x; this phenomenon is called the "O-equivalence" of velocity functions (Goldin, 1986). Thus, for processing purposes it is necessary to *assume* a certain velocity distribution $v(z)$.

If the composite layer in Figure 1 is treated as homogeneous and isotropic in accordance with the usual practice of velocity analysis, the reflection traveltime $T(x)$ is supposed to be a hyperbola parameterized by the vertical velocity V_0,

$$T^2(x) = T^2(0) + \frac{x^2}{V_0^2}\,. \tag{4}$$

The traveltime from equation (4), however, corresponds to ray ARB (Figure 1) that does not satisfy Fermat's principle. The actual traveltime $T(x)$ along the geometrical ray $A\,L_A R\,L_B B$ is smaller than that predicted by equation (4). As a result, the NMO velocity $V_{\rm nmo}$ for the model in Figure 1 is always *greater* than the vertical velocity V_0, and the data cannot be explained in terms of a homogeneous *isotropic* model.

The obtained relationship between $V_{\rm nmo}$ and V_0 is typical for *transversely isotropic* media with a positive value of δ [see equation (2)]. It can be shown that $V_{\rm nmo}^2/V_0^2$ is equal to the ratio of the arithmetic and harmonic averages of the interval velocities, which implies that the parameter δ of the effective VTI model is given by

$$1+2\,\delta = \frac{V_{\rm nmo}^2}{V_0^2} = \left[\frac{1}{D}\int\limits_0^D v(z)\,dz\right]\left[\frac{1}{D}\int\limits_0^D \frac{dz}{v(z)}\right] \geq 1\,. \tag{5}$$

This relationship might also explain a certain bias toward positive δ values derived from stacking (NMO) and vertical velocities for purposes of anisotropic parameter estimation.

Effective P-wave NMO ellipse from a dipping reflector

Next, consider P-wave reflection moveout from a plane *dipping* reflector beneath a vertically heterogeneous isotropic medium (Figure 2). The azimuthally dependent P-wave NMO velocity for this model is described by the NMO ellipse with the axes in the dip and strike directions of the reflector (Grechka and Tsvankin, 1998). Since the dip plane of the reflector represents a plane of symmetry for the whole model, the Dix-type averaging of the interval NMO ellipses (Grechka et al., 1999) reduces to the conventional Dix formula for the NMO velocities in the dip ($V_{\rm nmo,dip}$) and strike ($V_{\rm nmo,str}$) directions. In terms of the ray parameter p of the zero-offset ray, those velocities can be written as

$$V_{\rm nmo,dip}^2 = \frac{1}{T}\int\limits_0^T \frac{v^2\,dt}{1-p^2\,v^2}\,, \tag{6}$$

$$V_{\rm nmo,str}^2 = \frac{1}{T}\int\limits_0^T v^2\,dt\,; \tag{7}$$

T is the zero-offset traveltime. If the medium above the reflector were homogeneous, the dip and strike components of the NMO velocity would satisfy the following well-known (cosine-of-dip) relationship:

$$V_{\rm nmo,dip}^2\left(1-p^2\,V_{\rm nmo,str}^2\right) = V_{\rm nmo,str}^2\,. \tag{8}$$

Using equations (6) and (7), it can be shown that in the presence of vertical heterogeneity,

$$\tilde V_{\rm nmo,dip}^2 \equiv V_{\rm nmo,dip}^2\left(1-p^2\,V_{\rm nmo,str}^2\right) \geq V_{\rm nmo,str}^2 \tag{9}$$

for *any* interval-velocity function $v(t)$. $\tilde V_{\rm nmo,dip} = V_{\rm nmo,dip} = V_{\rm nmo,str}$ only in the special case of a horizontal reflector ($p=0$).

Figure 3 shows the two-way zero-offset traveltime T and the velocities $\tilde V_{\rm nmo,dip}$ and $V_{\rm str}$ computed for a model that contains a plane dipping reflector $z = 1 + Y\tan\phi$ (ϕ is the dip) beneath an isotropic medium with a constant vertical-velocity gradient [$v(z) = 1 + 0.6\,z$]. The common-midpoint (CMP) locations Y in Figure 3 record reflections from the segment of the interface within the depth range 1 km $\leq z \leq$ 2 km. Note that the zero-offset time T (Figure 3a) varies with Y because of the combined influence of the reflector dip and vertical heterogeneity. The gradual decrease of the reflection slope $p(Y) = (1/2)\,(dT/dY)$ in Figure 3a and the corresponding increase of the velocities in Figure 3b may lead to the conclusion that the subsurface is *laterally* heterogeneous.

Also, since $\tilde V_{\rm nmo,dip} \neq V_{\rm nmo,str}$, the medium above the reflector may be mistakenly identified as anisotropic.

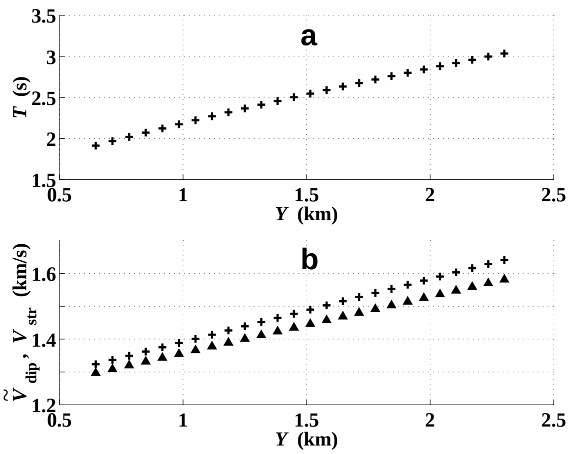

Figure 3. Normal moveout for a dipping reflector beneath a vertically heterogeneous isotropic medium. (a) Two-way zero-offset traveltime T as a function of the CMP coordinate Y in the dip plane of the reflector (the dip $\phi = 40°$); (b) quantity $\tilde{V}_{\rm nmo,dip}$ (crosses) defined by equation (9) and the strike-line NMO velocity $V_{\rm nmo,str}$ (triangles).

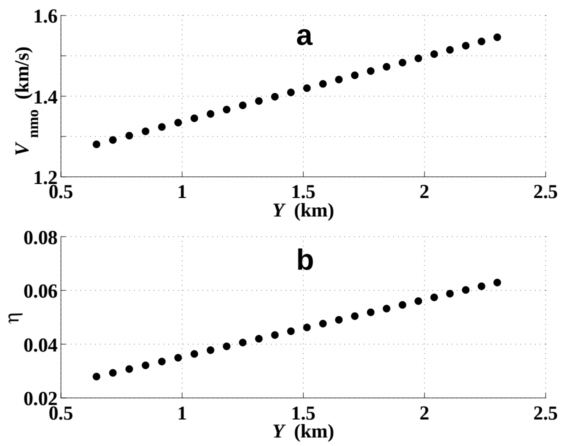

Figure 4. Effective zero-dip NMO velocity $V_{\rm nmo}$ (a) and the anellipticity coefficient η (b) estimated from the traveltimes and NMO velocities shown in Figures 3a,b under the assumption of a vertically homogeneous VTI model.

Below we invert the dip- and strike-components of the normal-moveout velocity for the effective anellipticity coefficient η under the assumption that the model is VTI and show that such η is always non-negative.

Estimation of the effective $V_{\rm nmo}$ and η

Clearly, the subsurface parameters obtained from the traveltimes and velocities in Figures 3a and 3b will depend on the selected model of the overburden. If the model is assumed (correctly) to be vertically heterogeneous and isotropic, the reflection data can be used to estimate the actual function $v(z)$ within the depth range 1 km $\le z \le$ 2 km covered by the reflection points (Goldin, 1986). Still, as mentioned above, the velocity $v(z)$ cannot be found uniquely for depths $z < 1$ km.

Therefore, it may be more attractive from the practical point of view to adopt a model that is vertically homogeneous but changes laterally. In this case, it is possible to estimate the medium parameters in a unique fashion for the whole overburden, although the inverted model cannot be isotropic (indeed, $\tilde{V}_{\rm nmo,dip} > V_{\rm nmo,str}$). If we ignore the influence of lateral heterogeneity on the NMO-velocity measurements on the scale of a single CMP gather, as is usually done in practice, the velocities $V_{\rm nmo,dip}(Y)$ and $V_{\rm nmo,str}(Y)$ can be inverted for the anisotropic parameters at each CMP location. Then, the dependence of the obtained parameters on Y can be interpreted in terms of lateral heterogeneity.

Since the reflection traveltimes are symmetric with respect to the dip plane, it is natural to use the azimuthally isotropic VTI model for the inversion. We applied the algorithm of Grechka and Tsvankin (1998) based on the exact NMO equations to estimate the zero-dip NMO velocity $V_{\rm nmo}(Y)$ and the anellipticity coefficient $\eta(Y)$ of the effective VTI medium. Both $V_{\rm nmo}$ and η vary with the CMP coordinate Y (Figure 4), which can be attributed to the influence of lateral heterogeneity.

It should be emphasized that the effective η is positive; this is a general result that follows directly from inequality (9) in the limit of weak anisotropy. The value of η is controlled not only by the magnitude of vertical heterogeneity, but also by the reflector dip. Clearly, for a horizontal reflector the NMO ellipse degenerates into a circle, and the effective η vanishes. With increasing reflector dip and eccentricity of the NMO ellipse, the isotropic relationship (8) between $V_{\rm nmo,dip}(Y)$ and $V_{\rm nmo,str}(Y)$ becomes less accurate, which leads to higher values of η (Figure 5). It can be also shown that the effective NMO velocity $V_{\rm nmo}$ decreases with dip.

It might be thought that the non-negligible values of η for the model from Figure 3a,b are associated with the monotonic increase in velocity with depth. However, since the apparent anisotropy is caused by the different types of averaging applied to the vertically varying velocity to obtain the measured (effective) quantities, the phenomena discussed above can be observed in any $v(z)$ media. For example, Figure 6 shows the inverted effective $V_{\rm nmo}$ and η for the isotropic velocity $v(z)$ specified as a random Gaussian function (the mean is 1 km/s, the standard deviation is 0.15 km/s). As for the model from Figure 3, the velocities $V_{\rm nmo}$ are smaller than the mean of $v(z)$ (Figure 6b) and the coefficients η are positive (Figure 6c).

Discussion and conclusions

Complicated, spatially varying isotropic velocity fields are sometimes kinematically equivalent to simpler

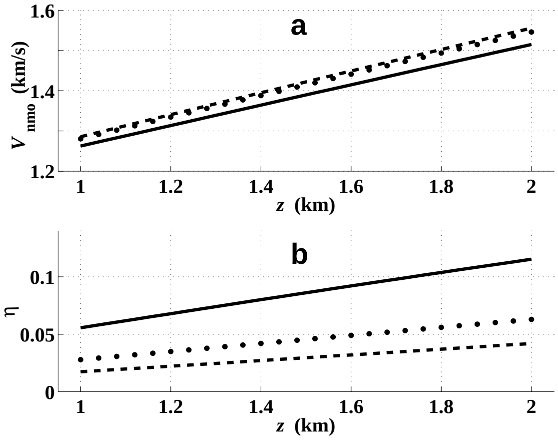

Figure 5. Effective V_{nmo} (a) and η (b) estimated for reflectors with the dips $\phi = 30°$ (dashed), $\phi = 40°$ (dots), and $\phi = 50°$ (solid) overlaid by a heterogeneous isotropic medium with the velocity function $v(z) = 1 + 0.6\,z$. The depths of the reflection points range from 1 to 2 km.

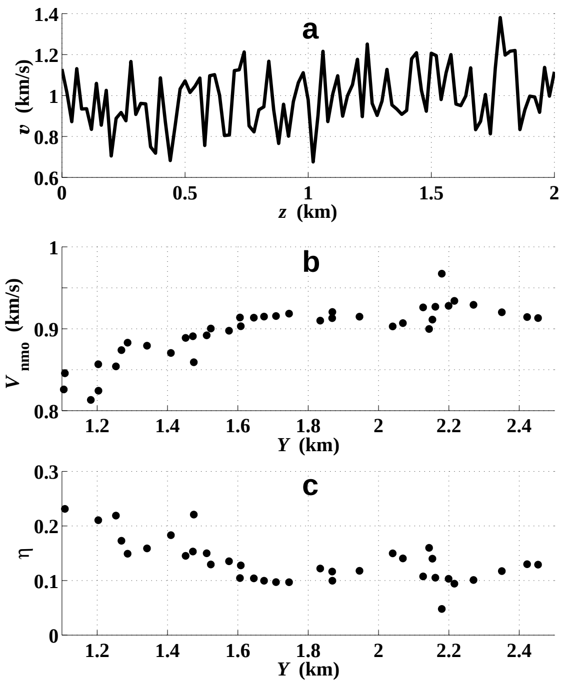

Figure 6. Isotropic velocity model (a) and the inverted effective V_{nmo} (b) and η (c). The reflector dip is $\phi = 45°$, the depths of the reflection points range from 1 to 2 km.

effective anisotropic models, which poses a serious challenge for anisotropic velocity analysis. Here, we examined one of the consequences of approximate treatment of vertical heterogeneity in estimating the subsurface velocity field. If a heterogeneous medium between reflectors is treated as a homogeneous layer, the traveltime measurements cannot be fit without introducing "apparent" (non-existent) aisotropy.

The apparent (or effective) VTI model, equivalent to a vertically heterogeneous isotropic medium above a horizontal reflector, has a non-negative coefficient δ. If the reflector is dipping, the relationship between the semi-axes of the P-wave NMO ellipse yields a non-negative effective anellipticity coefficient η. It is interesting to note that although the inequality $\eta \geq 0$ was obtained from 3-D inversion using the dip- and strike-components of the P-wave NMO ellipse, the same result follows from the 2-D dip-moveout method of η-estimation developed by Alkhalifah and Tsvankin (1995).

Therefore, ignoring vertical velocity variation in isotropic media creates an apparent VTI model in which $\epsilon \geq \delta \geq 0$. This result also provides insight into potential biases in anisotropic velocity analysis. For example, ignoring vertical velocity gradient between reflectors in VTI media should lead to overestimating the parameters η and δ. This may partially explain the discrepancy between predominantly positive δ values estimated from reflection data (e.g., Alkhalifah et al., 1996; Williamson et al., 1997) and sometimes negative values of δ derived from core measurements and VSP surveys (e.g., Thomsen, 1986; Vernik and Liu, 1997).

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