Multicomponent stacking-velocity tomography for transversely isotropic media

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Summary

Accurate estimation of the velocity field is the most difficult step in imaging of seismic data from anisotropic media. Here, the velocity-analysis problem is examined for the most common anisotropic model of sedimentary formations – transverse isotropy (TI) with arbitrary orientation of the symmetry axis. We show that supplementing wide-azimuth reflected PP data with mode-converted (PS) waves yields more stable estimates of the anisotropic coefficients and, in many cases, helps to constrain the depth scale of the model.

An important processing step preceding the inversion is reconstruction of the traveltimes of the pure SS-waves from those of the PP- and PS-waves based on the technique recently developed by Grechka and Tsvankin. This procedure allows us to replace PS-wave moveout, which is generally asymmetric with respect to zero offset, with the symmetric (hyperbolic on short spreads) moveout of the pure SS reflection. Then, generalizing the algorithm previously suggested for PP data, we develop a joint tomographic inversion of the NMO ellipses and zero-offset traveltimes of the PP- and SS-waves. Application of the method to wide-azimuth PP and PS reflections from a dipping interface beneath a homogeneous TI layer shows that for a range of reflector dips and tilt angles of the symmetry axis it is possible to build the anisotropic velocity field in the depth domain. We also extend the tomographic procedure to layered TI media with curved interfaces and study its stability in the presence of noise.

Introduction

Acquisition and processing of multicomponent seismic data demonstrated that seismic anisotropy has a first-order influence on the signatures of mode-converted PS reflections. For example, the velocity anisotropy of SV- and PSV-waves in TI media is mostly controlled by the coefficient $\sigma \equiv (V_{P0}/V_{S0})^2(\epsilon - \delta)$, which is typically much larger that the Thomsen parameters ϵ and δ governing P-wave data (V_{P0} and V_{S0} are the symmetry-direction P- and S-wave velocities, respectively). Mis-ties between PP and PS sections routinely produced by conventional isotropic imaging methods indicate the need for joint anisotropic velocity analysis of PP and PS reflection events. As shown by Tsvankin and Grechka (2000, 2001), wide-azimuth reflection traveltimes of PP- and PSV-waves from a single mildly dipping reflector are sufficient for estimating all relevant parameters (V_{P0}, V_{S0}, ϵ and δ) of VTI media. It should be emphasized that the vertical velocity and reflector depth are difficult to constrain using PP-wave moveout alone (Grechka et al., 2000).

Here we extend our previous results on the inversion of PP and PS data by introducing the methodology of anisotropic multicomponent stacking-velocity tomography and applying it to TI media with an arbitrary tilt of the symmetry axis. Rather than working with PS data directly, we combine them with PP data to compute the traveltimes of the pure SS (SV or SH for TI media) reflections from the same interface using the algorithm of Grechka and Tsvankin (2001). The reconstruction of SS traveltimes is entirely data-driven and does not require knowledge of the velocity model. In contrast to the more complicated moveout of mode conversions, reflection traveltimes of pure SS-waves are symmetric with respect to zero offset and, for moderate offset-to-depth ratios, can be described by the NMO ellipse (Grechka and Tsvankin, 1998). The tomographic inversion procedure operates with the NMO ellipses, the zero-offset traveltimes, and the reflection slopes (measured on zero-offset time sections) of PP-waves and the reconstructed SS-waves. We examine a wide range of TI models with a tilted symmetry axis (including horizontal transverse isotropy) and establish the conditions needed for stable parameter estimation.

Methodology of stacking-velocity tomography

The goal of the tomographic algorithm introduced here is to estimate the anisotropic subsurface model using wide-azimuth measurements of stacking (moveout) velocities of PP- and SS-waves on moderate-length CMP spreads (i.e., spreads close to the reflector depth). Therefore, this approach can be classified as *anisotropic multicomponent stacking-velocity tomography.* Although limiting the input data to stacking velocities excludes the far-offset information from analysis, it has important advantages over conventional tomography. First, azimuthally-varying moveout velocity, described by the NMO ellipse, can be computed by tracing only one zero-offset ray per common midpoint and per reflector, which makes anisotropic tomography computationally feasible (Grechka et al., 2000). Second, even for lower-symmetry systems NMO ellipses can be described by semi-analytic expressions providing valuable insight into the parameter combinations constrained by a certain set of input data. Third, stacking-velocity tomography can be performed locally on the horizontal scale of a single CMP gather, and the velocity field can be estimated separately for relatively small blocks containing several adjacent common midpoints. Then global smoothing can be applied to build the laterally varying anisotropic velocity field and reflecting interfaces.

We implemented the multicomponent tomographic procedure for TI media composed of homogeneous layers separated by plane or smooth curved interfaces. The algorithm includes the following main steps:

- Picking PP and PS traveltimes from pre-stack 3-D data volumes and identifying the events reflected from the same interface.
- Computation of the traveltimes of the pure SS reflections from PP and PS data using the method of Grechka and Tsvankin (2001).
- Azimuthal velocity analysis to reconstruct the NMO ellipses of the PP- and SS-waves.
- Inversion of the NMO ellipses, zero-offset traveltimes, and reflection slopes for the interval anisotropic parameters by extending to multicomponent data the approach of Grechka et al. (2000).

Joint inversion of P- and S-wave traveltime data in a single TI layer

Consider the model of a single homogeneous TI layer with a plane lower boundary (horizontal or dipping) and arbitrary orientation of the symmetry axis. The problem addressed here is whether or not wide-azimuth reflection traveltimes of PP- and SS-waves (i.e., SV reflections reconstructed from PP and PSV data) can be inverted for the symmetry-direction velocities V_{P0} and V_{S0} and the parameters ϵ and δ. It is convenient to study the feasibility of parameter estimation by applying the weak-anisotropy approximation to the NMO ellipses and zero-offset traveltimes. The derivation has to be performed for P-waves only, because any kinematic signature of SV-waves for weak transverse isotropy can be obtained from the corresponding P-wave signature by making the following substitutions: $V_{P0} \to V_{S0}$, $\delta \to \sigma$ and $\epsilon \to 0$ (Tsvankin, 2001). The inversion for a VTI layer is discussed in Tsvankin and Grechka (2001) who show that the parameter estimation becomes sufficiently stable if the dip exceeds 15°. Below, we discuss the inversion results for TI models with a horizontal (HTI) and tilted (TTI) symmetry axis.

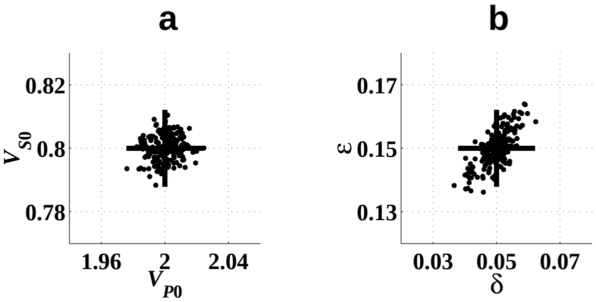

Figure 1. Results of the inversion (dots) of PP and SS traveltime data for parameters of HTI layer above the interface with dip $\phi = 25°$ using the exact equations for the NMO ellipses. The data were contaminated by Gaussian noise with the standard deviation equal to 2% for the NMO velocities and 1% for the zero-offset traveltimes and reflection slopes. Each dot corresponds to the inverted parameters for a certain realization of the noise. The correct layer parameters are marked by the crosses. V_{P0} and V_{S0} are the velocities in the symmetry-axis direction in km/s; the azimuth of the symmetry axis with respect to the reflector dip plane is $\beta = 40°$.

HTI layer

Contreras et al. (1999) studied the inversion of wide-azimuth P-wave data for HTI media and showed that the symmetry-direction velocity V_{P0}, the coefficients ϵ and δ, and the azimuth β of the (horizontal) symmetry axis can be found using the P-wave NMO ellipses from a horizontal and a dipping reflector. However, the need to use two dips for each depth interval makes this algorithm difficult to implement in practice. Our approach is designed to estimate the HTI parameters using the NMO ellipses of PP- and $SS(SVSV)$-waves from a *single* reflector that can be either horizontal or dipping. (Note that by the "SV-wave" in HTI and TTI media we mean the mode polarized in the plane formed by the slowness vector and the symmetry axis.)

The inversion for a *horizontal* HTI layer confirms the results of Tsvankin (1997) who pointed out that the combination of wide-azimuth PP- and SS-wave moveout data in should be sufficient for estimating the symmetry-direction velocities V_{P0} and V_{S0} and the parameters ϵ and δ. To examine the inversion for *dipping* interfaces, we adapted for SS-waves (see the substitution rule above) the weak-anisotropy approximations for PP-wave NMO ellipses given by Contreras et al. (1999). Both the analytic results and the inversion based on the exact equations prove that the parameter estimation remains stable for the whole range of dips from 0° to 90°. Numerical inversion of noise-contaminated data in Figure 1 yields small standard deviations in all parameters, including the azimuth β of the symmetry axis not shown on the plot (the standard deviation in β is 0.8°).

TTI layer

The parameter-estimation problem for TTI media includes only one additional unknown compared to the HTI case – the tilt ν. This, however, makes the inversion substantially more ill-posed than that for HTI media because the NMO ellipses are nonlinear functions of ν,

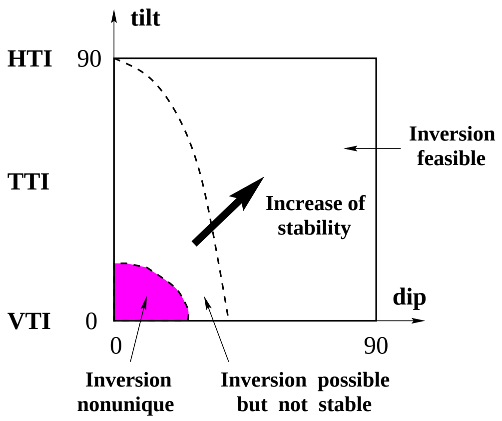

Figure 2. Stability of depth-domain parameter estimation in TI media for the full range of reflector dips and tilt angles of the symmetry axis.

even for weak anisotropy. Grechka and Tsvankin (2000) found a nonlinear dependence on the tilt in the weak-anisotropy approximations for the PP-wave NMO ellipse in HTI media, and adaptation of their equations for shear modes leads to the same result for both SV- and SH-waves. Therefore, the misfit function for the NMO ellipses may have local minima. The multimodal nature of the misfit (objective) function usually requires performing several inversions starting from different points in the model space.

The schematic summary of our numerical results is given in Figure 2. When the tilt is small, the properties of TTI media are similar to those for vertical transverse isotropy, and the inversion cannot be carried out for small reflector dips. With increasing tilt, the TTI model approaches HTI, and the parameter estimation generally becomes more stable. However, to achieve the stability comparable to that for HTI media, it is necessary to fix the tilt of the symmetry axis at the correct value (in HTI media, the tilt is *known* to be equal to 90°).

As expected, there is a relatively broad intermediate range of tilts and dips where the parameter estimation is theoretically possible but relatively unstable. The stability of the inversion in this area may be substantially increased by adding SH-wave NMO ellipses and zero-offset traveltimes to the input data. The SH traveltimes can be obtained using PSH conversions generated for source-receiver azimuths outside of the vertical symmetry plane(s) of the model.

Parameter estimation for layered TI media

The multicomponent tomographic procedure was implemented for homogeneous TI layers with arbitrary orientation of the symmetry axis separated by smooth curved interfaces. Suppose the input data include the wide-azimuth traveltimes of PP- and PSV-waves reflected from all three interfaces of the model in Figure 3. After reconstructing the traveltimes of the pure SS reflections using the method of Grechka and Tsvankin (2001), we collect the PP and SS data into CMP gathers for azimuthal velocity analysis. Estimating the PP- and SS-wave NMO ellipses and reflection slopes at four CMP locations (Figure 3), we obtain the data vector

$$\boldsymbol{d}(Q,\,\boldsymbol{Y},\,n) \equiv \{\tau_Q(\boldsymbol{Y},\,n),\,\boldsymbol{p}_Q(\boldsymbol{Y},\,n),\,\boldsymbol{W}_Q(\boldsymbol{Y},\,n)\}\,, \quad (1)$$

where $Q = PP$ or SS is the mode type, $\boldsymbol{Y} = [Y_1, Y_2]$ is the CMP coordinate, $n = 1, 2, 3$ is the reflector number, τ_Q is the zero-offset traveltime, $\boldsymbol{p}_Q$ is the reflection slope on zero-offset time sections, and $\boldsymbol{W}$ are the $2{\times}2$ matrices (Grechka and Tsvankin, 1998) describing the NMO ellipses. Our goal is to find the model vector $\boldsymbol{m}$ which contains the interval anisotropic parameters and the coefficients of the polynomials used to approximate the model interfaces.

In general, the parameter-estimation algorithm is organized in the same way as that introduced for PP-waves by Grechka et al. (2000). For a given set of trial interval anisotropic parameters, the zero-offset traveltimes τ_Q and the reflection slopes $\boldsymbol{p}_Q$ are used to compute the one-way zero-offset rays for all reflection events. Then the interfaces for the trial model are reconstructed by fitting 2-D polynomials to the termination points of the zero-offset rays. Finally, the interval parameters are obtained by minimizing the following objective function:

$$\mathcal{F}(\boldsymbol{m}) \equiv \sum_{Q,\boldsymbol{Y},n} ||\boldsymbol{W}_Q^{\rm calc}(\boldsymbol{Y}, n, \boldsymbol{m}) - \boldsymbol{W}_Q^{\rm meas}(\boldsymbol{Y}, n)||\,. \quad (2)$$

The norms in the function (2) contain the differences between the computed and measured NMO ellipses $\boldsymbol{W}$ for all modes and all reflectors at each CMP location.

To examine the sensitivity of the estimated interval parameters to random errors, we contaminated the data vector (1) for four CMP locations in Figure 3 with Gaussian noise (the standard deviations were 2% for the NMO velocities and 1% for the zero-offset traveltimes and reflection slopes). Repeating the inversion 100 times for different realizations of the noise produced the standard deviations in the interval parameters shown in Figure 4. Although the error bars become larger for deeper horizons, the overall stability of the algorithm is satisfactory. A general increase in errors with depth, caused by the relatively small contribution of the deeper layers to the reflection traveltimes from their lower boundaries, is typical for all Dix-type algorithms.

Conclusions

We introduced a tomographic algorithm designed to invert reflection traveltimes acquired in wide-azimuth

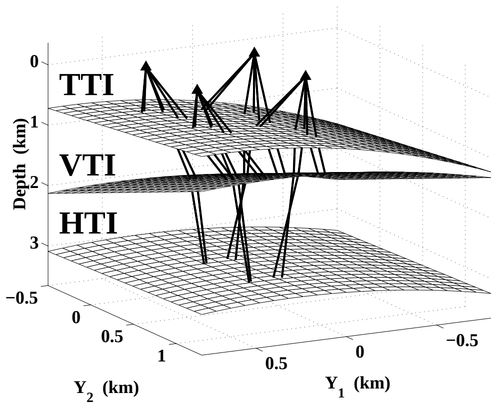

Figure 3. Zero-offset rays of PP- and $SS(SVSV)$-waves in a model composed of TTI, VTI and HTI layers.

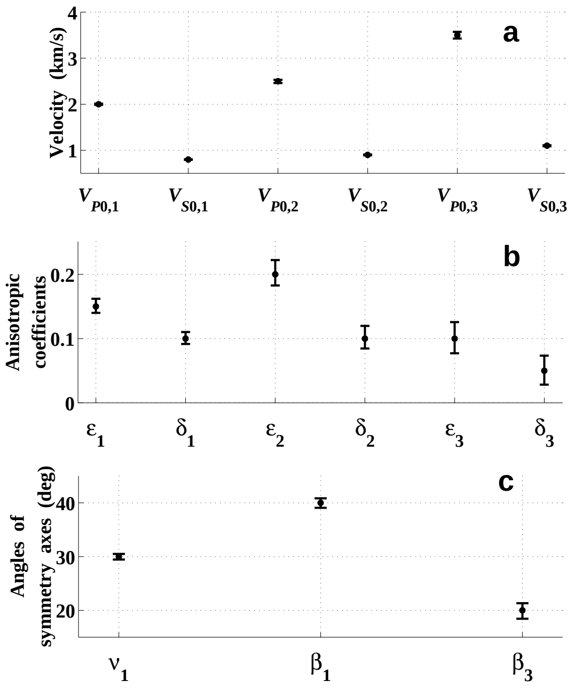

Figure 4. Results of stacking-velocity tomography for the model in Figure 3. The dots mark the exact parameter values, the bars correspond to the ± standard deviation in each parameter.

multicomponent surveys for the interval parameters of transversely isotropic media. The method operates with reflection moveout of PP-waves and converted PS-waves, so it can be applied in ocean-bottom surveys. The traveltimes of pure SS-wave reflections are *reconstructed* from PP and PS data using the model-independent kinematic algorithm of Grechka and Tsvankin (2001). Azimuthal semblance analysis of PP and SS traveltimes on CMP gathers produces the NMO ellipses (i.e., azimuthally varying stacking velocities) and zero-offset traveltimes which serve as the input to the tomographic inversion. This "stacking-velocity tomography" makes anisotropic parameter estimation computationally feasible because it eliminates the need for multioffset and multiazimuth ray tracing.

Here, the multicomponent tomography was implemented for a stack of homogeneous TI layers separated by smooth interfaces. Our results show that for a range of reflector dips and tilt angles of the symmetry axis the combination of PP and SS ($SVSV$) data can be used to build anisotropic models for *depth* processing. The most notable exception is horizontally layered VTI media, where even long-spread (nonhyperbolic) moveout of PP- and PSV-waves does not constrain the vertical velocities. It should be emphasized that the parameter-estimation results can be compromised by assuming the wrong anisotropic symmetry (e.g., HTI instead of TTI). In principle, such errors can be avoided by using the most general TTI model in the inversion, but the need to estimate the tilt reduces the stability of the algorithm.

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