

# Inversion of $P$ -wave data in laterally heterogeneous VTI media.

## Part II: Irregular interfaces

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### Summary

$P$ -wave reflection traveltimes may be used to constrain the depth scale of transversely isotropic models with a vertical symmetry axis (VTI) if the medium above the reflector is *laterally heterogeneous*. In Part I of this work we examined the inversion of  $P$ -wave normal-moveout (NMO) ellipses for the vertical velocity  $V_{P0}$  and anisotropic coefficients  $\epsilon$  and  $\delta$  in VTI models with plane dipping interfaces in the overburden. Here, we extend these results to stratified VTI media with *irregular* (e.g., curved) interfaces. Despite the higher complexity of the model, non-planar interfaces increase the angle coverage of reflected rays, which helps to resolve trade-offs between the medium parameters.

By employing singular-value decomposition (SVD), we show that there exists a subset of VTI models for which all parameters needed for anisotropic *depth* processing can be obtained solely from  $P$ -wave reflection traveltimes. The inversion methodology is based on an extension of the theory of NMO-velocity surfaces to wave propagation through irregular interfaces. We develop a tomographic procedure to reconstruct both the interval VTI parameters and the shape of the interfaces and demonstrate its reliable performance on input data contaminated by noise.

### Introduction

Depth imaging in laterally homogeneous VTI media above the target reflector is hampered by the fact that  $P$ -wave reflection traveltimes are controlled by just two parameter combinations: the NMO velocity from a horizontal reflector  $V_{\text{nmo}}(0) = V_{P0} \sqrt{1 + 2\delta}$  and the anisotropic coefficient  $\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}$  ( $V_{P0}$  is the vertical velocity,  $\epsilon$  and  $\delta$  are Thomsen's anisotropic coefficients). However, this result of Alkhalifah and Tsvankin (1995) is no longer valid if the overburden contains non-horizontal interfaces or lateral velocity gradient (Grechka and Tsvankin, 1999; Le Stunff et al., 1999). Although lateral heterogeneity introduces additional unknowns to be estimated from reflection data, it may also provide information about the individual values of  $V_{P0}$ ,  $\epsilon$  and  $\delta$ .

In Paper I (Grechka et al., 2000) we evaluated the feasibility of parameter estimation for VTI media composed of homogeneous layers separated by plane dipping interfaces. If the interfaces do not intersect each other, unambiguous inversion for the depth model re-

quires minimal *a priori* information, such as knowledge of a single VTI parameter in one of the layers. Here, we carry out singular-value decomposition and tomographic inversion of  $P$ -wave reflection traveltimes for more complicated VTI models with irregular interfaces. Our results show that for some models with smooth curved interfaces, parameter estimation in depth can be accomplished without any *a priori* information.

### NMO velocities in anisotropic media with irregular interfaces

We begin by extending the theory of NMO-velocity surfaces (Grechka and Tsvankin, 1999) to anisotropic media with irregular interfaces. The NMO velocity  $V_{\text{nmo}}(\mathcal{L})$  measured in 3-D space along a CMP line specified by the unit vector  $\mathcal{L}$  is given by

$$V_{\text{nmo}}^{-2}(\mathcal{L}) = \mathcal{L} \mathbf{U} \mathcal{L}^T, \quad (1)$$

where  $\mathbf{T}$  denotes transposition. The  $3 \times 3$  symmetric matrix  $\mathbf{U}$  is expressed in terms of the spatial derivatives of the one-way traveltime  $\tau$  or the slowness vector  $\mathbf{p}$ :

$$U_{km} = \tau_0 \left. \frac{\partial^2 \tau(\mathbf{x})}{\partial x_k \partial x_m} \right|_{\mathbf{x}=\mathbf{Y}} = \tau_0 \left. \frac{\partial p_k(\mathbf{x})}{\partial x_m} \right|_{\mathbf{x}=\mathbf{Y}}. \quad (2)$$

( $k, m = 1, 2, 3$ ).

Here  $\tau_0$  is the one-way zero-offset traveltime, and the derivatives are evaluated at the CMP location  $\mathbf{x} = \mathbf{Y}$ .

The matrix  $\mathbf{U}$  describes a quadratic *NMO-velocity surface* obtained by plotting the NMO velocity as the radius-vector along all possible directions of CMP lines. Grechka and Tsvankin (1999) showed that if the medium in the vicinity of the common midpoint is homogeneous, the NMO-velocity surface is always a cylinder; otherwise, it can be an ellipsoid or a one-sheeted hyperboloid. The NMO ellipse  $\mathbf{W}$  recorded in the plane  $\mathcal{P}$  can be viewed as the intersection of the NMO-velocity surface  $\mathbf{U}$  and  $\mathcal{P}$ :

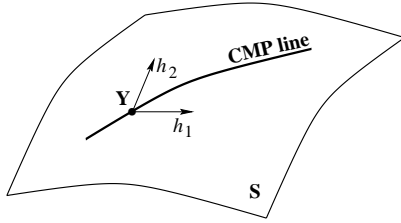
$$\mathbf{W} = \mathbf{U} \cap \mathcal{P}. \quad (3)$$

If a CMP line lies on an irregular surface  $\mathbf{S}$  (Figure 1) and is characterized by the unit tangent  $\ell$  at the common midpoint  $\mathbf{Y}$ , the azimuthally varying NMO velocity  $V_{\text{nmo}}(\mathbf{S}, \ell)$  can be found as

$$V_{\text{nmo}}^{-2}(\mathbf{S}, \ell) = \ell (\mathbf{W} + \tau_0 \mathbf{p} \cdot \boldsymbol{\kappa}) \ell^T. \quad (4)$$

$\mathbf{W}$  is the NMO ellipse obtained as the intersection of the NMO-velocity surface  $\mathbf{U}$  with the plane  $\mathcal{P}$  tangent to the surface  $\mathbf{S}$  at  $\mathbf{Y}$ . In equation (4) it is assumed

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**Figure 1.** CMP line at an irregular surface described by the radius-vector  $\mathbf{S}(h_1, h_2)$ .  $\mathbf{Y}$  is the common midpoint,  $h_1$  and  $h_2$  denote the half-offsets in the coordinate directions  $x_1$  and  $x_2$ ; at the CMP location,  $h_1 = h_2 = 0$ .

that the slowness vector  $\mathbf{p}$  is continuous across  $\mathbf{S}$ . The quantity

$$\kappa_{ij} = \left. \frac{\partial^2 \mathbf{S}}{\partial h_i \partial h_j} \right|_{h_1=h_2=0}, \quad (i, j = 1, 2) \quad (5)$$

is the matrix of the second-order derivatives of  $\mathbf{S}(h_1, h_2)$  with respect to the half-offsets  $h_1$  and  $h_2$  (Figure 1).

If the surface  $\mathbf{S}$  represents an interface with a jump in the elastic properties, the NMO ellipse in the plane  $\mathcal{P}$  (denoted as  $\mathbf{W}^{(+)}$ ) will depend on the difference between the slowness vector above ( $\mathbf{p}^{(+)}$ ) and below ( $\mathbf{p}^{(-)}$ ) the interface. It is convenient to introduce an imaginary NMO ellipse  $\mathbf{W}^{(-)}$  that would be measured in  $\mathcal{P}$  if  $\mathbf{p}^{(+)}$  were equal to  $\mathbf{p}^{(-)}$ . Then equation (4) leads to the following expression for  $\mathbf{W}^{(+)}$ :

$$\mathbf{W}^{(+)} = \mathbf{W}^{(-)} - \tau_0 (\mathbf{p}^{(+)} - \mathbf{p}^{(-)}) \cdot \boldsymbol{\kappa}, \quad (6)$$

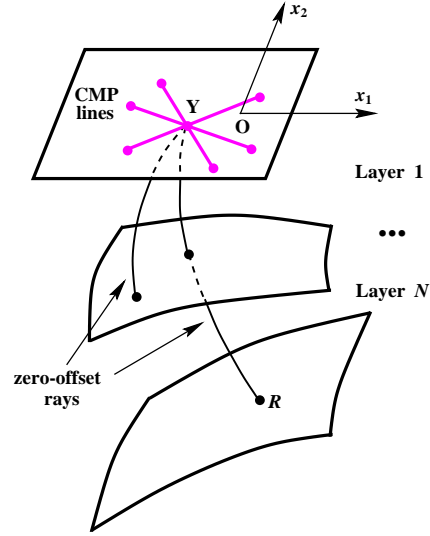
where the term  $\tau_0 (\mathbf{p}^{(+)} - \mathbf{p}^{(-)}) \cdot \boldsymbol{\kappa}$  represents a correction for the interface curvature.

The above results allow us to adapt the generalized Dix formula of Grechka et al. (1999),

$$[\mathbf{W}(N)]^{-1} \sum_{n=1}^N \tau_{0,n} = \sum_{n=1}^N [\mathbf{W}_n^{(+)}]^{-1} \tau_{0,n}, \quad (7)$$

to layered media with irregular interfaces.  $\mathbf{W}(N)$  is the effective NMO ellipse from the  $N$ th reflector measured in the acquisition plane, and  $\tau_{0,n}$  are the interval zero-offset traveltimes. The quantities  $\mathbf{W}_n^{(+)}$  are computed by applying equation (6) at each interface encountered by the zero-offset ray.  $\mathbf{W}_n^{(-)}$  is obtained as the intersection of the interval NMO cylinder  $\mathbf{U}_n$  with the plane  $\mathcal{P}_n$  tangent to the interface at the transmission point of the zero-offset ray.

This theory allows us to compute effective NMO ellipses  $\mathbf{W}$  which can be obtained from 3-D multi-azimuth reflection data. Since it is necessary to trace only one (zero-offset) ray per common midpoint, the hyperbolic portion of pure-mode reflection moveout can be modeled without the time-consuming calculation of multi-offset and multi-azimuth traveltimes.



**Figure 2.** Multi-azimuth CMP recording over a layered VTI model with irregular interfaces.

### Inversion of $P$ -wave reflection traveltimes

#### Model representation and data for inversion

As in Paper I, it is assumed that at each CMP location  $\mathbf{Y}$  we can measure the zero-offset  $P$ -wave traveltimes  $\tau_0(n)$ , the reflection slopes (horizontal slownesses)  $\mathbf{p}(n) = [p_1(n), p_2(n)]$ , and the NMO ellipses  $\mathbf{W}(n)$  from all interfaces  $n = 1, \dots, N$  (Figure 2). Our goal is to determine whether or not the data

$$\mathbf{d}(\mathbf{Y}, n) \equiv \{\tau_0(\mathbf{Y}, n), \mathbf{p}(\mathbf{Y}, n), \mathbf{W}(\mathbf{Y}, n)\}, \quad (8)$$

acquired at a number of CMP locations  $\mathbf{Y} = [Y_1, Y_2]$  can be inverted for all relevant model parameters

$$\mathbf{m} \equiv \{V_{P0,n}, \epsilon_n, \delta_n, \zeta_{j_1 j_2, n}\}, \quad (9)$$

$$(n = 1, \dots, N; j_1 = 1, \dots, J_1; j_2 = 1, \dots, J_2).$$

Here  $V_{P0,n}$ ,  $\epsilon_n$ ,  $\delta_n$  are the interval VTI parameters, and  $\zeta_n$  are the matrices of the coefficients describing the depths  $z_n(Y_1, Y_2)$  of the model interfaces. We elected to use the following polynomial representation:

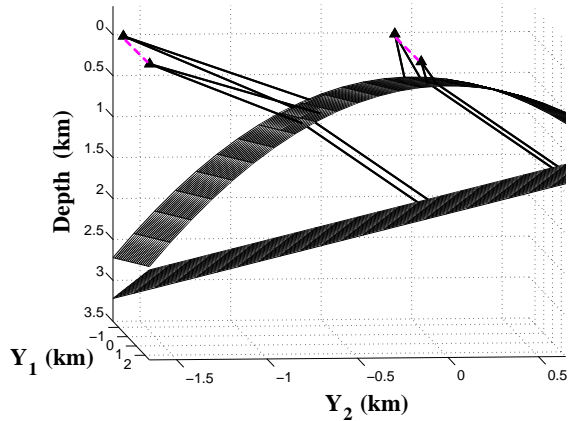
$$z_n(Y_1, Y_2) = \sum_{j_1=1}^{J_1} \sum_{j_2=1}^{J_2} \zeta_{j_1 j_2, n} Y_1^{j_1-1} Y_2^{j_2-1}. \quad (10)$$

#### Feasibility study

For models with plane interfaces, the dependence of the data vector (8) on the CMP coordinate  $\mathbf{Y}$  does not provide any new information about the model parameters (Paper I). This is no longer the case in the presence of interface curvature, because zero-offset reflection rays change direction with the CMP location. Therefore, the spatial variation of the data may help to constrain the inversion and determine the depth scale of the model.

The zero-offset traveltimes  $\tau_0(n)$  and the reflection

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**Figure 3.** Zero-offset rays in the two-layer VTI model used in SVD analysis. CMP locations are marked by triangles. The relevant layer parameters are  $V_{P0,1} = 1$  km/s,  $\epsilon_1 = 0.20$ ,  $\delta_1 = 0.10$ ,  $V_{P0,2} = 2$  km/s,  $\epsilon_2 = 0.15$ ,  $\delta_2 = 0.05$ . The interfaces are described by 2-D quadratic polynomials, so  $\zeta_1$  and  $\zeta_2$  are  $3 \times 3$  matrices.

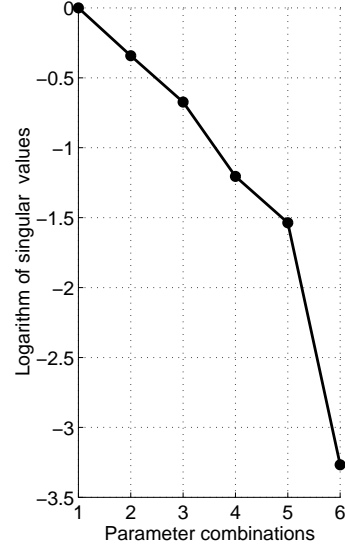
slopes  $\mathbf{p}(n)$  can be used to find the shape of the interfaces  $z_n(\mathbf{Y})$  for any given estimate of the layer parameters

$$\mathbf{l} = \{V_{P0,n}, \epsilon_n, \delta_n\}, \quad (n = 1, \dots, N). \quad (11)$$

This is achieved by tracing zero-offset rays downward and computing the coordinates of the reflection point and the corresponding interface normal. The resulting equations can be solved (in the least-squares sense) for the coefficients  $\zeta_n$  in equation (10). Since the traveltimes  $\tau_0(n, \mathbf{Y})$  and slopes  $\mathbf{p}(n, \mathbf{Y})$  are used to obtain  $z_n(\mathbf{Y})$ , the layer parameters  $\mathbf{l}$  [equation (11)] should be found from the NMO ellipses  $\mathbf{W}(n)$ .

To prove that the NMO ellipses might constrain the layer parameters uniquely, we present an example of SVD analysis for a two-layer VTI model shown in Figure 3. The ellipses measured from two reflectors at four CMP locations (triangles in Figure 3) provide  $2 \times 4 \times 3 = 24$  equations for the six components of the vector  $\mathbf{l}$ . Thus, the feasibility of the inversion can be verified by applying SVD to the  $24 \times 6$  matrix of Fréchet derivatives  $\mathcal{F} = \frac{\partial \mathbf{W}(n, \mathbf{Y})}{\partial \mathbf{l}}$  computed for the correct model parameters  $\mathbf{m}$ . Since none of the singular values vanishes (Figure 4), the input data provide sufficient information for parameter estimation, and this VTI model can be fully reconstructed in the depth domain from  $P$ -wave reflection traveltimes.

Clearly, it is not always possible to obtain VTI parameters using just  $P$ -wave data. For example, when the curvature of the intermediate interface in Figure 3 decreases, our model approaches that with plane interfaces where at least one singular value is zero (Paper I). One



**Figure 4.** Normalized singular values for the two-layer VTI model from Figure 3.

$V_{P0,1}$ (km/s)	$\epsilon_1$	$\delta_1$	$V_{P0,2}$ (km/s)	$\epsilon_2$	$\delta_2$
1.02	0.17	0.07	2.05	0.11	0.03

**Table 1.** Inversion results for the two-layer VTI model from Figure 3. The standard deviations of Gaussian noise added to the NMO velocities and zero-offset traveltimes are 2.0% and 1.0%, respectively.

of the singular values also vanishes when either layer is elliptically anisotropic (i.e.,  $\epsilon_n = \delta_n$ ).

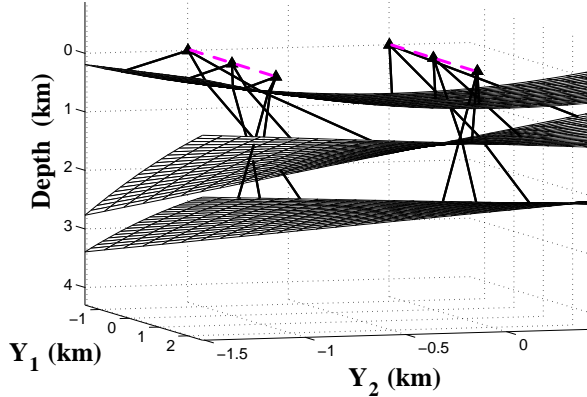
### Numerical examples

To confirm the SVD results, we carried out actual inversion of  $P$ -wave data for the model in Figure 3 in the presence of noise. Reflection traveltimes were computed for 240 common midpoints placed at every 25 m along the two dashed lines between the triangles in Figure 3. (Thus, we have an overdetermined system of  $240 \times 3 = 720$  equations for reconstructing the interface parameters  $\zeta_n$ .) We added Gaussian noise to the NMO velocities and zero-offset traveltimes determined from the data and obtained the model vector  $\mathbf{m}$  [equation (9)] using least-squares fitting of the data  $\mathbf{d}$  [equation (8)]. Comparing the results of this test (Table 1) with the actual parameters (see the caption to Figure 3), we conclude that all three parameters in both layers were found with good accuracy. The relative errors in the interval vertical velocities  $V_{P0,n}$  and absolute errors in the anisotropic coefficients are comparable to the standard deviation of the noise added to the NMO velocities. The inversion results do not depend (for the same input data) on the initial guess for the model pa-

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	$V_{P0,1}$ (km/s)	$\epsilon_1$	$\delta_1$	$V_{P0,2}$ (km/s)	$\epsilon_2$	$\delta_2$	$V_{P0,3}$ (km/s)	$\epsilon_3$	$\delta_3$
Correct	1.80	0.20	0.10	2.00	0.15	0.05	2.30	0.10	0.03
Inverted	1.77	0.23	0.12	1.97	0.17	0.07	2.26	0.12	0.05

**Table 2.** Comparison of the correct and inverted parameters for the three-layer VTI model from Figure 5. The errors in the inverted quantities are due to Gaussian noise with the standard deviation 2.0% (for the NMO velocities) and 1.0% (for the zero-offset traveltimes).



**Figure 5.** Zero-offset rays in a three-layer VTI model. Gray dashed lines mark the CMP lines used in the inversion.

rameters, which suggests that the least-squares objective function in a certain vicinity of the correct solution is unimodal. Knowledge of the interval VTI parameters  $V_{P0,n}$ ,  $\epsilon_n$  and  $\delta_n$  is sufficient to reconstruct the depth and shape of the interfaces and build the whole VTI model *in depth*.

Another example, this time for a three-layer VTI model with a more complicated shape of the interfaces is shown in Figure 5 and Table 2. The data vector  $\mathbf{d}(\mathbf{Y}, n)$  was determined from the traveltimes computed at 600 CMP's located along two lines with a spacing of 15 m. The accuracy of parameter estimation (Table 2) is comparable to that in the previous example. Therefore, we identified a subset of layered VTI models with irregular interfaces for which  $P$ -wave reflection traveltimes provide sufficient information for the inversion in the depth domain.

### Discussion and conclusions

The possibility of inverting  $P$ -wave reflection data for the interval parameters of layered VTI media strongly depends on the geometry of intermediate interfaces. While only the zero-dip NMO velocity  $V_{nmo}(0)$  and  $\eta$  can be obtained for laterally homogeneous VTI media above the reflector, the presence of dipping or irregular interfaces may help to estimate all three relevant

anisotropic parameters ( $V_{P0}$ ,  $\epsilon$  and  $\delta$ ). For plane interfaces which do not intersect each other, unambiguous inversion requires *a priori* knowledge of a single interval VTI parameter in at least one layer (Paper I). Here we showed that if the interfaces are irregular, it may be possible to reconstruct the model in depth from  $P$ -wave reflection traveltimes alone.

The most critical assumption that ensured the success of the inversion procedure is that the model is composed of *homogeneous* layers. Allowing for a variation in the VTI parameters within some of the layers may prevent us from resolving the three principal components of the model: anisotropy, irregular interfaces and heterogeneity. Even for isotropic models with lateral velocity variation and irregular interfaces traveltimes inversion is generally non-unique (Goldin, 1986). Still, in some special cases it may be possible to separate the contributions of each of those three factors to the reflection traveltimes; this topic requires further investigation.

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