

Inversion of P -wave data in laterally heterogeneous VTI media.

Part I: Plane dipping interfaces

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Summary

A major complication caused by anisotropy in velocity analysis and imaging is the uncertainty in estimating the vertical velocity and depth scale of the model from surface data. For laterally homogeneous VTI (transversely isotropic with a vertical symmetry axis) media above the target reflector, P -wave reflection traveltimes alone are insufficient to obtain the vertical velocity V_{P0} and Thomsen anisotropic parameters ϵ and δ . As a result, P -wave surface data have to be combined with other information (e.g., borehole data or converted waves) to build velocity models for depth imaging.

The presence of lateral heterogeneity in the overburden may create the dependence of P -wave reflection data on all three relevant parameters (V_{P0} , ϵ and δ) and, therefore, may help to determine the depth scale of the velocity field. Here, we examine parameter estimation in VTI media composed of homogeneous layers separated by plane dipping interfaces. In general, for non-intersecting interfaces the interval parameters cannot be recovered from P -wave moveout in a unique way. Still, if the reflectors have sufficiently different azimuths, *a priori* knowledge of any *single* interval parameter makes it possible to reconstruct the whole model in depth. Furthermore, such a priori information is not needed for some types of VTI media with irregular interfaces or faults.

Introduction

Vertical transverse isotropy, which is believed to be the most common anisotropic model for sedimentary basins, may have a significant impact on velocity analysis and imaging of reflection data. P -wave kinematic signatures in VTI media are governed by the vertical velocity V_{P0} and the anisotropic coefficients ϵ and δ . However, only two combinations of these parameters – the NMO velocity from a horizontal reflector $V_{\text{nmo}}(0) = V_{P0} \sqrt{1 + 2\delta}$ and the anellipticity coefficient $\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}$ – are constrained by P -wave reflection traveltimes if the medium above the reflector is laterally homogeneous (Alkhalifah and Tsvankin, 1995). Therefore, the vertical velocity V_{P0} remains undetermined, and P -wave reflection data provide sufficient information only for time processing.

The two-parameter description of P -wave time-domain signatures, however, breaks down in the presence of lateral heterogeneity above the reflector (Grechka and Tsvankin, 1999). Le Stunff et al. (1999) presented an example of successful inversion of P -wave

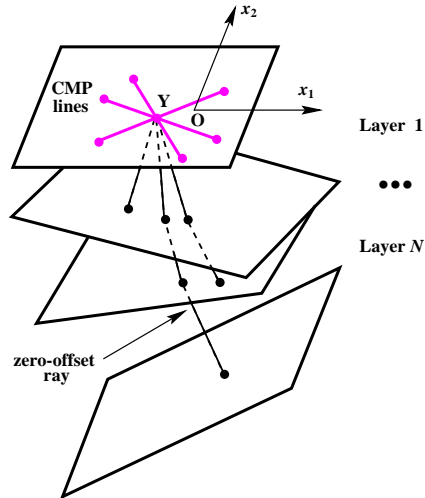


Figure 1. Azimuthally-dependent reflection traveltimes are recorded over a stack of VTI layers separated by plane dipping interfaces.

traveltimes for the parameters V_{P0} , ϵ and δ in a model that included a dipping VTI layer overlying a purely isotropic medium. Here we discuss P -wave moveout inversion for more complicated, multilayered media with plane dipping interfaces above the reflector.

VTI model and data for inversion

We consider a model composed of N homogeneous VTI layers (some of them may be isotropic) separated by plane dipping non-intersecting interfaces (Figure 1). The model parameters responsible for P -wave kinematics include the interval vertical velocities $V_{P0,n}$, anisotropic coefficients ϵ_n and δ_n , the interface dips ϕ_n , azimuths ψ_n and depths z_n (the depth can be measured, for example, under the coordinate origin \mathbf{O}). Thus, the model vector

$$\mathbf{m} \equiv \{V_{P0,n}, \epsilon_n, \delta_n, \phi_n, \psi_n, z_n\}, \quad (n = 1, \dots, N) \quad (1)$$

is characterized by $6N$ independent quantities. It is convenient to split the vector \mathbf{m} into two vectors \mathbf{l} and \mathbf{i} , where \mathbf{l} contains the layer parameters,

$$\mathbf{l} \equiv \{V_{P0,n}, \epsilon_n, \delta_n\}, \quad (n = 1, \dots, N), \quad (2)$$

and \mathbf{i} describes the interfaces,

$$\mathbf{i} \equiv \{\phi_n, \psi_n, z_n\}, \quad (n = 1, \dots, N). \quad (3)$$

Velocity analysis of 3-D multi-azimuth P -wave data recorded at common midpoints (CMP) with the coordinates $\mathbf{Y} = [Y_1, Y_2]$ can provide the one-way zero-offset

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reflection traveltimes $\tau_0(\mathbf{Y}, n)$ from all interfaces and the corresponding NMO ellipses represented by 2×2 symmetric matrices $\mathbf{W}(\mathbf{Y}, n)$. Using the zero-offset (or stacked) time sections of reflection events, we can pick the reflection slopes and determine the zero-offset ray parameters $\mathbf{p}(\mathbf{Y}, n) = [p_1(\mathbf{Y}, n), p_2(\mathbf{Y}, n)]$. Hence, the traveltime data to be used for parameter estimation can be written as

$$\mathbf{d}(\mathbf{Y}, n) \equiv \{\tau_0(\mathbf{Y}, n), \mathbf{p}(\mathbf{Y}, n), \mathbf{W}(\mathbf{Y}, n)\}, \quad (4)$$

where $n = 1, \dots, N$.

In the model from Figure 1, zero-offset ray trajectories are parallel to each other for different \mathbf{Y} , and the reflection slopes \mathbf{p} are independent of \mathbf{Y} . From the theory of Grechka and Tsvankin (1999) it also follows that the NMO ellipses $\mathbf{W}(\mathbf{Y}, n)$ can be expressed in terms of the ellipses $\mathbf{W}(\mathbf{O}, n)$ recorded, for example, at the coordinate origin \mathbf{O} . This means that equation (4) becomes

$$\mathbf{d}(\mathbf{Y}, n) = \{\mathbf{d}(\mathbf{O}, n), \mathbf{Y} - \mathbf{O}\}. \quad (5)$$

Feasibility of parameter estimation

It is clear from equation (5) that although traveltime data from different common midpoints may be useful in practice to suppress noise, they give the same information about the medium parameters as the data vector

$$\mathbf{d}(\mathbf{O}, n) = \{\tau_0(\mathbf{O}, n), \mathbf{p}(n), \mathbf{W}(\mathbf{O}, n)\} \quad (6)$$

at a single CMP. Thus, analyzing the dependence of the vector $\mathbf{d}(\mathbf{O}, n)$ on the parameter vector \mathbf{m} [equation (1)] should be sufficient for evaluating the feasibility of the inversion. For brevity, henceforth we will omit the CMP coordinate.

For an N -layered VTI model, the vectors \mathbf{d} and \mathbf{m} contain $6N$ components each. Therefore, the vector \mathbf{m} can be obtained from the data \mathbf{d} only if all components of \mathbf{d} are independent. Unfortunately, this is not the case for VTI media. The P -wave NMO ellipse $\mathbf{W}(1)$ from a dipping reflector overlaid by a homogeneous VTI medium provides only two equations for the medium parameters (Grechka and Tsvankin, 1998) because its orientation is fixed by the reflector azimuth (determined by the slope \mathbf{p}). As a result, the data vector in the top layer contains only five components. SVD analysis performed below shows that this is the only relationship between the components of the data vector, if all interfaces have different azimuths.

For inversion purposes, it is convenient to split the vector $\mathbf{d}(\mathbf{O}, n)$ into two parts. Indeed, for a given (“trial”) set of the interval VTI parameters \mathbf{l}_n [equation (2)], the values of $\tau_0(n)$ and $\mathbf{p}(n)$ can be used to determine the depths, dips and azimuths of the interfaces \mathbf{i}_n [equation (3)]. Then the best-fit vector \mathbf{l}_n is found by

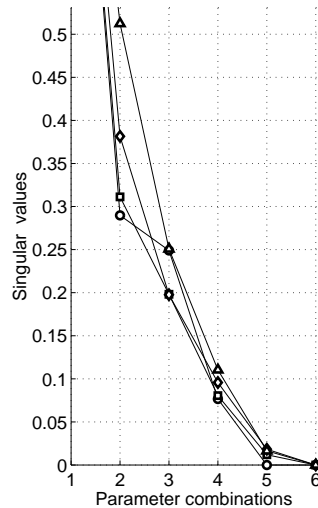


Figure 2. SVD analysis for a two-layer VTI model with the parameters $V_{P0,1} = 2$ km/s, $\epsilon_1 = 0.15$, $\delta_1 = 0.05$, $V_{P0,2} = 3$ km/s, $\epsilon_2 = 0.25$, $\delta_2 = 0.10$. The singular values are normalized by the greatest one. The interface depths under the CMP location are $z_1 = 1$ km and $z_2 = 3$ km, the dips $\phi_1 = 40^\circ$ and $\phi_2 = 20^\circ$, and the azimuth of the bottom interface is $\psi_2 = 0^\circ$. The curves correspond to different azimuths of the intermediate (first) interface: \circ ($\psi_1 = 0^\circ$), \square ($\psi_1 = 30^\circ$), \diamond ($\psi_1 = 60^\circ$) and \triangle ($\psi_1 = 90^\circ$).

inverting the NMO ellipses $\mathbf{W}(n)$. Therefore, it is sufficient to perform SVD analysis of the $3N \times 3N$ matrix of Fréchet derivatives $\mathcal{F} = \frac{\partial \mathbf{W}(n)}{\partial \mathbf{l}_k}$, ($k, n = 1, \dots, N$), as long as the vector \mathbf{i} yields the measured traveltimes $\tau_0(n)$ and reflection slopes $\mathbf{p}(n)$. The NMO ellipses are computed using the formalism developed by Grechka and Tsvankin (1999).

Figure 2 shows a typical result of SVD analysis of the NMO ellipses with respect to the parameters $\mathbf{l} = \{V_{P0,1}, \epsilon_1, \delta_1, V_{P0,2}, \epsilon_2, \delta_2\}$ in a two-layer model. While the last singular value is always equal to zero, the other five do not vanish if the azimuths of the interfaces are different (see the curves marked by squares, diamonds and triangles). The presence of two vanishing singular values when the azimuths of both interfaces coincide (circles in Figure 2) is not surprising, because in this case the axes of the NMO ellipse from the bottom of the model are aligned with the dip and strike directions (i.e., the model becomes 2-D), and there is one less independent data component.

These observations can be extended to an arbitrary number of VTI layers. At a maximum, the NMO ellipses constrain $3N - 1$ combinations of $3N$ interval parameters $\{V_{P0,n}, \epsilon_n, \delta_n\}$ ($n = 1, \dots, N$), provided the model interfaces have different azimuths. Otherwise, P -wave traveltimes contain less information about the model parameters. For instance, if the azimuths of all

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	$V_{P0,1}$ (km/s)	ϵ_1	δ_1	$V_{P0,2}$ (km/s)	ϵ_2	δ_2	$V_{P0,3}$ (km/s)	ϵ_3	δ_3
Correct	1.00	0.08	0.04	2.00	0.20	0.10	3.00	0.10	0.05
Inverted	0.99	0.09	–	2.02	0.18	0.09	2.96	0.18	0.07

	z_1 (km)	ϕ_1 (deg)	ψ_1 (deg)	z_2 (km)	ϕ_2 (deg)	ψ_2 (deg)	z_3 (km)	ϕ_3 (deg)	ψ_3 (deg)
Correct	1.00	30.0	–10.0	2.00	30.0	30.0	4.00	10.0	70.0
Inverted	1.00	29.8	–10.0	1.98	29.9	30.4	3.98	10.3	70.5

Table 1. Comparison of the correct and inverted values of parameters for a three-layer VTI model (see Figure 3). The standard deviation of Gaussian noise added to the NMO velocities and zero-offset traveltimes is 2.0% and 0.5%, respectively.

interfaces are identical, the model degenerates into 2-D, and the NMO ellipses from different reflectors are co-oriented. Thus, only their semi-axes constrain the layer parameters, and the number of independent equations reduces to $2N$. In the limiting case of horizontal layers, the NMO ellipses become circles defined by N interval zero-dip NMO velocities.

Inversion using a priori information

Figure 2 suggests that *a priori* knowledge of a single layer parameter may be sufficient to overcome the ambiguity. To verify this conclusion, we performed several numerical tests, with typical results listed in Table 1. We traced reflected rays through a three-layer VTI model (Figure 3), added Gaussian noise to the computed NMO velocities and zero-offset traveltimes, and found the layer parameters by least-squares fitting of the NMO ellipses. To constrain the inversion, the parameter $\delta_1 = 0.04$ was assumed to be known, which allowed the other parameters to be estimated with good accuracy (Table 1). Holding any other interval parameter at the correct value produces similar results. It is always possible to reconstruct the whole model if the vertical velocity $V_{P0,n}$ or one of the anisotropic coefficients (ϵ_n or δ_n) is known. The inversion procedure also works well in the special case of isotropy, i.e., when ϵ_n and δ_n are set to zero in one or more layers.

Another way to reduce the number of unknowns is to impose an empirical relationship between ϵ and δ in at least one layer. We found that making ϵ a known function of δ [i.e., $\epsilon = \epsilon(\delta)$] in any layer generally makes the parameter estimation unique. One special case when this approach does *not* help is elliptical anisotropy. Even if all layers are assumed to be elliptically anisotropic ($\epsilon_n = \delta_n$), and the number of unknowns reduces to $2N$, the $3N - 1$ equations for the NMO ellipses do not constrain all layer parameters. This conclusion is in agreement with the results of Dellinger and Muir (1988) ob-

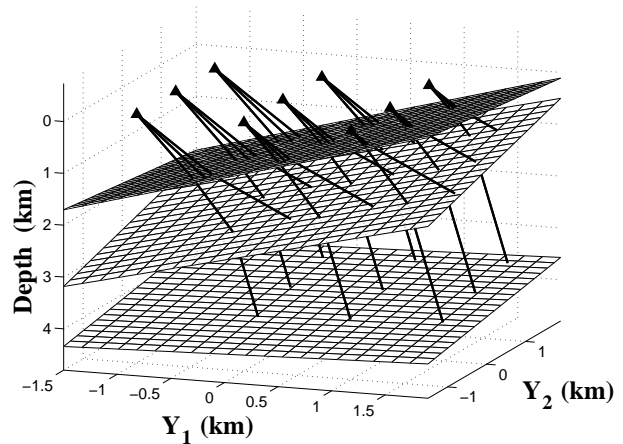


Figure 3. Zero-offset P -wave rays in the three-layer VTI model from Table 1.

tained using linear transformations (stretching) of the isotropic wave equation.

Models with curved interfaces or faults

A priori information may not be needed for models with more than one plane reflector in some of the layers. Let us suppose, for example, that the intermediate interface in the two-layer VTI model from Figure 2 is bent in such a way that it has two plane portions with the same dip $\phi_1 = 40^\circ$ but different azimuths $\psi_1 = 30^\circ$ and $\psi_1 = 90^\circ$. Recording rays from the bottom reflector which cross those two portions of the intermediate interface yields an additional NMO ellipse (i.e., three more equations). The absence of vanishing singular values for this problem (Figure 4) shows that all parameters can be resolved uniquely.

Parameter estimation also becomes feasible if the model contains a dipping fault, and the data include the reflections from both the fault plane and layer boundaries. In such a case, the VTI parameters may be ob-

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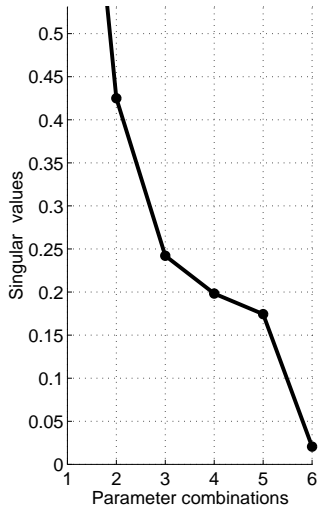


Figure 4. SVD analysis for a two-layer VTI model similar to the one in Figure 2. This time, however, the intermediate interface contains two segments with the same dip $\phi_1 = 40^\circ$ but different azimuths $\psi_1 = 30^\circ$ and $\psi_1 = 90^\circ$.

tained even for 2-D models where the axes of all NMO ellipses point in the dip and strike directions. For example, the semi-axes of the NMO ellipses corresponding to the zero-offset rays in Figure 5 provide us with six equations which constrain all six layer parameters (Figure 6).

Conclusions

P -wave NMO ellipses measured from multi-azimuth 3-D reflection data over layered VTI media depend on all three relevant Thomsen parameters ($V_{0,n}$, ϵ_n , and δ_n), if the overburden contains dipping interfaces. However, if the interfaces do not cross each other, the $3N$ parameters of an N -layer model cannot be found in a unique fashion because the NMO ellipses yield only $3N - 1$ independent equations. Remarkably, the spatial variation of the NMO ellipses for this model does not provide any additional information for the inversion procedure.

This ambiguity can be overcome if a single parameter in any layer is known *a priori*. For example, knowledge of the vertical velocity in the subsurface layer is sufficient to make the inversion unique. Another possibility is to introduce a certain relationship between the parameters (e.g., between ϵ and δ) in one of the layers. The only model for which this approach fails to remove the ambiguity is elliptical anisotropy ($\epsilon_n = \delta_n$). The VTI parameters are generally better constrained by P -wave reflection data if the medium contains a through-going fault plane or curved interfaces. On the whole, our results indicate that for a range of laterally heterogeneous VTI models it is possible to build velocity models in depth using surface P -wave data.

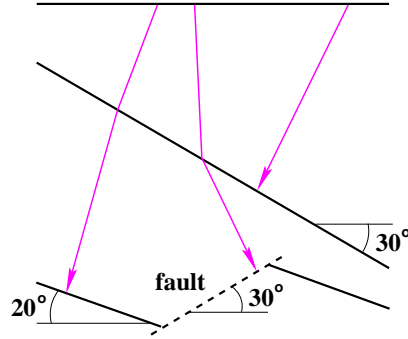


Figure 5. The presence of fault-plane reflections in layered VTI medium might be sufficient to obtain the model in depth using P -wave reflection data.

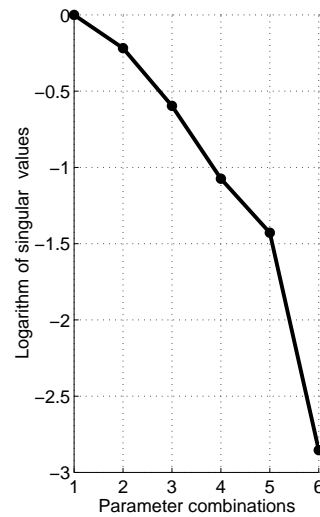


Figure 6. SVD analysis for the VTI model from Figure 5. The layer parameters are the same as those in Figure 2.

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