Two approaches to anisotropic velocity analysis of converted waves

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Summary

P-wave reflection traveltimes in anisotropic media are generally insufficient to determine the full set of anisotropic parameters required for depth imaging. Here, we discuss two different ways of performing anisotropic parameter estimation using joint inversion of P-waves and converted (PS) modes.

The first method operates with PS traveltimes originally collected into common-midpoint (CMP) gathers and later resorted into common-conversion-point (CCP) gathers. It is based on a 3-D parametric description of PS moveout that can be used to generate areal (multiazimuthal) CMP gathers without time-consuming two-point ray tracing. The idea of the second approach is to make converted-wave moveout symmetric by a special resorting of PS data that does not require knowledge of medium parameters. In contrast to CMP and CCP gathers, the resulting RTM (resorting to minimum traveltime) gathers do not suffer from the low amplitude and polarity reversals of PS arrivals and can be processed using conventional velocity-analysis techniques developed for P-waves.

Application of our algorithms to transversely isotropic media with a vertical symmetry axis (VTI) yields a model suitable for both P-wave depth migration and processing (e.g., transformation to zero offset) of converted waves. For wide-azimuth multicomponent 3-D surveys it is possible to estimate all VTI parameters (including the SH-wave coefficient γ) by combining P and PS moveout from a *single* mildly dipping reflector.

Introduction

In several exploration scenarios, mode-converted waves were effectively used to image targets poorly illuminated by P-wave data (e.g., Thomsen, 1999). For anisotropic media, PS-waves also play a crucial role in parameter estimation because it is often impossible to find the vertical velocity and build a velocity model in depth using P-wave reflections alone. The main difficulty in operating with PS data is the asymmetry of common-midpoint moveout with respect to zero offset and a significant reflection-point dispersal on CMP gathers. The dispersal is usually mitigated by resorting PS data into CCP gathers, but this operation requires knowledge of the velocity model and still does not necessarily make the moveout symmetric.

The first velocity-analysis method introduced here employs analytic 3-D traveltime-offset relationships for PS moveout to perform the inversion in the conventional CMP and CCP geometry. The second method is designed to remove the asymmetry of PS-moveout by resorting PS data around selected source-receiver pairs into the so-called RTM gathers.

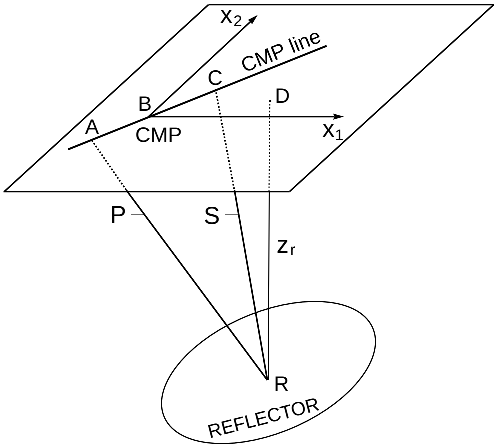

Figure 1. Converted PS-wave (ray ARC) recorded on an arbitrary oriented common-midpoint line over a homogeneous anisotropic layer with a dipping lower boundary. z_r is the depth of the conversion point.

Modeling and inversion on CMP and CCP gathers

3-D parametric representation of PS traveltime

Consider a PS-wave formed by the mode conversion at a plane dipping interface underlying an arbitrary anisotropic homogeneous medium (Figure 1). The formalism introduced below is valid for either split PS-wave with the substitution of the appropriate slowness vector. Extending the approach of Tsvankin and Grechka (1999) to 3-D, the traveltime of the wave reflected (converted) at the depth z_r can be written as

$$\begin{aligned} t_{PS} = t_P + t_S = z_r\,(& q_P - p_{1P}\, q_{,1P} - p_{2P}\, q_{,2P} \\ & + q_S - p_{1S}\, q_{,1S} - p_{2S}\, q_{,2S}), \end{aligned} \tag{1}$$

where q_P and q_S are the vertical slownesses of the P- and S-waves, p_{iP} and p_{iS} ($i = 1, 2$) are the horizontal slownesses (ray parameters), $q_{,iP} \equiv \partial q_P / \partial p_{iP}$ and $q_{,iS} \equiv \partial q_S / \partial p_{iS}$. The source-receiver vector $\mathbf{x}_{PS} = \mathbf{AC}$ (Figure 1) is given by

$$\mathbf{x}_{PS} = z_r \left\{ (q_{,1P} - q_{,1S}),\, (q_{,2P} - q_{,2S}) \right\}. \tag{2}$$

Equations (1) and (2) are sufficient for generating CCP gathers of the PS-wave. Common-midpoint moveout can be modeled by replacing z_r with the reflector depth beneath the CMP location:

$$z_r = \frac{z_{\mathrm{CMP}}}{1 + \frac{\tan\phi}{2}\left[(q_{,1P} + q_{,1S})\,\zeta_1 + (q_{,2P} + q_{,2S})\,\zeta_2\right]}\,, \qquad (3)$$

where ζ_1 and ζ_2 are the components of a horizontal unit vector that points in the updip direction, and ϕ is the reflector dip.

We also obtained the corresponding 3-D expressions for *layered* arbitrary anisotropic media. The parametric moveout equations yield an areal (multiazimuthal) CMP gather without tracing multiple rays for each source-receiver pair.

3-D inversion of wide-azimuth P and PS data

The general expressions given above were applied to develop a moveout-inversion algorithm for transversely isotropic media with a vertical symmetry axis (VTI). If a model contains horizontal VTI layers above a dipping reflector, P-wave traveltimes are controlled by the NMO velocity from a horizontal reflector $[V_{\mathrm{nmo},P}(0)]$ and the anellipticity coefficient $\eta \equiv (\epsilon - \delta)/(1+2\delta)$, where ϵ and δ are Thomsen's anisotropic coefficients. As demonstrated by Tsvankin and Grechka (1999), the interval parameters ϵ and δ, along with the vertical velocities V_{P0} and V_{S0}, can be determined by combining the moveout of P- and $PS(PSV)$-waves on the dip line of the reflector from two (horizontal and dipping) interfaces. Here we show that it is possible to obtain all VTI parameters and constrain the depth scale of the model using a *single* dipping reflector, if P and PS data are recorded for a wide range of source-receiver azimuths.

For simplicity, we restrict ourselves to a homogeneous VTI layer with a dipping lower boundary. Azimuthally varying NMO velocity of any pure mode in VTI media is described by an ellipse with the axes parallel to the dip and strike directions of the reflector:

$$V_{\mathrm{nmo}}^{-2}(\alpha, p) = W_{11}(p)\,\cos^2\alpha + W_{22}(p)\,\sin^2\alpha\,, \qquad (4)$$

where α is the azimuth with respect to the dip plane, p is the ray parameter (horizontal slowness) of the zero-offset ray, and $W_{ii}\,(i = 1, 2)$ determine the elliptical semi-axes. For P-waves, W_{ii} and the NMO velocity as a whole depend only on the parameters $V_{\mathrm{nmo},P}(0)$ and η (Grechka and Tsvankin, 1998).

The first step of the inversion procedure is to specify a trial value of one of the parameters responsible for P-wave kinematics (e.g., δ) and find the other two (V_{P0} and ϵ) using the values of $V_{\mathrm{nmo},P}(0)$ and η determined from the P-wave NMO ellipse (4). Then we obtain the horizontal slownesses (ray parameters) p_{1P} and p_{2P} of the zero-offset P-reflection from the reflection slopes on the zero-offset (stacked) section and use them (along with the zero-offset traveltime) to compute the dip, azimuth and depth of the reflecting interface for the trial model. Then δ and the vertical shear-wave velocity V_{S0} (a parameter not constrained by P-wave moveout) can be estimated from 3-D PS-wave data.

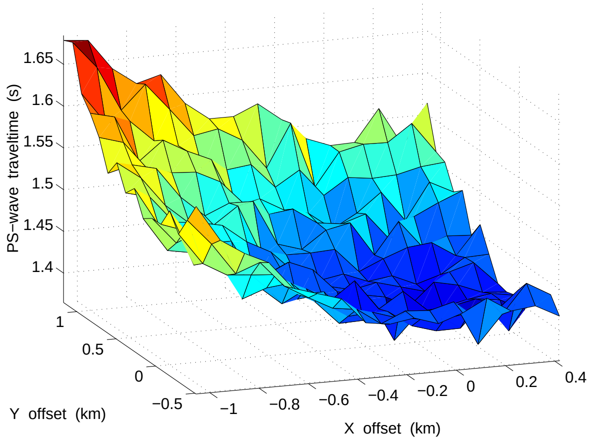

Figure 2. Reflection traveltimes of the PS-wave (after the addition of Gaussian noise) on a multiazimuthal CMP gather above a VTI layer. The VTI parameters are $V_{P0} = 2$ km/s, $V_{S0} = 1$ km/s, $\epsilon = 0.3$, $\delta = 0.1$; the reflector dip is 15°, the azimuth with respect to the x-axis is 0°, the depth under the CMP ($x = y = 0$) is 1 km.

We search for the best-fit pair $\{\delta, V_{S0}\}$ by matching the traveltime surface of the PS-wave on an areal CMP gather. The objective function is defined as the rms difference between the measured traveltimes and those computed for a trial model using the parametric traveltime-offset relationships (1), (2) and (3). The minimization of the objective function is carried out using the simplex method. Since the forward-modeling operation does not involve multiazimuth and multioffset two-point ray tracing, the algorithm allows for a fast examination of a relatively wide range of both unknown parameters.

To test the 3-D inversion procedure, we generated ray-traced P and PS data reflected from the bottom of a VTI layer and contaminated the PS traveltimes by Gaussian noise with a standard deviation of 1% (Figure 2). The traveltime surface of the PS-wave computed for each trial model was approximated by a 2-D quartic polynomial in the horizontal coordinates to find the times at the source and receiver locations. The results of the inversion are quite close to the actual values of the VTI parameters: $V_{P0} = 2.02$ km/s (error=0.02 km/s), $V_{S0} = 1.01$ km/s (error=0.01 km/s), $\epsilon = 0.29$ (error=–0.01), $\delta = 0.09$ (error=–0.01). The dip, azimuth and depth of the reflector were also reconstructed with high accuracy. It should be emphasized that the reflector dip in this test was quite mild (15°).

To reduce the influence of reflection-point dispersal, the model can be refined by resorting the PS data into areal common-conversion-point (CCP) gathers and

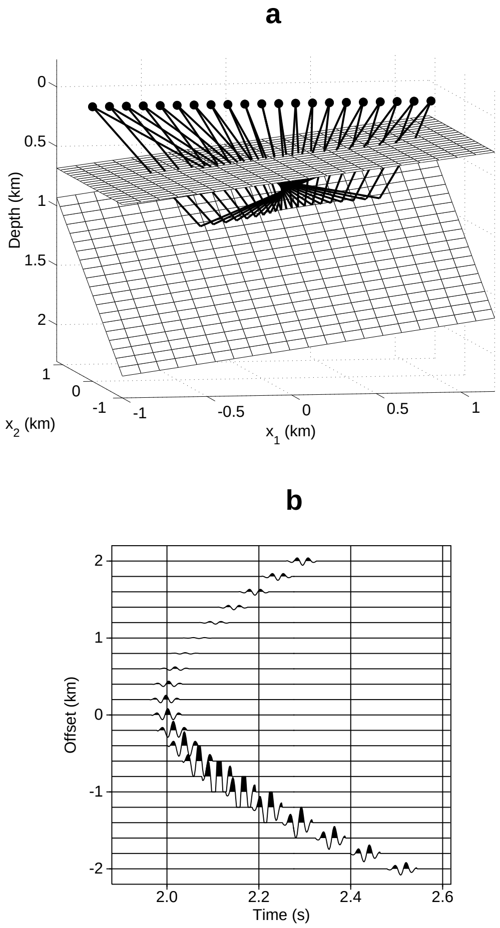

Figure 3. Rays (a) and the horizontal in-line displacement (b) of a PSV-wave reflected from a dipping interface overlaid by two VTI layers with typical moderate values of ϵ and δ.

repeating the inversion. This procedure is similar to the 2-D parameter-estimation algorithm described by Tsvankin and Grechka (1999).

Modeling and inversion on RTM gathers

Resorting of PS data into RTM gathers

Despite the generally successful results of the inversion on CMP and CCP gathers, application of the above methodology is hampered by the difficulties in reconstructing and processing of PS traveltime surfaces from reflection data. To illustrate those difficulties, we computed rays and ray-traced synthetic seismograms of a $PS(PSV)$-wave recorded on a CMP line above a two-layer VTI model (Figure 3). This example clearly shows three undesirable features of common-midpoint PS-data: moveout asymmetry with respect to zero offset, substantial reflection-point dispersal and low amplitudes along one of the branches of the gather compounded by a polarity reversal.

To overcome those problems, we suggest to resort PS data in such a way that would make the moveout symmetric and exclude source-receiver pairs in the low-amplitude area. First, we select a portion of the PS-moveout where the signal is sufficiently strong and the polarity does not change. Second, the data are resorted to make the moveout symmetric in the vicinity of a chosen (generally nonzero) offset. This procedure does not require knowledge of any model parameters because it is based solely on the reflection slopes picked from common-shot and common-receiver sections (see below). Third, we perform conventional hyperbolic velocity analysis on the resorted gathers and estimate the NMO velocity $V_{\text{nmo},PS}$. Fourth, $V_{\text{nmo},PS}$ is combined with the P-wave traveltimes in the inversion for the relevant medium parameters. The reconstructed depth model is then used in depth migration of the PS data.

The new resorting procedure can be applied to converted-wave data acquired on an *arbitrary* oriented line ℓ. Suppose the goal is to build a gather with a traveltime minimum corresponding to a given source-receiver pair $[s_\ell, r_\ell]$ (s_ℓ and r_ℓ are the source and receiver coordinates). Using a Taylor-series expansion of the traveltime, it can be shown that the displacements of the source ($s_\ell^\triangle$) and receiver ($r_\ell^\triangle$) with respect to their original positions $[s_\ell, r_\ell]$ should be related by

$$\frac{s_\ell^\triangle}{r_\ell^\triangle} = \frac{p_\ell^r}{p_\ell^s}\,, \tag{5}$$

where p_ℓ^s and p_ℓ^r are the slopes of the reflection event on the *common-shot* and *common-receiver* gathers formed at $[s_\ell, r_\ell]$. Therefore, the quantities p_ℓ^s and p_ℓ^r can be picked directly from the reflection data. The gather composed of source-receiver pairs that satisfy equation (5) will have a moveout minimum at $[s_\ell, r_\ell]$; we will call such a gather RTM (resorted to traveltime minimum).

Since the RTM gather is locally symmetric at $[s_\ell, r_\ell]$, it can be described by the NMO velocity defined by analogy with pure modes. The asymmetry of RTM moveout becomes noticeable at relatively large offsets and cannot prevent conventional velocity analysis from giving an accurate estimate of V_{nmo}. To give an analytic representation of NMO velocity on RTM gathers, we express it through the second derivatives of the traveltime with respect to the source and receiver coordinates. In contrast to the elliptical azimuthal dependence of the pure-mode NMO velocity (Grechka and Tsvankin, 1998), $V_{\text{nmo},PS}$ of mode conversions has an oval shape, which is not necessarily close to an ellipse. The oval does degenerate into an ellipse, however, if the RTM gather is constructed at zero offset and the medium above the reflector is homogeneous.

3-D inversion using RTM gathers

NMO velocities obtained on RTM gathers carry in-

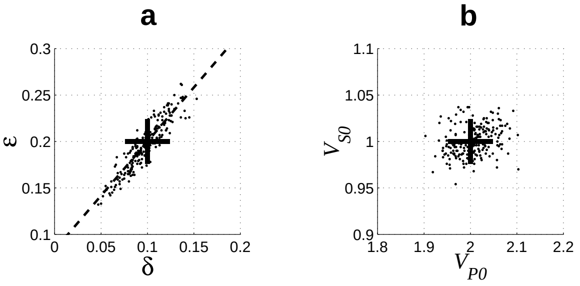

Figure 4. Anisotropic parameters ϵ and δ (a) and the vertical velocities V_{S0} and V_{P0} (b) determined by inverting P and PSV moveout data from a single dipping reflector. The input data were distorted by Gaussian noise with a standard deviation of 1% for the traveltimes and slownesses and 2% for the NMO velocities. Each dot represents the inversion result for a particular realization of the noise. The actual parameter values are marked by the crosses. The dashed line corresponds to the correct value of η.

formation about the anisotropic parameters and can be efficiently used in velocity analysis. Here, we present an inversion algorithm for VTI media that operates with wide-azimuth P and PS data reflected from a dipping interface. Whereas the input data are the same as those in the method operating in CMP geometry (see above), this time the PS arrivals are resorted into RTM gathers. To increase the angle coverage of reflected rays, we use $V_{\text{nmo},PS}$ determined on RTM gathers built for a range of initial source-receiver offsets.

Figure 4 shows the inversion results for a homogeneous VTI layer with the lower boundary (reflector) dipping at 30°. The input data included the following:

(i) P-wave NMO ellipse and zero-offset traveltimes.

(ii) PSV-wave NMO ellipse for RTM gathers built around zero offset.

(iii) PSV-wave NMO velocities on RTM gathers in the dip plane of the reflector at four initial offsets ($-z/2$, $-z/4$, $z/4$, $z/2$, where z is the reflector depth).

(iv) Horizontal slownesses (reflection slopes) of the PSV-wave on the common-shot and common-receiver gathers and the reflection traveltimes at the offsets $-z/2$, $-z/4$, 0, $z/4$, $z/2$.

The VTI parameters were obtained for 200 realizations of noise-contaminated input data using least-squares minimization. Sufficient stability of the inversion procedure is confirmed by the relatively low standard deviations of V_{P0}, V_{S0}, ϵ and δ (2.0%, 1.6%, 0.03 and 0.02, respectively). The reflector depth z and the dip ϕ were estimated with the standard deviations 2.1% and 0.4°. Note that the values of ϵ and δ are scattered around the line corresponding to the correct value of η which is tightly constrained by the P-wave NMO ellipse.

The remaining anisotropic coefficient γ can be found from PSH reflections which exist for all azimuthal directions outside the dip plane. Supplementing the input data listed above with the PSH-wave NMO ellipse for RTM gathers built around zero offset allowed us to estimate γ with a standard deviation comparable to that for ϵ and δ.

Discussion

The two methods of velocity analysis of mode-converted waves introduced here are based on different ways of resorting PS data. One method employs the traditional CMP geometry, with the multiazimuthal CMP gather generated by concise parametric expressions which eliminate the need for two-point ray tracing. After the initial inversion of PS data on CMP gathers, the results can be updated by repeating parameter estimation on CCP gathers. This two-step algorithm minimizes the influence of reflection-point dispersal, but it has to operate with asymmetric moveout and may suffer from the absence of reliable data and polarity reversals in areas of low-amplitude PS arrivals.

The key idea of the second approach is to resort PS-wave data into the so-called RTM gathers where the traveltime is symmetric in the vicinity of a certain offset. In contrast to CCP gathers, building of RTM gathers is a model-independent procedure that requires only the reflection slopes on the common-shot and common-receiver sections. RTM moveout is well-described by the NMO velocity which can be estimated from conventional velocity analysis and inverted for the medium parameters. Also, resorting into RTM gathers can be performed around selected source-receiver pairs with a strong PS arrival. The disadvantages of the RTM approach include a significant reflection-point dispersal on RTM gathers, the need for additional preprocessing (determination of reflection slopes, building of RTM gathers) and relatively complicated expressions for NMO velocity in RTM geometry.

Although the inversion examples were given for VTI media, both methods can be also applied to multicomponent 3-D data acquired over *lower-symmetry* anisotropic formations (e.g., fractured reservoirs).

References

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