Two prestack converted-wave migration algorithms for vertical transverse isotropy

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Summary

Because the Earth's subsurface is generally anisotropic, data processing techniques that take anisotropy into account are required when the magnitude of anisotropy is so large that it cannot be ignored. Also, with ocean bottom seismic (OBS) data, converted-wave sections may be superior to conventional P-wave data in some situations. Here, I present two 2D algorithms for prestack depth migration of converted-wave data in vertical transversely isotropic (VTI) media. Derived from the isotropic phase-shift-plus-interpolation (PSPI) and implicit finite-difference (FD) methods, these two algorithms inherit the accuracy of these wavefield-extrapolation migration methods. Based on analytic solution of the Christoffel equation, the anisotropic PSPI algorithm can handle arbitary strength of anisotropic parameters. The anisotropic FD algorithm, however, requires the weak-anisotropy assumption and thus is less accurate when anisotropy is strong. Migration examples on synthetic data show the success of the PSPI converted-wave algorithm for strongly VTI media. In contrast, application of the FD algorithm gives a less accurate result for comparable strong anisotropy.

Introduction

Conventional seismic data processing techniques are based on the assumption of isotropy. Such an assumption works fine when the anisotropic influence is negligible; ignoring anisotropy, however, may lead to migration errors, especially in imaging dipping reflectors, as studied by Larner & Cohen (1993) and Jaramillo & Larner (1995) for both poststack and prestack migration. To obtain a focused and correctly positioned subsurface image, conventional isotropic algorithms need to be extended to accommodate anisotropy.

In recent years, multicomponent OBS data have shown, for certain situations, better quality than that of pure P-P data. Granli et al. (1999) show an example of imaging through a gas zone with marine converted-wave data. Even though the conventional isotropic, converted-wave data processing used in their example enhanced the imaging quality significantly over that of the P-P data processing, anisotropy should be taken into account in areas that are strongly anisotropic. Thomsen (1999) shared his experience in the theoretical aspects of converted-wave data processing in areas of heterogeneous, anisotropic media, indicating that such processing is more dependent on physical assumptions concerning rock velocities than is pure mode processing.

The goal here is to develop 2D prestack, converted-wave algorithms for migration in VTI media. I initially intended to implement a Kirchhoff-type algorithm, but, given the complexity of implementing an anisotropic ray-tracing code, I turned instead to wavefield-extrapolation techniques, which are capable of generating an accurate image for complicated geological models. Among such methods, I singled out the phase-shift-plus-interpolation (PSPI) and implicit finite-difference (FD) methods as candidates for implementation of converted-wave migration in VTI media.

Here, I describe the two anisotropic 2D prestack, converted-wave migration algorithms, which I have tested on a simple synthetic data set.

P- and SV-wave anisotropy

As migration concentrates on kinematic aspects of wavefield propagation, I will concentrate on the anisotropic dispersion relationship and phase velocities for P- and SV-waves in VTI media.

Similar to the isotropic case, the anisotropic dispersion relationship is

$$k_z = \pm\sqrt{\frac{\omega^2}{v(\theta)^2} - k_x^2}, \tag{1}$$

where ω is frequency, v is velocity, and k_x and k_z are horizontal and vertical wavenumbers. This relationship differs from the isotropic case in that velocity v is no longer a constant at any location (x, z); now it depends on θ, the phase angle with respect to the symmetry axis in the transversely isotropic medium.

For all phase-shift-based migration algorithms, k_x is known; the issue is how to relate k_z to k_x. For isotropic media, ω and v are specified, so k_z can be readily computed from k_x, ω, and v using the isotropic dispersion relationship. For anisotropic media, however, the angle-dependence of velocity makes the computation more complicated.

According to Tsvankin (1996), the phase velocity for P- and SV-waves can be written in terms of Thomsen parameters ϵ, δ, plus Vp_0 and Vs_0 (the vertical phase velocities for P- and SV-waves) as

$$\frac{V^2(\theta)}{V_{P0}^2} = 1 + \epsilon \sin^2\theta - f/2$$

$$\pm f/2\sqrt{(1+\frac{2\epsilon\sin^2\theta}{f})^2-\frac{2(\epsilon-\delta)\sin^2 2\theta}{f}}, \qquad (2)$$

where the + and − signs are for P- and SV-waves respectively, and

$$f \equiv 1 - V_{S0}^2/V_{P0}^2. \qquad (3)$$

Expression (2) is obtained by solving for phase velocity as the eigenvalue of the Christoffel equation,

$$(a_{ijkl}n_jn_k - \delta_{il}V^2)U_i = 0, \qquad (4)$$

where a_{ijkl} is density-normalized stiffness tensor, equal to c_{ijkl}/ρ, with ρ the bulk density; n_j is the component of the unit vector in the j direction, and δ_{il} is the Kronecker delta function. For the 2D problem here, $j = 1$ or 3.

With the weak-anisotropy assumption, we further simplify the expression for the anisotropic phase velocity, yielding

$$\frac{V_P^2(\theta)}{V_{P0}^2} = 1 + 2\delta\sin^2\theta\cos^2\theta + 2\epsilon\sin^4\theta, \qquad (5)$$

and

$$\frac{V_S^2(\theta)}{V_{S0}^2} = 1 + 2\frac{V_{P0}^2}{V_{S0}^2}(\epsilon-\delta)\sin^2\theta\cos^2\theta. \qquad (6)$$

Implementation of anisotropic converted wave migration

Even though the goal here is to develop imaging tools for converted waves, because prestack migration of common-shot (or common-receiver) gathers involves separate one-way propagation of source and receiver wavefields, we need do nothing more than pure-mode data processing for each of the wavefields. That is, we can propagate one wavefield with P-wave velocity, and the other with SV-wave velocity.

Over the years, efforts have been made to develop migration algorithms for anisotropic media, especially transversely isotropic media. Kitchenside (1991) developed a phase-shift-based algorithm for TI media that used interpolating tables to link horizontal wavenumber to vertical wavenumber. Le Rousseau (1997) further used that idea for a table-driven PSPI (phase-shift-plus-interpolation) algorithm.

Finite-difference algorithms have been developed as well. Uzcategui (1994) extended the explicit downward-continuation method to TI media, using laterally-variant convolution operators, and allowing for lateral variation in both velocity and anisotropy parameters. Ristow and Rühl (1997) developed an implicit finite-difference operator founded on the simplifying assumption of weak anisotropy.

1. PSPI algorithm

Using equation (2) as the starting point, Le Rousseau (1997) developed an anisotropic PSPI algorithm for P-P migration wherein he precomputes a table of $k_z(\theta)$ and $k_x(\theta)$ for supplied sets of anisotropic parameters, locates (interpolates) a given input k_x in the table, and finds the corresponding k_z. The accuracy of this table-driven algorithm is directly related to the size of the table; the finer the increment in θ and the more sets of reference anisotropic parameters, the better the results. With a large table, however, searching is time-consuming. Also, since the code runs in parallel across a network, transferring a large table to all the slave processors puts a heavy burden on the network.

Different from the above PSPI algorithm, I derived my algorithm directly from the Christoffel equation. With $p_j = n_j/V$, the Christoffel equation can be rewritten as

$$(a_{ijkl}p_jp_k - \delta_{il})U_i = 0, \qquad (7)$$

where p_1 is the horizontal slowness, and p_3 is the vertical slowness.

The solution, $p_3(p_1)$, for both P- and SV-waves can be obtained simultaneously by solving the quadratic equation

$$Det[a_{ijkl}p_jp_k - \delta_{il}] = 0. \qquad (8)$$

Also since $p_1 = k_x/\omega$, and $p_3 = k_z/\omega$, k_x and k_z are connected through the relationship $p_3(p_1)$.

Because I use an analytical solution for Christoffel equation in VTI media, no interpolation table is needed. Also I make no assumption about the strength of anisotropy; i.e., no weak-anisotropy assumption is used in the PSPI algorithm.

As in the isotropic PSPI algorithm, sets of reference parameters must be used for the migration. At each depth step, I scan the P-wave velocity profile with the statistical method introduced by Bagaini et al. (1995) to determine the reference P-wave velocities, then choose reference values for SV-wave velocity, ϵ, and δ that are tied to the reference values for P-wave velocity.

2. Implicit FD algorithm

Ristow and Rühl (1997) developed a P-wave implicit finite-difference algorithm wherein the coefficients of the one-way finite-difference operator are expressed as a combination of V_{p0}, V_{s0}, ϵ and δ in each local cell of the model, thus permitting variations in all four parameters. In deriving the analytic expression of the finite-difference operator, Taylor expansions are used in several steps, but this approach requires the weak-anisotropy assumption.

Under the weak-anisotropy assumption, all the SV-wave algorithms can be obtained by simply replacing parameters in the P-wave formulas with corresponding ones for SV-waves (Tsvankin, 1996). The replacements are

$$V_{P0} \longrightarrow V_{S0} \tag{9}$$

$$\delta \longrightarrow \sigma \equiv \frac{V_{p0}^2}{V_{s0}^2}(\epsilon - \delta) \tag{10}$$

and

$$\epsilon \longrightarrow 0. \tag{11}$$

As with the P-wave algorithm, the implicit FD converted-wave algorithm will break down where anisotropy is strong.

Prestack migration example with anisotropic P-S data

I generated anisotropic prestack converted-wave data using a ray-theoretical method for the VTI media (Alkhalifah, 1993). The model used in the experiment is relatively simple, containing two horizontal and two dipping reflectors, with dip less than 30 degrees. The anisotropy parameters are constant across the model, with $V_{p0} = 1500$ m/s, $V_{s0} = 750$ m/s, $\epsilon = 0.2$, $\delta = 0.05$.

The prestack converted-wave PSPI migration, shown in Figure 1a, used the exact anisotropy parameters listed above. This excellent result can be used as a reference for comparison with other results. The isotropic PSPI result shown in Figure 1b was obtained by setting ϵ and δ to zero in the anisotropic PSPI algorithm. Because the correct vertical velocities were used in the migration, this result shows the correct imaged depth for the horizontal reflectors. In practice, however, the image depth will generally be incorrect because conventional velocity analysis does not yield true vertical velocity. Because anisotropy is ignored for the migration shown in Figure 1b, the two dipping reflectors are poorly imaged, and mis-positioned.

The anisotropic FD results are shown in Figure 2. Figure 2a was obtained using the same anisotropic parameters as in the anisotropic PSPI algorithm. Even though the reflectors are imaged at the correct location, the image quality is poorer than that of the anisotropic PSPI result, particularly for the dipping reflectors. Since the model is simple, the likely source of the problem

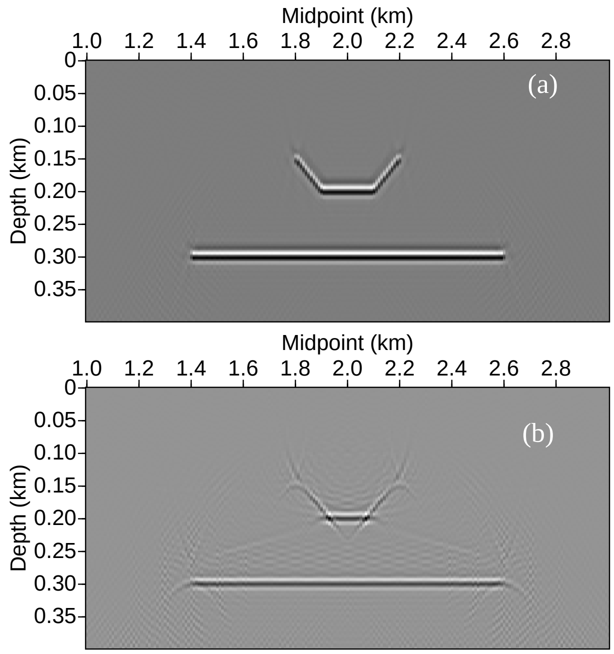

Figure 1. PSPI results; (a) anisotropic result using the exact parameters; (b) result with anisotropy ignored .

is that the anisotropy parameters violate the weak-anisotropy assumption in the anisotropic FD algorithm. For a model with ϵ reduced to 0.1, but keeping other parameters untouched, the anisotropic FD result (Figure 2b) shows much improved amplitude for the dipping reflectors. For another model ($\epsilon = 0.05$ and $\delta = 0.2$), the anisotropic FD result (Figure 2c) has even more artifacts than that shown in Figure 2a.

Conclusions

The tests on a simple anisotropic model show successful migration of anisotropic, 2D converted-wave data using the PSPI and FD algorithms. The anisotropic PSPI algorithm gives an excellent image for the data tested because no approximation for the anisotropic dispersion relationship is made in this algorithm. The FD algorithm achieved acceptable results as long as the approximation of weak-anisotropy in the FD algorithm was not violated.

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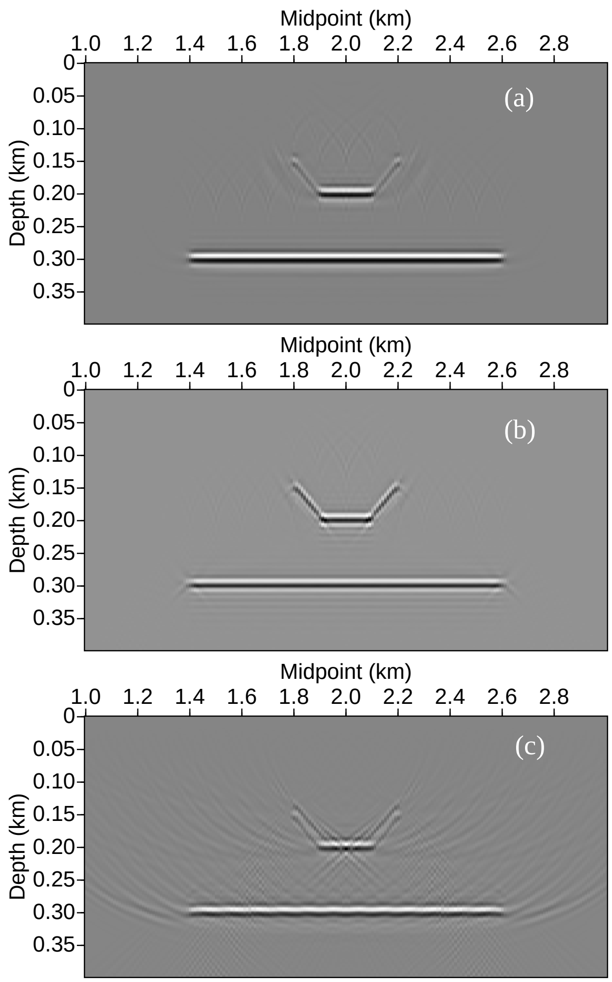

Figure 2. Anisotropic FD results: (a) FD result using the same parameters as in anisotropic PSPI; (b) FD result with ϵ reduced from 0.2 to 0.1, keeping other parameters untouched; (c) FD result with $\epsilon = 0.05$ and $\delta = 0.2$.

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