Estimation of fracture parameters of monoclinic media from reflection seismic data

Andrey Bakulin[⋆], *Vladimir Grechka*[†] *and Ilya Tsvankin*[†]
[⋆]*Schlumberger Cambridge Research, High Cross, Madingley Road, Cambridge, CB3 0EL, England*
[†]*Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines, Golden, CO 80401*

Summary

Geophysical and geological data acquired over naturally fractured reservoirs often reveal the presence of multiple subvertical fracture sets, which make the effective medium monoclinic. Here, we discuss modeling and inversion of the anisotropic parameters for two types of fractured media with monoclinic symmetry.

The first model is formed by two different non-orthogonal sets of rotationally invariant vertical fractures in an isotropic host rock. Using exact NMO equations, we devise a complete fracture-characterization procedure based on the vertical velocities of the P- and two split S-waves (or converted PS-waves) and their NMO ellipses from a horizontal reflector. Our algorithm yields the azimuths and compliances of both fracture systems, as well as the P- and S-wave velocities in the isotropic background medium.

In the second model, which contains a single set of *micro-corrugated* fractures, monoclinic symmetry stems from the coupling between the normal and tangential (to the fracture faces) slips. This coupling causes a dependence of the shear-wave splitting coefficient at vertical incidence on the *fluid content* of the fractures – a phenomenon that cannot be explained by conventional fracture models.

Introduction

This work continues our series of publications aimed at developing efficient fracture-characterization methodologies based on seismic reflection data. The previous two papers (Bakulin et al. 1999a; Bakulin et al. 1999b) were devoted to fractured models with HTI (transversely isotropic with a horizontal symmetry axis) and orthorhombic symmetry.

Here, the linear-slip theory of Schoenberg and co-workers (e.g., Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995) is used to extend our previous results to two types of more complicated monoclinic models. We relate the Thomsen-type anisotropic coefficients, which govern the influence of anisotropy on various seismic signatures, to the inherent parameters of the linear-slip theory – the background stiffnesses and fracture compliances and azimuths. This formalism allows us to invert seismic signatures for the fracture parameters and make inferences about the physical properties of fracture networks.

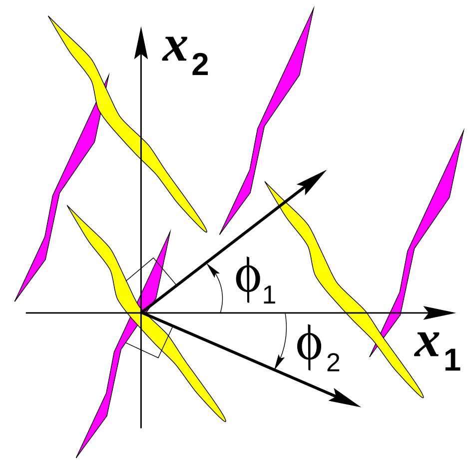

Figure 1. Two sets of parallel vertical fractures form an effective monoclinic medium. The fracture normals make the angles ϕ_1 and ϕ_2 with the axis x_1 that coincides with the polarization direction of the vertically traveling S_1-wave. To define ϕ_1 and ϕ_2 in a unique fashion, we assume that $-\pi/2 < \phi_1 < \pi/2$ and $-\pi/2 < \phi_2 < \pi/2$.

Two sets of vertical fractures

Compliance formalism

Within the framework of the linear-slip theory, the effective compliance matrix $\mathbf{s}$ of a rock with multiple fracture sets can be determined as the sum of the compliance $\mathbf{s}_b$ of the background medium and the fracture compliances $\mathbf{s}_f$ (e.g., Schoenberg and Sayers, 1995). For two different arbitrarily oriented fracture sets with the compliances $\mathbf{s}_{f1}$ and $\mathbf{s}_{f2}$ in a purely isotropic background (Figure 1), the effective compliance $\mathbf{s}$ and stiffness $\mathbf{c}$ are given by

$$\mathbf{s} = \mathbf{s}_b + \mathbf{s}_{f1} + \mathbf{s}_{f2} \equiv \mathbf{c}^{-1}\,. \qquad (1)$$

In general, the effective medium has monoclinic symmetry. Assuming that both fracture sets are rotationally invariant, their matrices $\mathbf{s}_{fi}$ $(i = 1, 2)$ can be described by the normal and tangential (with respect to the crack faces) compliances K_{Ni} and K_{Ti}. If the normal $\mathbf{n}_i$ to the ith fracture set points in the direction of the x_1-axis, the only non-zero elements of the compliance matrix are $s_{fi,11} = K_{Ni}$,

$s_{fi,55} = s_{fi,66} = K_{Ti}$. The matrix $\mathbf{s}_{fi}$ in a rotated coordinate system can be found using the so-called Bond transformation.

A simplified form of the effective stiffness matrix $\mathbf{c}$ can be obtained by aligning the horizontal coordinate axes with the polarization directions of the vertically traveling S-waves. In this natural coordinate frame the stiffness coefficient c_{45} vanishes and the number of independent stiffnesses reduces from thirteen to twelve.

It is convenient to replace the compliances by the dimensionless fracture "weaknesses" Δ_{Ni} and Δ_{Ti} $(i = 1, 2)$:

$$\Delta_{Ni} = \frac{(\lambda + 2\mu)\, K_{Ni}}{1 + (\lambda + 2\mu)\, K_{Ni}}\,; \quad \Delta_{Ti} = \frac{\mu\, K_{Ti}}{1 + \mu\, K_{Ti}}\,. \quad (2)$$

The condition $c_{45} = 0$ leads to the following constraint:

$$K_{T1} \sin 2\phi_1 + K_{T2} \sin 2\phi_2 = 0\,, \quad (3)$$

or

$$\frac{\Delta_{T1}}{1 - \Delta_{T1}} \sin 2\phi_1 + \frac{\Delta_{T2}}{1 - \Delta_{T2}} \sin 2\phi_2 = 0\,. \quad (4)$$

According to equation (3), the polarization direction of the fast (S_1) shear wave (i.e., the x_1-axis) is always closer to the strike of the more compliant fractures. Quantitative estimates in Figure 2 show that for $K_{T1}/K_{T2} > 3$ the S_1-polarization direction does not deviate by more than 10° from the strike of the first fracture system (see also Sayers, 1998).

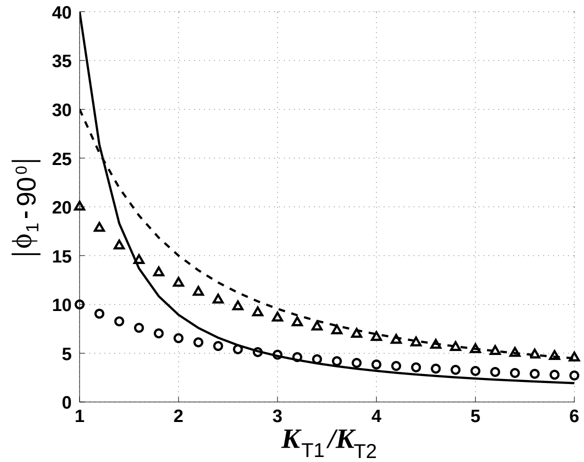

Figure 2. Azimuth of first fracture system ($|\phi_1 - 90^\circ|$) as a function of the ratio of the tangential compliances K_{T1}/K_{T2}. Each curve corresponds to a different angle between the fracture systems: $\phi_1 - \phi_2 = 20^\circ$ (circles), 40° (triangles), 60° (dashed line), and 80° (solid line).

Estimation of fracture parameters

Grechka et al. (1999) suggested to replace the stiffness coefficients of monoclinic media by Thomsen-type anisotropic parameters which control the NMO ellipses (i.e., conventional-spread reflection traveltimes) of P- and S-waves. The first parameter group contains the vertical velocities of P-waves (V_{P0}) and one of the S-waves (V_{S0}) and the coefficients $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$ and $\gamma^{(1,2)}$ defined exactly in the same way as Tsvankin's (1997) parameters for orthorhombic media. These nine quantities mainly control the semi-axes of the NMO ellipses of waves P, S_1, and S_2 reflected from horizontal interfaces. The second group includes three coefficients $\zeta^{(1,2,3)}$ (which vanish in orthorhombic media) responsible for the *rotation* of the NMO ellipses with respect to the coordinate axes. Grechka et al. (1999) showed that eleven parameters of monoclinic media (all except $\delta^{(3)}$) can be estimated in a stable way using the vertical velocities and NMO ellipses from horizontal reflectors of the P- and two split S-waves; pure shear reflections can be replaced by the converted waves PS_1 and PS_2. Here we express these parameters through the fracture compliances and background velocities by applying their definitions in terms of the stiffnesses to the effective matrix $\mathbf{c}$ given by equation (1).

Our goal is to invert the eleven parameters (V_{P0}, V_{S0}, $\epsilon^{(1,2)}$, $\delta^{(1,2)}$, $\gamma^{(1,2)}$ and $\zeta^{(1,2,3)}$) and the constraint (4) for the eight physical parameters of the model: the P- and S-wave velocities V_P and V_S in the isotropic background, the azimuths of the fracture sets ϕ_1 and ϕ_2 and the weaknesses Δ_{Ni} and Δ_{Ti} $(i = 1, 2)$. Only in the limit of weak anisotropy can the inversion be carried out without applying numerical methods. Another favorable case for parameter estimation is that of equal tangential compliances, when the S-wave polarization direction bisects the angle between the fracture sets (see Figure 2 for $K_{T1}/K_{T2} = 1$). For such a model, the fracture parameters can be obtained in a relatively straightforward fashion just from the vertical velocities of the P- and S-waves and the P-wave NMO ellipse.

To examine the stability of the inversion algorithm in the general case, we introduced errors in the input parameters by adding Gaussian noise with the following variances: 2% in V_{P0} and V_{S0}, 0.01 in $\zeta^{(1)}$ and $\zeta^{(2)}$, and 0.03 in all other anisotropic coefficients. The fracture parameters that give the best fit to the error-contaminated values of the vertical velocities and anisotropic coefficients are found by minimization of the exact equations using the simplex method. Figure 3 displays typical inversion re-

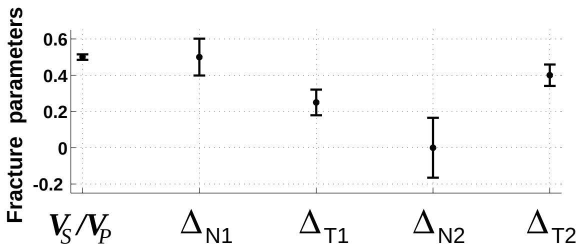

Figure 3. Results of the inversion for the parameters of two fracture systems and the background velocities. The dots mark the actual values of the fracture parameters; the bars correspond to ± one standard deviation in the inverted quantities. Not shown are the standard deviations in the estimated background velocities V_P and V_S (2.0% and 2.5%, respectively) and in the fracture azimuths (9°; the actual values are $\phi_1 = 30°$ and $\phi_2 = -12.8°$).

sults for the V_S/V_P ratio The standard deviation in the estimated V_S/V_P ratio (3.1%) is somewhat higher than that in the input vertical velocities V_{P0} and V_{S0} but still is quite acceptable. Although the errors in the weaknesses are much larger than those in the input anisotropic coefficients, it is possible to distinguish between dry and fluid-filled fractures using the normal weaknesses. The first system (dry fractures) has $\Delta_{N1} = 0.5$, while fluid-filled fractures of the second system are characterized by $\Delta_{N2} \approx 0$. The unphysical values $\Delta_{N2} < 0$ appear in Figure 3 because random errors added to the data may produce anisotropic coefficients which do not correspond to any fractured medium.

Single set of micro-corrugated fractures

It has been common knowledge among researchers working on fracture characterization that for media with vertical fractures shear-wave splitting at vertical incidence depends only on the fracture density and does not contain any information about the fluid content of the fracture network. The magnitude of splitting is usually described by the parameter $\gamma^{(S)}$ defined through the vertical shear-wave velocities V_{S1} and V_{S2} as

$$\gamma^{(S)} \equiv \frac{V_{S1}^2 - V_{S2}^2}{2\,V_{S2}^2}\,. \qquad (5)$$

Recently, Guest et al. (1998) presented a case study where $\gamma^{(S)}$ for gas-filled cracks proved to be significantly higher than that for brine-filled ones. Here, we give a possible theoretical explanation of these observations by obtaining the effective parameters of a fracture set with *micro-corrugated* faces.

We consider a single system of parallel fractures

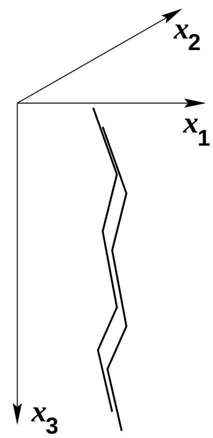

Figure 4. Vertical fracture with micro-corrugated faces. This microstructure leads to the coupling between the normal and tangential slips.

with the normal $\mathbf{n} = [1, 0, 0]$ in an isotropic background medium. The matrix of the excess fracture compliance in this case has the form (Schoenberg and Douma, 1988):

$$\mathbf{s}_f = \begin{pmatrix} K_N & 0 & 0 & 0 & K_{NV} & K_{NH} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K_{NV} & 0 & 0 & 0 & K_V & K_{VH} \\ K_{NH} & 0 & 0 & 0 & K_{VH} & K_H \end{pmatrix}. \qquad (6)$$

The compliance K_{NV} couples the normal slip to the tangential vertical stress or, equivalently, the tangential slip in the direction $[0, 0, 1]$ to the normal stress. Likewise, the compliances K_{NH} and K_{VH} couple the horizontal stress in the $[0, 1, 0]$ direction to the normal and vertical slips. The conventional conclusion about the shear-wave splitting coefficient $\gamma^{(S)}$ being independent of fracture infill is based on the *assumption* that the normal and tangential slips are decoupled ($K_{NV} = K_{NH} = K_{VH} = 0$). For fractures with micro-corrugated faces (Figure 4), however, the compliance component K_{NV} does not vanish because the stresses in either the normal (x_1) or vertical (x_3) direction applied to such a fracture produce slips in both x_1 and x_3 directions.

Normalizing the compliances by introducing $E_V = \mu K_V$ and $E_{NV} = \sqrt{\mu(\lambda + 2\mu)}K_{NV}$ (λ and μ are the Lamé parameters) and keeping only the linear and quadratic terms in E_V and E_{NV}, we obtain the shear-wave splitting coefficient [equation (5)] as

$$\gamma^{(S)} \approx \frac{E_V}{2} - \frac{E_{NV}^2\, g\,(3 - 4\,g)}{2\,(1 - g)}\,, \qquad (7)$$

where $g \equiv V_S^2/V_P^2$. Equation (7) shows that the cou-

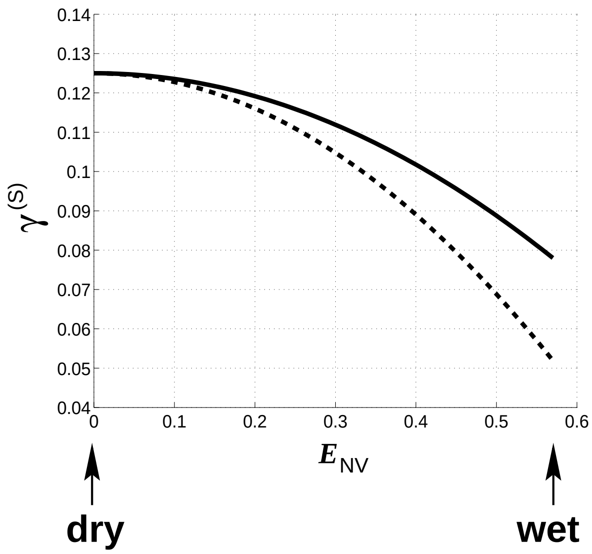

Figure 5. Shear-wave splitting coefficient $\gamma^{(S)}$ computed from the exact equations (solid) and the approximation (7) (dashed) as a function of the dimensionless compliance E_{NV}. The model parameters are $g = 0.16$, $E_N = 1.3$, and $E_V = 0.25$.

pling between the normal and tangential slips that leads to $E_{NV} \neq 0$ always *reduces* the value of $\gamma^{(S)}$.

To analyze the influence of fluid content on shear-wave splitting for fractures with micro-corrugated faces, we generalized the criterion originally formulated by Schoenberg and Sayers (1995) to distinguish between dry and fluid-filled penny-shaped cracks. The splitting coefficients for dry and fluid-filled micro-corrugated fractures are given by

$$\gamma^{(S)}_{\text{dry}} \approx \frac{E_V}{2} \tag{8}$$

and

$$\gamma^{(S)}_{\text{wet}} \approx \frac{E_V}{2} \left[1 - \frac{E_N\, g\,(3-4\,g)}{1-g}\right] , \tag{9}$$

where $E_N = (\lambda + 2\mu)K_N$. Hence, for micro-corrugated fractures the splitting coefficient is always higher for dry than for fluid-filled fractures, which is consistent with the observations of Guest et al. (1998). In Figure 5, $\gamma^{(S)}$ decreases by about 30% as fluid saturation changes from zero ($E_{NV} = 0$) to 100%.

Conclusions

The linear-slip theory and the methodology of 3-D (azimuthal) moveout analysis were applied to develop an inversion procedure for an effective monoclinic medium caused by two non-orthogonal sets of rotationally invariant fractures. Fracture orientation in this model cannot be determined directly from the data because the axes of the NMO ellipses and the polarization directions of the vertically traveling S-waves generally do not coincide with each other and with the strike of either fracture system. The weaknesses and azimuths of both fracture systems, along with the velocities in the isotropic background, can be obtained using the vertical velocities and NMO ellipses (from horizontal interfaces) of the P- and two split S-waves [or two converted (PS) waves]. Numerical analysis shows that the tangential compliances are generally estimated with a higher accuracy than the normal ones, but the difference between the normal compliances of dry and fluid-filled cracks can still be detected in the presence of noise in the data.

We also examined another monoclinic model that contains a single system of vertical fractures with micro-corrugated faces in an isotropic host rock. A simple approximation for the shear-wave splitting coefficient $\gamma^{(S)}$ predicts a substantial decrease in $\gamma^{(S)}$ with fluid saturation, which agrees with the experimental results of Guest et al. (1998).

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