## Introduction

Efficient application of the modern technology of wide-azimuth and multi-azimuth surveys requires development of interval velocity-analysis methods that take azimuthal anisotropy into account. Here, we discuss moveout inversion of P-wave data for orthorhombic models which describe, for example, naturally fractured formations (e.g., Schoenberg and Helbig 1997). The interval parameters of anisotropic media are often estimated using Dix-type layer stripping or tomographic techniques. In particular, the Dix-type formalism (Dix 1955; Grechka et al. 1999; Tsvankin 2005) helps to obtain the interval normal-moveout (NMO) ellipses and anellipticity parameters $\eta^{(1,2,3)}$ of orthorhombic layers from azimuthally varying, long-spread P-wave traveltimes (Vasconcelos and Tsvankin 2006). However, the tradeoff between the NMO velocity and the quartic moveout coefficient may cause significant errors in the effective $\eta$-parameters, which are greatly amplified in the layer-stripping process (Grechka and Tsvankin 1998).

Here, we suggest an alternative approach based on the velocity-independent layer-stripping method (VILS) of Dewangan and Tsvankin (2006). Instead of the effective moveout parameters used by Dix-type techniques, VILS operates directly with reflection traveltimes, which are estimated with much higher accuracy. We introduce the 3D version of VILS and apply it to nonhyperbolic moveout inversion of wide-azimuth synthetic and field P-wave data from orthorhombic media.

## 3D layer-stripping methodology

The layer-stripping algorithm developed by Dewangan and Tsvankin (2006) is entirely data-driven and can be applied prior to velocity analysis. The 3D version of VILS does not impose any restrictions on the properties (anisotropy, heterogeneity) of the target zone, but each layer in the overburden has to be laterally homogeneous with a horizontal symmetry plane. Equalizing time slopes (horizontal slownesses) on common-source or common-receiver gathers helps to identify the reflections from the top of the target horizon that share the downgoing and upgoing ray segments in the overburden with the target event (Figure 1). Note that for wide-azimuth data it is necessary to estimate two orthogonal horizontal slowness components from reflection time slopes. The interval traveltime in the target layer can then be obtained as

$$t^{\rm int}(T,R) = t^{\rm eff}(x^{(1)},x^{(2)}) - \frac{1}{2}\left[t^{\rm ovr}(x^{(1)},x^{(3)}) + t^{\rm ovr}(x^{(2)},x^{(4)})\right], \qquad (1)$$

where the superscripts "eff" and "ovr" refer to the target event $x^{(1)}$TQR$x^{(2)}$ and the reflections from the bottom of the overburden, respectively. Since $\mathbf{T}$ and $\mathbf{R}$ represent the midpoints of the corresponding source-receiver pairs, their horizontal coordinates can be easily found from $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$, and $\mathbf{x}^{(4)}$. Thus, VILS makes it possible to construct the interval moveout function from 3D wide-azimuth data.

## Moveout inversion for orthorhombic media

To describe P-wave reflection traveltimes in an orthorhombic layer with a horizontal symmetry plane, we use the generalized version of the nonhyperbolic moveout equation suggested by Alkhalifah and Tsvankin (1995) for VTI media (Vasconcelos and Tsvankin 2006):

$$t^2(x,\alpha) = t_0^2 + \frac{x^2}{V_{\rm nmo}^2(\alpha)} - \frac{2\eta(\alpha)\,x^4}{V_{\rm nmo}^2(\alpha)[\,t_0^2\,V_{\rm nmo}^2(\alpha) + (1+2\eta(\alpha))x^2]}, \qquad (2)$$

where $x$ is the offset, $\alpha$ is the azimuth of the source-receiver line, $t_0$ is the zero-offset time, $V_{\rm nmo}$ is the normal-moveout (NMO) velocity, and $\eta$ is the anellipticity parameter. The velocity $V_{\rm nmo}$ is obtained from the equation of the NMO ellipse:

$$V_{\rm nmo}^{-2}(\alpha) = \frac{\sin^2(\alpha-\varphi)}{\left[V_{\rm nmo}^{(1)}\right]^2} + \frac{\cos^2(\alpha-\varphi)}{\left[V_{\rm nmo}^{(2)}\right]^2}, \qquad (3)$$

and the parameter $\eta$ is approximately given by

$$\eta(\alpha) = \eta^{(1)}\sin^2(\alpha-\varphi) + \eta^{(2)}\cos^2(\alpha-\varphi) - \eta^{(3)}\sin^2(\alpha-\varphi)\cos^2(\alpha-\varphi), \qquad (4)$$

where $\varphi$ is the azimuth of the $[x_1, x_3]$ symmetry plane, $V_{\rm nmo}^{(1)}$ and $V_{\rm nmo}^{(2)}$ are the symmetry-plane NMO velocities, and $\eta^{(1,2,3)}$ are the anellipticity parameters defined in the symmetry planes of the model

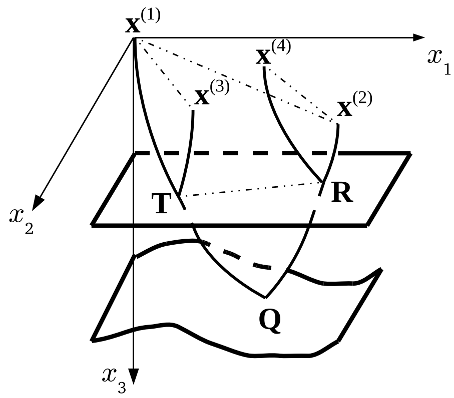


Figure 1: 3D diagram of the layer-stripping algorithm. Points $\mathbf{T}$ and $\mathbf{R}$ are located at the bottom of the laterally homogeneous overburden. The sources and receivers ($\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$ and $\mathbf{x}^{(4)}$) are placed at the surface but not necessarily along a straight line. The reflection point $\mathbf{Q}$ is located at the bottom of the target layer, which can be arbitrarily anisotropic and heterogeneous. The leg $\mathbf{x}^{(1)}\mathbf{T}$ is shared by the target event $\mathbf{x}^{(1)}\mathbf{TQRx}^{(2)}$ and the overburden reflection $\mathbf{x}^{(1)}\mathbf{Tx}^{(3)}$; the leg $\mathbf{Rx}^{(2)}$ is shared by the reflections $\mathbf{x}^{(1)}\mathbf{TQRx}^{(2)}$ and $\mathbf{x}^{(2)}\mathbf{Rx}^{(4)}$.

(Grechka and Tsvankin 1999; Vasconcelos and Tsvankin 2006). For layer-cake orthorhombic media, all moveout parameters become effective quantities. If the vertical symmetry planes in different layers are misaligned, the principal directions for the effective $\eta(\alpha)$ are described by a separate azimuth, $\varphi_1$.
To implement both VILS and the Dix-type inversion for layered orthorhombic media, we use the 3D nonhyperbolic semblance algorithm of Vasconcelos and Tsvankin (2006) based on equations (2-4). The best-fit effective moveout parameters $V_{\rm nmo}^{(1,2)}$, $\eta^{(1,2,3)}$, $\varphi$ and $\varphi_1$ for the top and bottom of the target layer are found by multidimensional semblance search using the full range of available offsets and azimuths. For purposes of the Dix-type layer stripping, the interval NMO ellipse is obtained from the generalized Dix equation (Grechka et al. 1999), and the interval $\eta$ value for each azimuth is computed from the VTI equation given in Tsvankin (2005). Finally, the interval parameters $\eta^{(1,2,3)}$ are estimated by fitting the azimuthally varying $\eta$ values to equation (4). Although equations (2-4) provide a good approximation for nonhyperbolic moveout, the parameters $\eta^{(1,2,3)}$ are sensitive to small errors in $V_{\rm nmo}^{(1,2)}$ even if the maximum offset-to-depth ratio ($x_{\rm max}/h$) is between two and three. This trade-off causes substantial instability in the $\eta$ estimation, which is amplified in the Dix-type layer stripping.
The long-spread, wide-azimuth reflection traveltimes reconstructed by the nonhyperbolic semblance analysis also serve as the input data for VILS. To apply VILS to 3D wide-azimuth data (Figure 1), it is necessary to estimate two horizontal slowness components at the source and receiver locations. For layer-cake media, the horizontal slownesses can be computed from the best-fit moveout parameters using equation (2).

## Application to synthetic data

We tested our layer-stripping algorithm on wide-azimuth, long-spread P-wave data generated by anisotropic ray tracing. For the model in Figure 2, the azimuthal anisotropy in the orthorhombic target layer makes long-offset traveltimes from its bottom vary noticeably with azimuth. Without traveltime noise, both methods give similar accuracy in the interval moveout parameters. To simulate the influence of long- and short-period statics errors, we added linear and sinusoidal error functions to the reflection traveltimes from the bottom of the target layer. For an azimuthally invariant linear error that changes from 6 ms at zero offset to -6 ms at the maximum offset, the interval parameters $V_{\rm nmo}^{(1,2)}$ and $\eta^{(1,2,3)}$ estimated by VILS are distorted by no more than 3% and 0.06, respectively. The Dix-type method produces much larger errors reaching 9% in $V_{\rm nmo}^{(1,2)}$ and 0.16 in $\eta^{(1,2,3)}$. For the noise function of the form $t(x,\alpha) = A\sin(n\pi x/x_{\rm max})\sin m\alpha$ with $A = 10\,{\rm ms}$ the Dix-type method distorts the NMO velocities by up to 10% and the $\eta$-parameters by 0.22, while the errors of VILS are limited to 2.4% and 0.09,

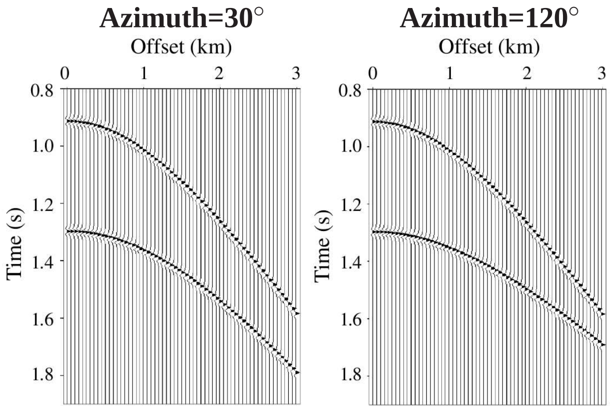


Figure 2: Long-spread P-wave reflections from the top and bottom of the target orthorhombic layer in a three-layer model. The subsurface layer is isotropic with $V_{\rm nmo} = V_P = 2$ km/s; the second layer is VTI with $V_{\rm nmo} = 2.49$ km/s and $\eta = 0.05$; the third (target) layer is orthorhombic with $V_{\rm nmo}^{(1)} = 3.18$ km/s, $V_{\rm nmo}^{(2)} = 2.64$ km/s, $\eta^{(1)} = 0.2$, $\eta^{(2)} = 0.06$, $\eta^{(3)} = 0.13$, and $\varphi = 30°$. The thickness of each layer is the same (0.5 km). The seismograms are computed in the orthogonal vertical symmetry planes of the orthorhombic layer.

respectively.

Tests for more complicated models that include orthorhombic layers with misaligned vertical symmetry planes also demonstrate the superior stability of VILS in the presence of typical correlated noise. Even relatively small time errors can cause substantial distortions in the effective moveout parameters, which propagate with amplification into the interval $\eta$-values. In contrast, percentage errors in the traveltimes themselves are insignificant, which ensures the high accuracy of the interval moveout produced by VILS. Still, as any other layer-stripping technique, VILS breaks down for relatively thin target layers (i.e., the thickness-to-depth ratio should exceed 15-20%).

**Field-data example**

The VILS algorithm was applied to wide-azimuth P-wave data acquired by the Reservoir Characterization Project (a CSM research consortium) at Rulison field, a tight-sand gas field in Piceance Basin, Colorado. Since the subsurface structure is close to layer-cake (Figure 3), equations (2-4) give an accurate description of reflection traveltimes. To increase the azimuth and offset coverage, we combined CMP gathers into superbins, as suggested by Xu and Tsvankin (2007). The average offset-to-depth ratio at the bottom of the reservoir is close to unity, which is insufficient for nonhyperbolic moveout analysis. Therefore, we chose the UMV shale (cap rock; Figure 3) as the target layer; in the center of the study area, the offset-to-depth ratio at the bottom of the shale is between 1.9 and 2.2.

The interval parameters $\eta^{(1,2,3)}$ for two superbin gathers near the center of the area are listed in Table 1. Although there is no independent information about the actual anellipticity parameters in the field, the values produced by VILS are much more plausible than those computed from the Dix-type equations. First, the $\eta$-parameters obtained by the Dix-type method are implausibly large for shale formations (Tsvankin 2005), while VILS yields much smaller magnitudes of $\eta^{(1,2,3)}$. Second, available data indicate that the symmetry of the shale is close to VTI. This implies that the difference between the parameters $\eta^{(1)}$ and $\eta^{(2)}$, as well as the magnitude of $\eta^{(3)}$, should be relatively small, which is in agreement with the output of VILS. The values of $\eta^{(2)}$ produced by the Dix-type method, however, are much larger than those of $\eta^{(1)}$. Near the east boundary of the study area, where the offset-to-depth ratio is smaller and the $\eta$-parameters are poorly constrained, application of VILS helps to obtain a more stable estimate of the interval NMO ellipse.

**Conclusions**

We extended the velocity-independent layer-stripping method (VILS) of Dewangan and Tsvankin (2006) to wide-azimuth data and applied it to nonhyperbolic moveout inversion for layered orthorhombic media. If the overburden is laterally homogeneous and has a horizontal symmetry plane, VILS generates

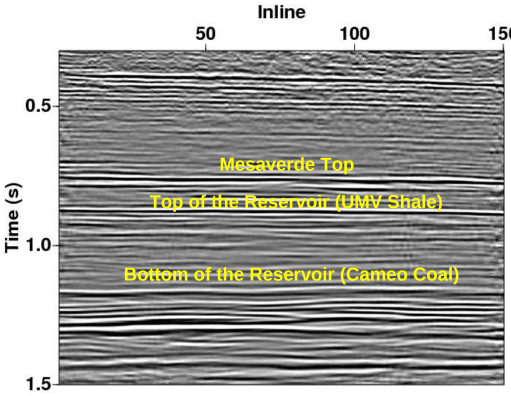


Figure 3: Typical seismic section at Rulison field (after Xu and Tsvankin 2007). The UMV shale layer (target) is beneath Mesaverde Top.

| Superbin | 1 | | | 2 | | |
|---|---|---|---|---|---|---|
| | $\eta^{(1)}$ | $\eta^{(2)}$ | $\eta^{(3)}$ | $\eta^{(1)}$ | $\eta^{(2)}$ | $\eta^{(3)}$ |
| VILS | 0.38 | 0.47 | -0.18 | 0.24 | 0.31 | -0.15 |
| Dix | 0.74 | 1.24 | -0.35 | 0.31 | 0.62 | -0.19 |

Table 1: Interval parameters $\eta^{(1,2,3)}$ estimated for two superbin gathers in the center of the study area.

the interval traveltime without information about the velocity field. Because effective traveltimes are much better constrained by reflection data than effective moveout parameters, VILS does not suffer from the inherent instability of Dix-type algorithms. Synthetic tests on noise-contaminated P-wave data confirm that VILS substantially increases the accuracy of nonhyperbolic moveout inversion for the interval anellipticity parameters $\eta^{(1,2,3)}$. The method was also successfully used to estimate the parameters of an anisotropic shale layer from wide-azimuth field data acquired at Rulison field in Colorado, USA.

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