Wave Equation Migration Using Isochron Rays

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Summary

The migration of a single seismic trace produces an image where the amplitudes are distributed along isochron surfaces. This image can be interpreted as a superposition of snapshots taken from a propagating isochron in an equivalent velocity media, i.e. an isochron-field. We present an isochron-field migration method which is comprised of three steps: equivalent velocity computation, data conditioning, and application of a standard zero-offset wave-field extrapolation migration algorithm. We suggest the term equivalent exploding reflector model for the hypothetical model that theoretically supports our imaging approach. The presented methodology extends the use of zero-offset wave-equation imaging algorithms to finite-offset gathers. We successfully applied the approach for depth migrate a common-offset gather. The methodology presents some features that can facilitate the implementation of parallelized migration velocity analysis algorithms.

Introduction

Isochron surfaces play an important role in seismic imaging. Hubral et al. (1996) show how a weighted Kirchhoff-type isochrone-stack integral can be applied in true-amplitude algorithms for both modeling and data transformation. The general theory of data mapping, presented by Bleistein et al. (2000), evidences the importance of isochrons in the establishment of integral formulas for inversion. Iversen (2004) introduced the term isochron ray for trajectories associated with surfaces of equal two-way time, i.e. isochron surfaces. He pointed out the potencial use of the isochron rays in future implementations of prestack depth migration. We are introducing a methodology that makes use of the isochron ray concept to perform prestack depth migration. We consider as isochron rays the lines that are perpendicular to the isochrons associated with the image produced by the migration of a single finite-offset seismic trace. This concept differs from that one introduced by Iversen (2004), which comprises non-orthogonal trajectories.

Our imaging approach consists of a trace-by-trace algorithm, wherein each finite-offset input trace is first conditioned for a zero-offset extrapolation and then migrated using an equivalent velocity model. In the data conditioning, a gather is created by time-shifting the input trace and repeating it at every isochron ray starting point. The prestack depth migration is achieved by performing the downward continuation of the conditioned data along the isochron rays, followed by the application of a proper imaging condition. This imaging procedure is the same used to migrate zero-offset data by wave-field extrapolation. In fact, we have used a zero-offset algorithm to migrate finite-offset traces, supplying as input the conditioned gather and the equivalent velocity media.

The isochron ray concept

For a single trace composed of a sequence of impulses, the image produced by depth migration contains a set of isochrons. The longer the reflection time, the greater the distance between the source-receiver pair and the isochron. The opposite is true; the shorter the reflection time, the shorter the distance. Deep isochrons are connected to long reflection times and the shallow ones are associated with short times. If the seismic velocity is constant in the vicinity of the source-receiver pair, the shallowest isochron surface collapses into a line connecting the source-receiver pair. The shallowest isochron is hypothetical because it is associated with the direct arrival traveltime instead of the reflection traveltime.

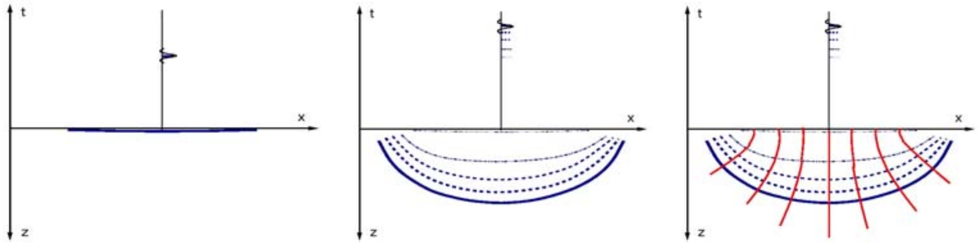


Figure 1: Propagating isochron sequence.

Imagine a movie composed of a sequence of depth migrated images, where the input data corresponds to a seismic impulse with a varying reflection time. Notice in Figure 1 that the impulse time increases by the same amount in each experiment and it is a little longer than the direct arrival traveltime of the first experiment. The initial surface (first isochron) observed in the first image moves towards the interior of the model during the sequence of experiments, changing its shape and acting as a propagating wave. If a point of the initial surface is selected and followed during the sequence, its trajectory will define a line. We refer to this line as an isochron ray because while the moving isochron plays the role of a propagating wave, it acts as a ray.

Equivalent velocity media

The propagating isochron moves through the model with a varying speed that is different from the wave velocity propagation. The isochron velocity propagation depends on the source-receiver location and the medium velocity, and it varies even in isotropic-homogeneous media.

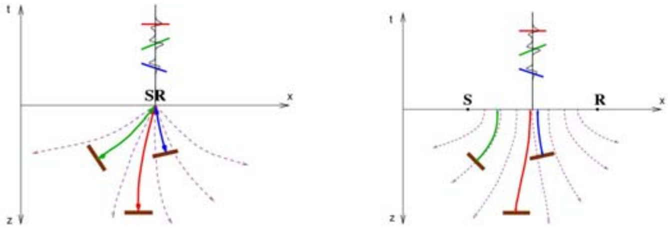
For a given source-receiver pair, we can imagine a hypothetical medium with the velocity distribution as the isochron velocity propagation. This will be referred to as the equivalent velocity medium. As the isochron velocity propagation depends on the isochron ray direction, the equivalent velocity media can assume multiple values in the presence of caustics. Another important feature of the equivalent velocity media is that they present a singularity in the line connecting the source and the receiver, which corresponds to the hypothetical starting isochron location.

The equivalent exploding reflector model

The exploding reflector model, which was introduced by Lowenthal (1976), has been widely applied in both seismic modeling and imaging algorithms. Although it is an approximation that cannot be reproduced by any experiment, it leads to simple, robust and efficient algorithms. Zero-offset seismic data can be modeled and migrated by a large number of approaches, such as Kirchhoff, finite-differences, and Gaussian beams.

All of the primary seismic reflections observed in zero-offset data have a reflection angle equal to zero, i.e., the wave achieves the reflector perpendicularly, reflects and travels back to the source-receiver position following the same trajectory which is called a normal ray. In the exploding reflector model, normal rays are traced in a half-velocity model by having the take-off direction perpendicular to the reflectors.

The isochron-rays play a role analogous to normal rays, i. e., they are perpendicular to the reflectors, and the traveltime measured along them comprises the two-way path: source-reflector-receiver. While the normal rays can be traced using the half-velocity medium, the isochron-rays need an equivalent velocity medium that depends on the source and receiver location, so we have an equivalent velocity medium for each source-receiver pair. Another important difference between normal and isochron rays is the take-off (or emergence) angle. While normal rays can assume any direction at the recording surface, the isochron rays are always perpendicular to it.



a) **b)**

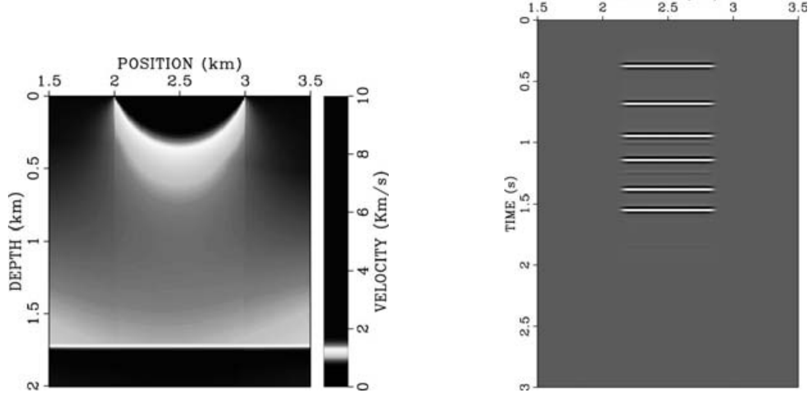
Figure 2: Exploding reflector model cartoon: a) zero-offset case, b) finite-offset case.

The application of the exploding reflector concept can be extended beyond the zero-offset case by making use of isochron rays and equivalent velocity media. We suggest the expression “equivalent exploding reflector model” to represent this extension of use. In principle, it is possible to extend the use of the whole family of modeling/imaging zero-offset algorithms to the finite-offset case.

Migration by isochron field extrapolation

In principle, all of the zero-offset migration methods based on the exploding reflector model can have their use extended to finite-offset gathers by making use of isochron rays and equivalent velocity media. In this article, only the use extension of the zero-offset wave-field extrapolation migration to common-offset gathers was addressed.

The isochron field extrapolation migration can be implemented in a trace-by-trace algorithm. The process starts with the creation of a zeroed image where the contribution of each input trace is accumulated. For each trace, the following steps are carried out: 1) computation of the equivalent velocity model, 2) creation of the conditioned data for extrapolation, 3) migration of the conditioned data by a zero-offset algorithm using the equivalent velocity model, and 4) addition of the migration result to the image. In the conditioning data step, a half-derivative and a time shift are applied to the input trace. This trace is repeated for all grid points located between the source-receiver pair, while the remaining grid positions are filled with a zeroed trace. Besides the required steps described above, a tapering should be applied to avoid the presence of artifacts at the final image.



a) b)

Figure 3: Equivalent velocity model (a) and conditioned data (b) for the central position of the seismic model described below.

Experiments

A 2D seismic model consisting of six interfaces immersed in a smooth velocity field was used to create a synthetic seismic dataset. Figure 4 is a superposition of the velocity model and the interfaces. A Kirchhoff modeling algorithm was used to create the common-offset gather presented in Figure 5a. Figure 5b is the depth image generated by the application of the isochron-field extrapolation migration algorithm described above.

Discussion

The computational cost of modeling or migrating a seismic trace using the isochron ray approach as presented above is close to the cost of modeling or migrating a zero-offset gather with the same size. Besides the high computational cost the described approach is restricted to smooth velocity models, which reduces the attractiveness of this approach. The cost can be reduced by using beams, redatuming, and limited aperture. The combination of these procedures can drastically decrease the processing time, especially for greater offsets. Problems due to triplication can be eliminated by representing the equivalent velocity in isochron ray coordinates.

The presented methodology is atractive because: it permits depth migrate common-offset gathers using wave-equation algorithms, it extends the use of robust zero-offset algorithms to the common-offset case, it produces algorithms easy to parallelize, it permits the creation of hybrid migration algorithms, and it is appropriated for migration velocity analysis.

Conclusions

We developed a method to depth migrate finite-offset seismic data by isochron ray extrapolation that was implemented as a three-step trace-by-trace algorithm. The methodology was successfully applied to depth migrate a common-offset gather using a standard zero-offset wave-field extrapolation program.

Acknowledgments

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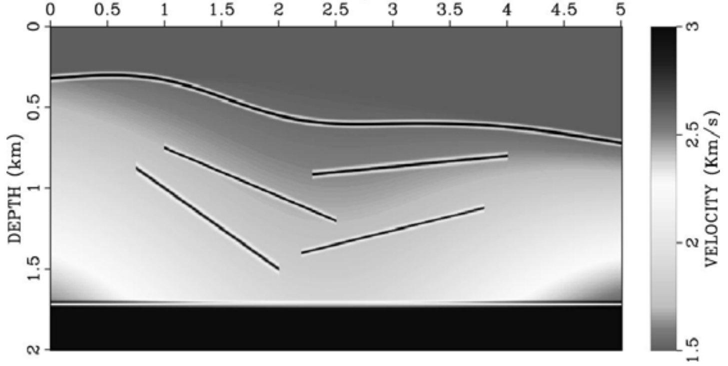
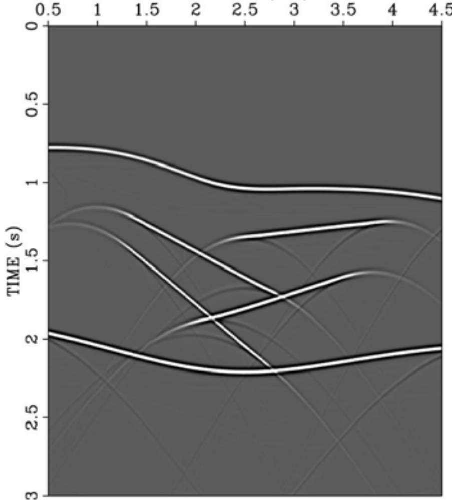


Figure 4: 2D seismic model.



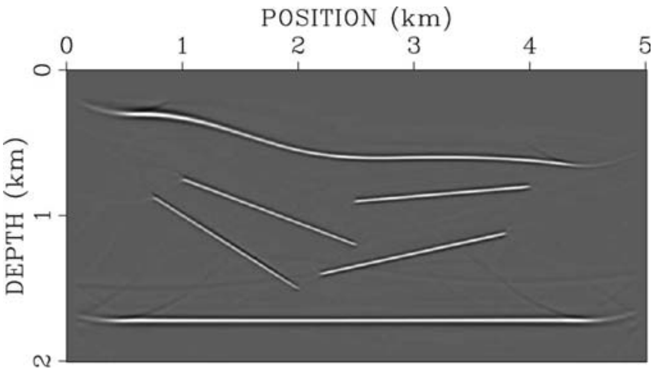


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Figure 5: a) Common-offset gather generated by Kirchhoff modeling (offset=1Km) b) Image generated by isochron-field extrapolation algorithm.